**PHY 381C (57710) Computational Physics, Fall 2019**

**Due Thursday, NOVEMEBR 7 by midnight.**

Consider a physical pendulum. Its dynamics is governed by the gravitational torque, frictional torque and external force torque. Let's assume that the external torque is sinusoidal and friction is proportional to velocity. Then the Newton's law gives us the following differential equation describing the dynamics of the pendulum:

$$\frac{d^2\theta}{dt^2} = -\omega_0^2 \sin\theta - \alpha\frac{d\theta}{dt} + f\cos\omega t$$

Here $\omega_0$ is the natural frequency of the system $\omega_0 = \frac{mgl}{I}$, where *l (mgl)* is the distance from the pivot point to the center of mass, and *I* (in the denominator) is the moment of inertia. $\beta$ and $\tau$ are the viscosity and the external force torque, respectively. The frequency of the external driving force is $\omega$. Take $\omega_0$=1, $\alpha$=0.2, $\phi$=0.52 and $\omega$=0.666.

Transform the equation into a system of two first order equations. Solve using your favorite method such as 4th order R-K.

First, explore the pendulum without friction or external forces. Try different initial conditions (including "over the top") and plot velocity as a function of position **v(t)** *vs* **x(t)** (this is known as a phase space trajectory). What can you say about the motion? Make a graph.

Now include friction. Try different initial conditions. What phase space trajectories do you see? Make a graph.

Now try no friction but small driving torque. Now try setting the driving frequency close to the natural frequency, what happens with the phase space trajectories? Make a graph.

Now include friction and driving torque, and try three different sets of the initial conditions ($x_0$,$v_0$):

(-0.0885,0.8) (-0.0883,0.8) (-0.0888,0.8)

What do you see? Describe the types of behavior you find looking at the phase space trajectories. Make a graph.

The report should have a structure of a research paper. It should have introduction, methods, results, discussion and conclusion sections. It should be typed. If you use literature, have a reference section after conclusions. If you have questions, see me or send me an e-mail. **Don't wait until the last minute.**

**Useful Hint:** Look in the second edition of "Computational Physics" by R. H. Landau, M. J. Paez and C. C. Bordianu (second edition chapter 19) or "Computational Physics" by R. H. Landau, M. J. Paez (chapter 14). Both were published by Wiley. You may find either book helpful.