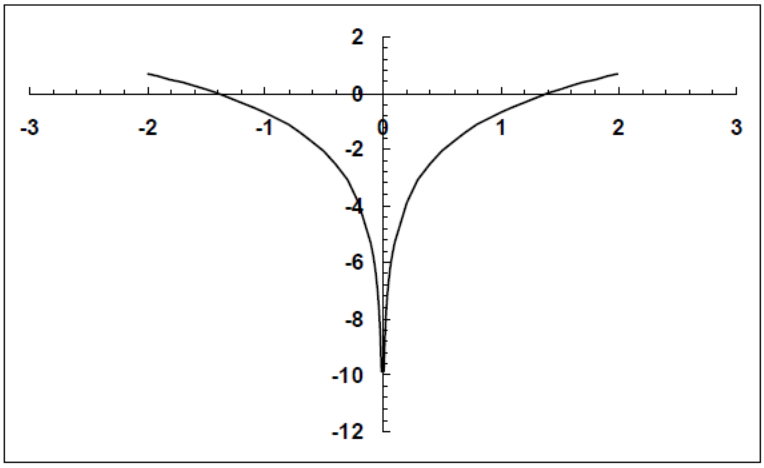
5.9 **(a)** A graph of the function indicates a positive real root at approximately $x = 1.4$.



(b) Using bisection, the first iteration is

$$x_r = \frac{0.5+2}{2} = 1.25$$
$$f(0.5)f(1.25) = -2.08629(-0.2537129) = 0.52932$$

Therefore, the root is in the second interval and the lower guess is redefined as $x_l = 1.25$. The second iteration is

$$x_r = \frac{1.25+2}{2} = 1.625$$
$$\varepsilon_a = \left|\frac{1.625-1.25}{1.625}\right| 100\% = 23.08\%$$
$$f(1.25)f(1.625) = -0.253713(0.2710156) = -0.06876$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 1.625$. The remainder of the iterations are displayed in the following table:

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$
1	0.5	−2.08629	2	0.6862944	1.25	−0.2537129	
2	1.25	−0.25371	2	0.6862944	1.625	0.2710156	23.08%
3	1.25	−0.25371	1.625	0.2710156	1.4375	0.025811	13.04%

Thus after three iterations, we obtain a root estimate of **1.4375** with an approximate error of 13.04%.

(c) Using false position, the first iteration is

$$x_r = 2 - \frac{0.6862944(0.5-2)}{-2.086294-0.6862944} = 1.628707$$
$$f(0.5)f(1.628707) = -2.086294(0.2755734) = -0.574927$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 1.628707$. The second iteration is

$$x_r = 1.628707 - \frac{0.275573(0.5-1.628707)}{-2.086294-0.275573} = 1.4970143$$
$$\varepsilon_a = \left|\frac{1.4970143-1.6287074}{1.4970143}\right| 100\% = 8.8\%$$
$$f(0.5)f(1.4970143) = -2.086294(0.1069453) = -0.223119$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 1.497014$. The remainder of the iterations are displayed in the following table:

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$
1	0.5	−2.08629	2	0.6862944	1.6287074	0.2755734	
2	0.5	−2.08629	1.628707	0.2755734	1.4970143	0.1069453	8.80%
3	0.5	−2.08629	1.497014	0.1069453	1.4483985	0.040917	3.36%

Therefore, after three iterations we obtain a root estimate of **1.4483985** with an approximate error of 3.36%.

19.22 The mass can be computed as

$$m = \int_0^r \rho(r) A_s(r)\, dr$$

The surface area of a sphere, $A_s(r) = 4\pi r^2$, can be substituted to give

$$m = \int_0^r \rho(r) 4\pi r^2\, dr$$

The average density is equal to the mass per volume, where the volume of a sphere is

$$V = \frac{4}{3}\pi R^3$$

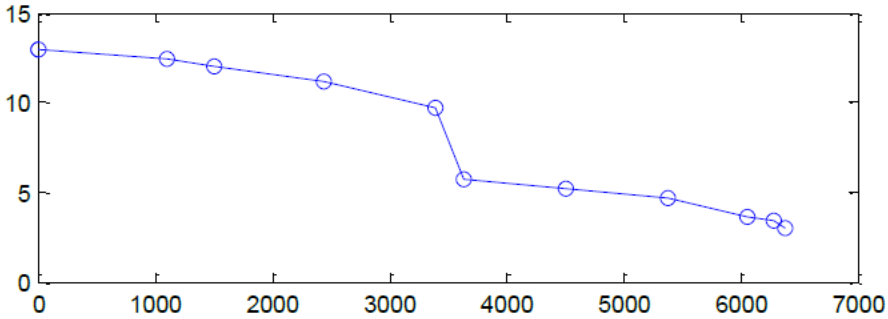
where R = the sphere's radius. For this problem, $V = 1.0878 \times 10^{21}$ m^3. The integral can be evaluated by a combination of trapezoidal and Simpson's rules as outlined in the following table. The following script can be used to implement these equations and solve this problem with MATLAB.

