9.7 (a) Pivoting is necessary, so switch the first and third rows,

$$
\begin{aligned}
-8x_1 + x_2 - 2x_3 &= -20 \\
-3x_1 - x_2 + 7x_3 &= -34 \\
2x_1 - 6x_2 - x_3 &= -38
\end{aligned}
$$

Multiply the first equation by $-3/(-8)$ and subtract the result from the second equation to eliminate the a_{21} term from the second equation. Then, multiply the first equation by $2/(-8)$ and subtract the result from the third equation to eliminate the a_{31} term from the third equation.

$$
\begin{aligned}
-8x_1 \quad + x_2 \quad - 2x_3 &= -20 \\
-1.375x_2 + 7.75x_3 &= -26.5 \\
-5.75x_2 \quad -1.5x_3 &= -43
\end{aligned}
$$

Pivoting is necessary so switch the second and third row,

$$
\begin{aligned}
-8x_1 \quad + x_2 \quad - 2x_3 &= -20 \\
-5.75x_2 \quad -1.5x_3 &= -43 \\
-1.375x_2 + 7.75x_3 &= -26.5
\end{aligned}
$$

Multiply pivot row 2 by $-1.375/(-5.75)$ and subtract the result from the third row to eliminate the a_{32} term.

$$
\begin{aligned}
-8x_1 \quad + x_2 \quad\quad - 2x_3 &= -20 \\
-5.75x_2 \quad\quad -1.5x_3 &= -43 \\
8.108696x_3 &= -16.21739
\end{aligned}
$$

At this point, the determinant can be computed as

$$
D = -8 \times -5.75 \times 8.108696 \times (-1)^2 = 373
$$

The solution can then be obtained by back substitution

$$
\begin{aligned}
x_3 &= \frac{-16.21739}{8.108696} = -2 \\
x_2 &= \frac{-43 + 1.5(-2)}{-5.75} = 8 \\
x_1 &= \frac{-20 + 2(-2) - 1(8)}{-8} = 4
\end{aligned}
$$

(b) Check:

$$2(4)-6(8)-(-2)=-38$$
$$-3(4)-(8)+7(-2)=-34$$
$$-8(4)+(8)-2(-2)=-20$$

10.3 (a) The coefficient a_{21} is eliminated by multiplying row 1 by $f_{21} = -0.3$ and subtracting the result from row 2. a_{31} is eliminated by multiplying row 1 by $f_{31} = 0.1$ and subtracting the result from row 3. The factors f_{21} and f_{31} can be stored in a_{21} and a_{31}.

$$\begin{bmatrix} 10 & 2 & -1 \\ -0.3 & -5.4 & 1.7 \\ 0.1 & 0.8 & 5.1 \end{bmatrix}$$

a_{32} is eliminated by multiplying row 2 by $f_{32} = -0.14815$ and subtracting the result from row 3. The factor f_{32} can be stored in a_{32}.

$$\begin{bmatrix} 10 & 2 & -1 \\ -0.3 & -5.4 & 1.7 \\ 0.1 & -0.14815 & 5.3519 \end{bmatrix}$$

Therefore, the LU decomposition is

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ -0.3 & 1 & 0 \\ 0.1 & -0.14815 & 1 \end{bmatrix} \qquad [U] = \begin{bmatrix} 10 & 2 & -1 \\ 0 & -5.4 & 1.7 \\ 0 & 0 & 5.3519 \end{bmatrix}$$

These two matrices can be multiplied to yield the original system. For example, using MATLAB to perform the multiplication gives

```
>> L =[1 0 0;-0.3 1 0;0.1 -0.14815 1]
>> U =[10 2 -1;0 -5.4 1.7;0 0 5.3519]
>> A=L*U

A =
   10.0000    2.0000   -1.0000
   -3.0000   -6.0000    2.0000
    1.0000    1.0000    5.0000
```

10.4 (a) Forward substitution: $[L]\{D\} = \{B\}$

$$\begin{bmatrix} 1 & 0 & 0 \\ -0.3 & 1 & 0 \\ 0.1 & -0.14815 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 27 \\ -61.5 \\ -21.5 \end{Bmatrix}$$

Solving yields $d_1 = 27$, $d_2 = -53.4$, and $d_3 = -32.11111$.

