PHY 381, Homework #2, Due October 1, 2019

5.9 Determine the positive real root of $\ln(x^2) = 0.7$ **(a)** graphically, **(b)** using three iterations of the bisection method, with initial guesses of $x_l = 0.5$ and $x_u = 2$, and **(c)** using three iterations of the false-position method, with the same initial guesses as in **(b)**.

19.22 As specified in the following table, the earth's density varies as a function of the distance from its center ($r = 0$):

r, km	0	1100	1500	2450	3400	3630
ρ(g/cm^3)	13	12.4	12	11.2	9.7	5.7
r, km	4500	5380	6060	6280	6380	
ρ(g/cm^3)	5.2	4.7	3.6	3.4	3	

Use numerical integration to estimate the earth's mass (in metric tonnes) and average density (in g/cm^3). Develop vertically stacked subplots of (top) density versus radius, and (bottom) mass versus radius. Assume that the earth is a perfect sphere.

19.26 The following data is provided for the velocity of an object as a function of time:

t, s	0	4	8	12	16	20	24	28	30
v, m/s	0	18	31	42	50	56	61	65	70

(a) Limiting yourself to trapezoidal rule and Simpson's 1/3 and 3/8 rules, make the best estimate of how far the object travels from $t = 0$ to 30 s?
(b) Employ the results of **(a)** to compute the average velocity.

20.29 Use the two-point Gauss quadrature approach to estimate the average value of the following function between $a = 1$ and $b = 5$

$$f(x) = \frac{2}{1 + x^2}$$

21.38 Evaluate $\partial f/\partial x$, $\partial f/\partial y$ and $\partial^2 f/(\partial x \partial y)$ for the following function at $x = y = 1$ (a) analytically and (b) numerically $\Delta x = \Delta y = 0.0001$:

$$f(x, y) = 3xy + 3x - x^3 - 3y^3$$

5.15 Figure P5.15*a* shows a uniform beam subject to a linearly increasing distributed load. The equation for the resulting elastic curve is (see Fig. P5.15*b*)

$$y = \frac{w_0}{120EIL}(-x^5 + 2L^2x^3 - L^4x) \qquad \text{(P5.15)}$$

Use bisection to determine the point of maximum deflection (i.e., the value of x where $dy/dx = 0$). Then substitute this value into Eq. (P5.15) to determine the value of the maximum deflection. Use the following parameter values in your computation: $L = 600$ cm, $E = 50{,}000$ kN/cm^2, $I = 30{,}000$ cm^4, and $w_0 = 2.5$ kN/cm.

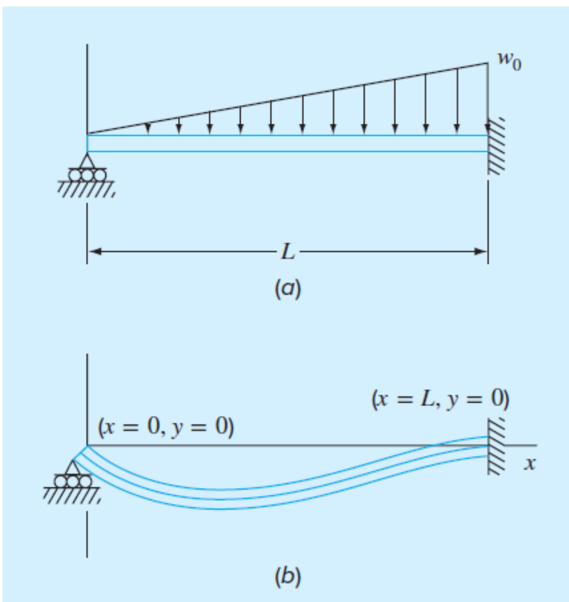

FIGURE P5.15

21.41 The velocity (m/s) of an object at time t seconds is given by

$$v = \frac{2t}{\sqrt{1+t^2}}$$

Using Richardson's extrapolation, find the acceleration of the particle at time $t = 5$ s using $h = 0.5$ and 0.25. Employ the exact solution to compute the true percent relative error of each estimate.

6.11 **(a)** Apply the Newton-Raphson method to the function $f(x) = \tanh(x^2 - 9)$ to evaluate its known real root at $x = 3$. Use an initial guess of $x_0 = 3.2$ and take a minimum of three iterations. **(b)** Did the method exhibit convergence onto its real root? Sketch the plot with the results for each iteration labeled.

6.12 The polynomial $f(x) = 0.0074x^4 - 0.284x^3 + 3.355x^2 - 12.183x + 5$ has a real root between 15 and 20. Apply the Newton-Raphson method to this function using an initial guess of $x_0 = 16.15$. Explain your results.

21.29 The enthalpy of a real gas is a function of pressure as described below. The data were taken for a real fluid. Estimate the enthalpy of the fluid at 400 K and 50 atm (evaluate the integral from 0.1 to 50 atm).

$$H = \int_0^P \left(V - T\left(\frac{\partial V}{\partial T}\right)_P\right) dP$$

	V, L		
P, atm	*T* = 350 K	*T* = 400 K	*T* = 450 K
0.1	220	250	282.5
5	4.1	4.7	5.23
10	2.2	2.5	2.7
20	1.35	1.49	1.55
25	1.1	1.2	1.24
30	0.90	0.99	1.03
40	0.68	0.75	0.78
45	0.61	0.675	0.7
50	0.54	0.6	0.62