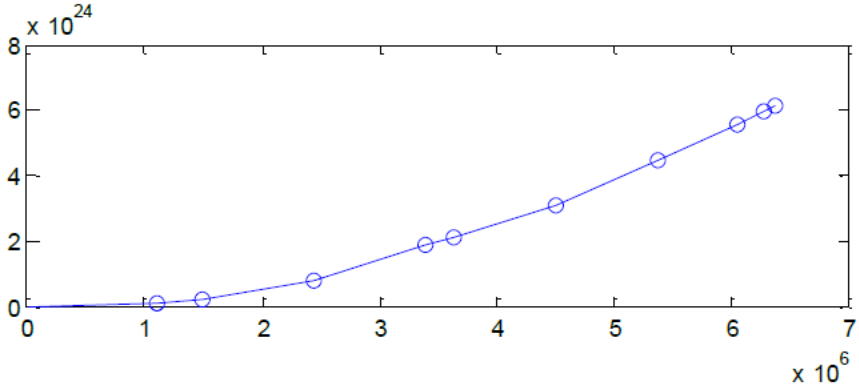
```
clear,clc,clf
format short g
r=[0 1100 1500 2450 3400 3630 4500 5380 6060 6280 6380];
rho=[13 12.4 12 11.2 9.7 5.7 5.2 4.7 3.6 3.4 3];
subplot(2,1,1)
plot(r,rho,'o-')
r=r*1e3;  %convert km to m
rho=rho*1e6/1e3;  %convert g/cm3 to kg/m3
integrand=rho*4*pi.*r.^2;

 mass=cumtrapz(r,integrand);
 EarthMass=max(mass)
 EarthVolume=4/3*pi*max(r).^3
 EarthDensity=EarthMass/EarthVolume/1e3
 subplot(2,1,2)
 plot(r,mass,'o-')
```

The results are

```
EarthMass =
  6.1087e+024
EarthVolume =
  1.0878e+021
EarthDensity =
       5.6156
```


15
10
5
0
0
1000
2000
3000
4000
5000
6000
7000


x 10^24
8
6
4
2
0
0
1
2
3
4
5
6
7
x 10^6

19.26 (a)

$$I=(8-0)\frac{0+4(18)+31}{6}+(16-8)\frac{31+4(42)+50}{6}+(28-16)\frac{50+3(56+61)+65}{8}+(30-28)\frac{65+70}{2}$$
$$=137.3333+332+699+135=1303.33\text{ m}$$

(b)

$$v=\frac{1303.33\text{ m}}{30\text{ s}}=43.4444\frac{\text{m}}{\text{s}}$$

20.29 Change of variable:

$$x=\frac{(5+1)+(5-1)x_d}{2}=3+2x_d$$

$$dx=\frac{5-1}{2}dx_d=2dx_d$$

$$\int_1^5\frac{2}{1+x^2}\,dx=\int_{-1}^1\frac{4}{1+(3+2x_d)^2}\,dx_d$$

Two-point formula:

$$I=\frac{4}{1+\ 3+2\ -1/\sqrt{3}\ ^2}+\frac{4}{1+\ 3+2\ 1/\sqrt{3}\ ^2}=0.908032+0.21904=1.127072$$

$$\text{average}=\frac{1.127072}{5-1}=0.281768$$

21.38 (a)

$$\frac{\partial f}{\partial x}=3y+3-3x^2=3(1)+3-3(1)^2=3$$
$$\frac{\partial f}{\partial y}=3x-9y^2=3(1)-9(1)=-6$$

$$\frac{\partial f}{\partial y} = 3x - 9y^2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 3$$