Back substitution:

$$\begin{bmatrix} 10 & 2 & -1 \\ 0 & -5.4 & 1.7 \\ 0 & 0 & 5.351852 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 27 \\ -53.4 \\ -32.11111 \end{Bmatrix}$$

$$x_3 = \frac{-32.11111}{5.351852} = -6$$

$$x_2 = \frac{-53.4 - 1.7(-6)}{-5.4} = 8$$

$$x_1 = \frac{27 + 1(-6) - 2(8)}{10} = 0.5$$

(b) Forward substitution: $[L]\{D\} = \{B\}$

$$\begin{bmatrix} 1 & 0 & 0 \\ -0.3 & 1 & 0 \\ 0.1 & -0.14815 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 12 \\ 18 \\ -6 \end{Bmatrix}$$

Solving yields $d_1 = 12$, $d_2 = 21.6$, and $d_3 = -4$.

Back substitution:

$$\begin{bmatrix} 10 & 2 & -1 \\ 0 & -5.4 & 1.7 \\ 0 & 0 & 5.351852 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 12 \\ 21.6 \\ -4 \end{Bmatrix}$$

$$x_3 = \frac{-4}{5.351852} = -0.7474$$

$$x_2 = \frac{-53.4 - 1.7(-0.7474)}{-5.4} = -4.23529$$

$$x_1 = \frac{27 + 1(-0.7474) - 2(-4.23529)}{10} = 1.972318$$

10.5 The system can be written in matrix form as

$$[A] = \begin{bmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{bmatrix} \qquad \{b\} = \begin{Bmatrix} -38 \\ -34 \\ -40 \end{Bmatrix} \qquad [\mathrm{P}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Partial pivot:

$$[A] = \begin{bmatrix} -8 & 1 & -2 \\ -3 & -1 & 7 \\ 2 & -6 & -1 \end{bmatrix} \qquad [\mathrm{P}] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Compute factors:

$$f_{21} = -3/-8 = 0.375 \qquad f_{31} = 2/(-8) = -0.25$$

Forward eliminate and store factors in zeros:

$$[A] = \begin{bmatrix} -8 & 1 & -2 \\ 0.375 & -1.375 & 7.75 \\ -0.25 & -5.75 & -1.5 \end{bmatrix}$$

Pivot again

$$[A] = \begin{bmatrix} -8 & 1 & -2 \\ -0.25 & -5.75 & -1.5 \\ 0.375 & -1.375 & 7.75 \end{bmatrix} \qquad [\mathrm{P}] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Compute factors:

$$f_{32} = -1.375/(-5.75) = 0.23913$$

Forward eliminate and store factor in zero:

$$[LU] = \begin{bmatrix} -8 & 1 & -2 \\ -0.25 & -5.75 & -1.5 \\ 0.375 & 0.23913 & 8.1087 \end{bmatrix}$$

Therefore, the LU decomposition is

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ 0.375 & 0.23913 & 1 \end{bmatrix} \qquad [U] = \begin{bmatrix} -8 & 1 & -2 \\ 0 & -5.75 & -1.5 \\ 0 & 0 & 8.1087 \end{bmatrix}$$

Forward substitution. First pre-multiply right-hand side vector $\{b\}$ by $[P]$ to give

$$[P]\{b\} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} -38 \\ -34 \\ -40 \end{Bmatrix} = \begin{Bmatrix} -40 \\ -38 \\ -34 \end{Bmatrix}$$

Therefore,

$$\begin{bmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ 0.375 & 0.23913 & 1 \end{bmatrix} \{d\} = \begin{Bmatrix} -40 \\ -38 \\ -34 \end{Bmatrix}$$

which can be solved for

$$d_1 = -40$$
$$d_2 = -38 - 0.25(-40) = -48$$
$$d_3 = -34 - 0.375(-40) - 0.23913(-48) = -7.52174$$

Back substitution:

$$\begin{bmatrix} -8 & 1 & -2 \\ 0 & -5.75 & -1.5 \\ 0 & 0 & 8.1087 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -40 \\ -48 \\ -7.52174 \end{Bmatrix}$$

$$x_3 = \frac{-7.52174}{8.1087} = -0.92761$$
$$x_2 = \frac{-48 + 1.5(-0.92761)}{-5.75} = 8.589812$$
$$x_1 = \frac{-40 + 2(-0.92761) - 1(8.589812)}{8} = 6.30563$$

11.5 The problem can be written in matrix form as

$$\begin{bmatrix} 0.866 & 0 & -0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.866 & 0 & 0 & 0 \\ -0.866 & -1 & 0 & -1 & 0 & 0 \\ -0.5 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & -0.866 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ H_2 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} F_{1,h} \\ F_{1,v} \\ F_{2,h} \\ F_{2,v} \\ F_{3,h} \\ F_{3,v} \end{Bmatrix}$$

MATLAB can then be used to solve for the matrix inverse,

```
>> A = [0.866 0 -0.5 0 0 0;
0.5 0 0.866 0 0 0;
-0.866 -1 0 -1 0 0;
-0.5 0 0 0 -1 0;
0 1 0.5 0 0 0;
0 0 -0.866 0 0 -1];
>> AI = inv(A)

AI =
    0.8660    0.5000         0         0         0         0
    0.2500   -0.4330         0         0    1.0000         0
   -0.5000    0.8660         0         0         0         0
   -1.0000    0.0000   -1.0000         0   -1.0000         0
   -0.4330   -0.2500         0   -1.0000         0         0
    0.4330   -0.7500         0         0         0   -1.0000
```

The forces in the members resulting from the two forces can be computed using the elements of the matrix inverse as in,

$$F_1 = a_{12}^{-1} F_{1,v} + a_{15}^{-1} F_{3,h} = 0.5(-2000) + 0(-500) = -1000 + 0 = -1000$$

$$F_2 = a_{22}^{-1} F_{1,v} + a_{25}^{-1} F_{3,h} = -0.433(-2000) + 1(-500) = 866 - 500 = 366$$

$$F_3 = a_{32}^{-1} F_{1,v} + a_{35}^{-1} F_{3,h} = 0.866(-2000) + 0(-500) = -1732 + 0 = -1732$$

12.3 The first iteration can be implemented as

$$x_1 = \frac{27 - 2x_2 + x_3}{10} = \frac{27 - 2(0) + 0}{10} = 2.7$$
$$x_2 = \frac{-61.5 + 3x_1 - 2x_3}{-6} = \frac{-61.5 + 3(2.7) - 2(0)}{-6} = 8.9$$
$$x_3 = \frac{-21.5 - x_1 - x_2}{5} = \frac{-21.5 - (2.7) - 8.9}{5} = -6.62$$

Second iteration:

$$x_1 = \frac{27 - 2(8.9) - 6.62}{10} = 0.258$$
$$x_2 = \frac{-61.5 + 3(0.258) - 2(-6.62)}{-6} = 7.914333$$
$$x_3 = \frac{-21.5 - (0.258) - 7.914333}{5} = -5.934467$$

The error estimates can be computed as

$$\varepsilon_{a,1} = \left| \frac{0.258 - 2.7}{0.258} \right| \times 100\% = 947\%$$
$$\varepsilon_{a,2} = \left| \frac{7.914333 - 8.9}{7.914333} \right| \times 100\% = 12.45\%$$
$$\varepsilon_{a,3} = \left| \frac{-5.934467 - (-6.62)}{-5.934467} \right| \times 100\% = 11.55\%$$

The remainder of the calculation proceeds until all the errors fall below the stopping criterion of 5%. The entire computation can be summarized as

iteration	unknown	value	ε_a	maximum ε_a
1	x_1	2.7	100.00%	
	x_2	8.9	100.00%	
	x_3	−6.62	100.00%	100%
2	x_1	0.258	946.51%	
	x_2	7.914333	12.45%	
	x_3	−5.93447	11.55%	947%
3	x_1	0.523687	50.73%	

	x_2	8.010001	1.19%	
	x_3	−6.00674	1.20%	50.73%
4	x_1	0.497326	5.30%	
	x_2	7.999091	0.14%	
	x_3	−5.99928	0.12%	5.30%
5	x_1	0.500253	0.59%	
	x_2	8.000112	0.01%	
	x_3	−6.00007	0.01%	0.59%

Thus, after 5 iterations, the maximum error is 0.59% and we arrive at the result: $x_1 = 0.500253$, $x_2 =$ 8.000112 and $x_3 = -6.00007$.

13.5 Here is a MATLAB session that uses `eig` to determine the eigenvalues and the natural frequencies:

```
>> k=2;
>> kmw2=[2*k,-k,-k;-k,2*k,-k;-k,-k,2*k];
>> [v,d]=eig(kmw2)

v =
    0.5774    0.7634    0.2895
    0.5774   -0.6325    0.5164
    0.5774   -0.1310   -0.8059
d =
    0.0000         0         0
         0    6.0000         0
         0         0    6.0000
```

Therefore, the eigenvalues are 0, 6, and 6. Setting these eigenvalues equal to $m\omega^2$, the three frequencies can be obtained.