(b)

$$\frac{\partial f}{\partial x} = \frac{f(x+\Delta x, y) - f(x-\Delta x, y)}{2\Delta x} = \frac{2.0003 - 1.9997}{2(0.0001)} = 3$$

$$\frac{\partial f}{\partial y} = \frac{f(x, y+\Delta y) - f(x, y-\Delta y)}{2\Delta y} = \frac{1.9994 - 2.0006}{2(0.0001)} = -6$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{f(x+\Delta x, y+\Delta y) - f(x+\Delta x, y-\Delta y) - f(x-\Delta x, y+\Delta y) + f(x-\Delta x, y-\Delta y)}{4\Delta x \Delta y}$$

$$= \frac{1.9997 - 2.0009 - 1.9991 + 2.0003}{4(0.0001)(0.0001)} = 2.99999998$$

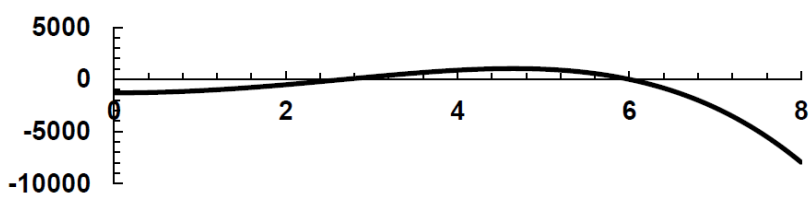
5.15 This problem can be solved by determining the root of the derivative of the elastic curve

$$\frac{dy}{dx} = 0 = \frac{w_0}{120EIL}(-5x^4 + 6L^2x^2 - L^4)$$

Therefore, after substituting the parameter values (in meter-kilogram-second units), we must determine the root of

$$f(x) = -5x^4 + 216x^2 - 1296 = 0$$

A plot of the function indicates a root at about $x = 2.6$ m.



Using initial guesses of 0 and 5 m, bisection can be used to determine the root. Here are the first few iterations:

i	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	$f(x_l)\times f(x_r)$	ε_a
1	0	5	2.5	−1296.00	−141.31	183141	
2	2.5	5	3.75	−141.31	752.73	−106370	33.33%
3	2.5	3.75	3.125	−141.31	336.54	−47557	20.00%
4	2.5	3.125	2.8125	−141.31	99.74	−14095	11.11%
5	2.5	2.8125	2.65625	−141.31	−20.89	2952	5.88%

After 22 iterations, the root is determined as $x = 2.683282852172852$ m. This value can be substituted into Eq. (P5.15) to compute the maximum deflection as –0.005151900620158 m.

21.41 The exact solution is

$$\frac{dv}{dt} = \frac{2}{\sqrt{1+t^2}} - \frac{2t^2}{(1+t^2)^{1.5}} = 0.015086$$

$h = 0.5$

$$\frac{dv}{dt} = \frac{v(t+h)-v(t-h)}{2h} = \frac{1.96774-1.952374}{1} = 0.015366 \quad \varepsilon_t = \left|\frac{0.015086-0.01536}{0.015086}\right| \times 100\% = 1.86\%$$

$h = 0.25$

$$\frac{dv}{dt} = \frac{v(t+h)-v(t-h)}{2h} = \frac{1.964677-1.9571}{0.5} = 0.015155 \quad \varepsilon_t = \left|\frac{0.015086-0.015155}{0.015086}\right| \times 100\% = 0.46\%$$

Richardson extrapolation:

$$\frac{dv}{dt} = \frac{4}{3}0.015155 - \frac{1}{3}0.015366 = 0.015085 \quad \varepsilon_t = \left|\frac{0.015086-0.015085}{0.015086}\right| \times 100\% = 0.006\%$$

6.11 **(a)** The formula for Newton-Raphson is

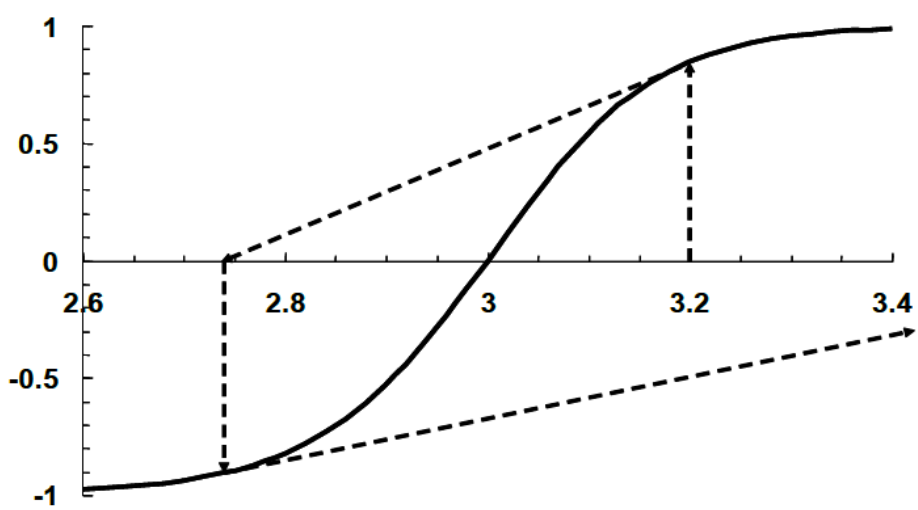
$$x_{i+1} = x_i - \frac{\tanh(x_i^2 - 9)}{2x_i \operatorname{sech}^2(x_i^2 - 9)}$$

Using an initial guess of 3.2, the iterations proceed as

iteration	x_i	$f(x_i)$	$f'(x_i)$	$\lvert\varepsilon_a\rvert$
0	3.2	0.845456	1.825311	
1	2.736816	–0.906910	0.971640	16.924%
2	3.670197	0.999738	0.003844	25.431%
3	–256.413			101.431%

Note that on the fourth iteration, the computation should go unstable.

(b) The solution diverges from its real root of $x = 3$. Due to the concavity of the slope, the next iteration will always diverge. The following graph illustrates how the divergence evolves.



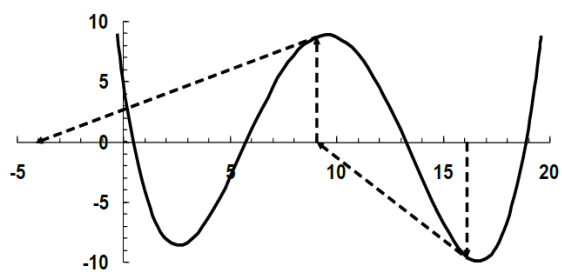
6.12 The formula for Newton-Raphson is

$$x_{i+1} = x_i - \frac{0.0074x_i^4 - 0.284x_i^3 + 3.355x_i^2 - 12.183x_i + 5}{0.0296x_i^3 - 0.852x_i^2 + 6.71x_i - 12.183}$$

Using an initial guess of 16.15, the iterations proceed as

iteration	x_i	$f(x_i)$	$f'(x_i)$	$\lvert\varepsilon_a\rvert$
0	16.15	−9.57445	−1.35368	
1	9.077102	8.678763	0.662596	77.920%
2	−4.02101	128.6318	−54.864	325.742%
3	−1.67645	36.24995	−25.966	139.852%
4	−0.2804	8.686147	−14.1321	497.887%
5	0.334244	1.292213	−10.0343	183.890%
6	0.463023	0.050416	−9.25584	27.813%
7	0.46847	8.81E−05	−9.22351	1.163%
8	0.46848	2.7E−10	−9.22345	0.002%

As depicted below, the iterations involve regions of the curve that have flat slopes. Hence, the solution is cast far from the roots in the vicinity of the original guess.



21.29 The following script solves the problem. Note that the derivative is calculated with a centered difference,

$$\frac{dV}{dT} = \frac{V_{450K} - V_{350K}}{100K}$$

The following script evaluates the derivative with centered finite differences and the integral with the `trapz` function,

```
V=[220 250 282.5;4.1 4.7 5.23;2.2 2.5 2.7;1.35 1.49 1.55;1.1 1.2
1.24;.9 .99 1.03;.68 .75 .78;.61 .675 .7;.54 .6 .62];
P=[0.1 5 10 20 25 30 40 45 50]';
T=[350 400 450]';
n=length(V);
dVdt=(V(1:n,3)-V(1:n,1))/(T(3)-T(1));
integrand=V(1:n,2)-T(2)*dVdt;
H=trapz(P,integrand)
```

When the script is run the result is $H = 21.4410$. The following table displays all the results

***P*,atm**	***T* = 350K**	***T* = 400K**	***T* = 450K**	***dVdT***	**$(V - T\,(dV/dT)p)$**	**Trap**
0.1	220	250	282.5	0.625	0	
5	4.1	4.7	5.23	0.0113	0.18	0.441
10	2.2	2.5	2.7	0.005	0.5	1.7
20	1.35	1.49	1.55	0.002	0.69	5.95
25	1.1	1.2	1.24	0.0014	0.64	3.325
30	0.9	0.99	1.03	0.0013	0.47	2.775
40	0.68	0.75	0.78	0.001	0.35	4.1
45	0.61	0.675	0.7	0.0009	0.315	1.6625
50	0.54	0.6	0.62	0.0008	0.28	1.4875
					Total Integral =	**21.441**