$m\omega_1^{\ 2} = 0 \Rightarrow \omega_1 = 0$ (Hz) 1st mode of oscillation

$m\omega_2^{\ 2} = 6 \Rightarrow \omega_2 = \sqrt{6}$ (Hz) 2nd mode

$m\omega_3^{\ 2} = 6 \Rightarrow \omega_3 = \sqrt{6}$ (Hz) 3rd mode

13.6 The solution along with its second derivative can be substituted into the simultaneous ODEs. After simplification, the result is

$$\begin{aligned}
&\left(\frac{1}{C_1}-L_1\omega^2\right)I_1 && -\frac{1}{C_1}I_2 && && =0\\
&-\frac{1}{C_1}I_1 && \left(\frac{1}{C_1}+\frac{1}{C_2}-L_2\omega^2\right)I_2 && -\frac{1}{C_2}I_3 && =0\\
& && -\frac{1}{C_2}I_2 && \left(\frac{1}{C_2}+\frac{1}{C_3}-L_3\omega^2\right)I_3 && =0
\end{aligned}$$

Thus, we have formulated an eigenvalue problem. Further simplification results for the special case where the *C*'s and *L*'s are constant. For this situation, the system can be expressed in matrix form as

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} \begin{Bmatrix} I_1 \\ I_2 \\ I_3 \end{Bmatrix} = \{0\} \tag{1}$$

where $\lambda = LC\omega^2$. MATLAB can be employed to determine values for the eigenvalues and eigenvectors :

```
>> a=[1 -1 0; -1 2 -1; 0 -1 2];
>> [v,d]=eig(a)

v =
   -0.7370   -0.5910    0.3280
   -0.5910    0.3280   -0.7370
   -0.3280    0.7370    0.5910
d =

    0.1981         0         0
         0    1.5550         0
         0         0    3.2470
```

The matrix v consists of the system's three eigenvectors (arranged as columns), and d is a matrix with the corresponding eigenvalues on the diagonal. Thus, the package computes that the eigenvalues are $\lambda =$ 0.1981, 1.555, and 3.247. These values in turn can be used to compute the natural circular frequencies of the system

$$\omega = \begin{cases} 0.4450/\sqrt{LC} \\ 1.2470/\sqrt{LC} \\ 1.8019/\sqrt{LC} \end{cases}$$

Aside from providing the natural frequencies, the eigenvalues can be substituted into Eq. 1 to gain further insight into the circuit's physical behavior. For example, substituting $\lambda =$ 0.1981 yields

$$\begin{bmatrix} 0.8019 & -1 & 0 \\ -1 & 1.8019 & -1 \\ 0 & -1 & 1.8019 \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ i_3 \end{Bmatrix} = \{0\}$$

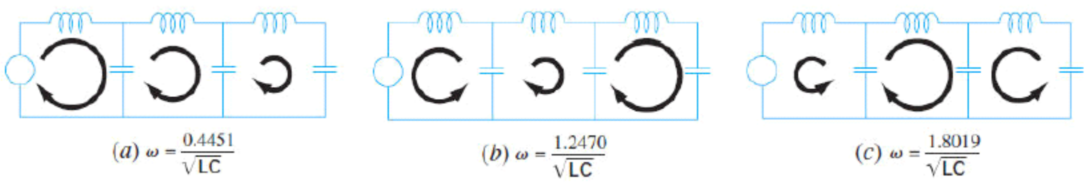
Although this system does not have a unique solution, it will be satisfied if the currents are in fixed ratios, as in

$$0.8019 i_1 = i_2 = 1.8019 i_3 \tag{2}$$

Thus, as depicted in (a) in the figure below, they oscillate in the same direction with different magnitudes. Observe that if we assume that $i_1 = 0.737$, we can use Eq. 2 to compute the other currents with the result

$$\{i\} = \begin{Bmatrix} 0.737 \\ 0.591 \\ 0.328 \end{Bmatrix}$$

which is the first column of the v matrix calculated with MATLAB.



In a similar fashion, the second eigenvalue of $\lambda = 1.555$ can be substituted and the result evaluated to yield

$$-1.8018 i_1 = i_2 = 2.247 i_3$$

As depicted in the above figure (b), the first loop oscillates in the opposite direction from the second and third. Finally, the third mode can be determined as

$$-0.445 i_1 = i_2 = -0.8718 i_3$$

Consequently, as in the above figure (c), the first and third loops oscillate in the opposite direction from the second.