PHY 381, Homework #3, Due October 22, 2019

9.7 Given the equations

$$\begin{aligned} 2x_1 - 6x_2 - x_3 &= -38 \\ -3x_1 - x_2 + 7x_3 &= -34 \\ -8x_1 + x_2 - 2x_3 &= -20 \end{aligned}$$

(a) Solve by Gauss elimination with partial pivoting. As part of the computation, use the diagonal elements to calculate the determinant. Show all steps of the computation.
(b) Substitute your results into the original equations to check your answers.

10.3 Use naive Gauss elimination to factor the following system according to the description in Sec. 10.2:

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27 \\ -3x_1 - 6x_2 + 2x_3 &= -61.5 \\ x_1 + x_2 + 5x_3 &= -21.5 \end{aligned}$$

Then, multiply the resulting $[L]$ and $[U]$ matrices to determine that $[A]$ is produced.

10.4 **(a)** Use LU factorization to solve the system of equations in Prob. 10.3. Show all the steps in the computation. **(b)** Also solve the system for an alternative right-hand-side vector

$$\{b\}^T = \lfloor 12 \quad 18 \quad -6 \rfloor$$

10.5 Solve the following system of equations using LU factorization with partial pivoting:

$$\begin{aligned} 2x_1 - 6x_2 - x_3 &= -38 \\ -3x_1 - x_2 + 7x_3 &= -34 \\ -8x_1 + x_2 - 2x_3 &= -40 \end{aligned}$$

11.15 A chemical constituent flows between three reactors as depicted in Fig. P11.15. Steady-state mass balances can be written for a substance that reacts with first-order kinetics. For example, the mass balance for reactor 1 is

$$Q_{1,\text{in}}c_{1,\text{in}} - Q_{1,2}c_1 - Q_{1,3}c_1 + Q_{2,1}c_2 - kV_1c_1 = 0 \qquad \text{(P11.15)}$$

where $Q_{1,\text{in}}$ = the volumetric inflow to reactor 1 (m^3/min), $c_{1,\text{in}}$ = the inflow concentration to reactor 1 (g/m^3), $Q_{i,j}$ = the flow from reactor i to reactor j (m^3/min), c_i = the concentration of reactor i (g/m^3), k = a first-order decay rate (/min), and V_i = the volume of reactor i (m^3).
(a) Write the mass balances for reactors 2 and 3.

(b) If $k = 0.1$/min, write the mass balances for all three reactors as a system of linear algebraic equations.
(c) Compute the *LU* decomposition for this system.
(d) Use the *LU* decomposition to compute the matrix inverse.
(e) Use the matrix inverse to answer the following questions: (*i*) What are the steady-state concentrations for the three reactors? (*ii*) If the inflow concentration to the second reactor is set to zero, what is the resulting reduction in concentration of reactor 1? (*iii*) If the inflow concentration to reactor 1 is doubled, and the inflow concentration to reactor 2 is halved, what is the concentration of reactor 3?

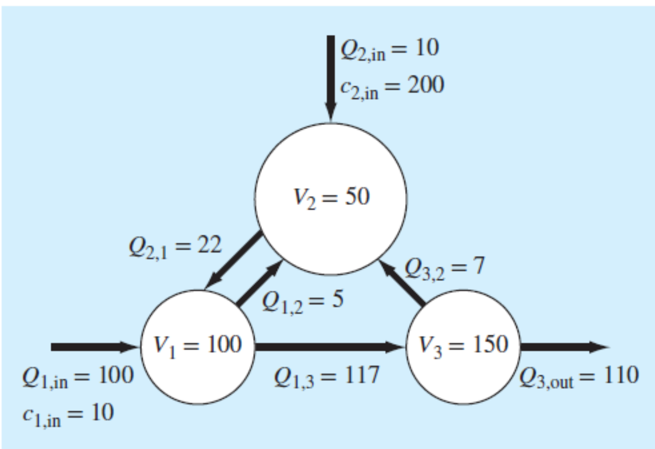

FIGURE P11.15

12.3 Use the Gauss-Seidel method to solve the following system until the percent relative error falls below $\varepsilon_s = 5\%$:

$$
\begin{aligned}
10x_1 + 2x_2 - x_3 &= 27 \\
-3x_1 - 6x_2 + 2x_3 &= -61.5 \\
x_1 + x_2 + 5x_3 &= -21.5
\end{aligned}
$$

13.5 Consider the mass-spring system in Fig. P13.5. The frequencies for the mass vibrations can be determined by solving for the eigenvalues and by applying $M\ddot{x} + kx = 0$, which yields

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{Bmatrix} 2k & -k & -k \\ -k & 2k & -k \\ -k & -k & 2k \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Applying the guess $x = x_0 e^{i\omega t}$ as a solution, we get the following matrix:

$$\begin{bmatrix} 2k - m_1\omega^2 & -k & -k \\ -k & 2k - m_2\omega^2 & -k \\ -k & -k & 2k - m_3\omega^2 \end{bmatrix} \begin{Bmatrix} x_{01} \\ x_{02} \\ x_{03} \end{Bmatrix} e^{i\omega t} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

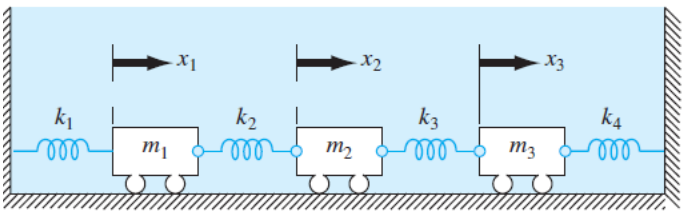

FIGURE P13.4

Use MATLAB's `eig` command to solve for the eigenvalues of the $k - m\omega^2$ matrix above. Then use these eigenvalues to solve for the frequencies (ω). Let $m_1 = m_2 = m_3 = 1$ kg, and $k = 2$ N/m.

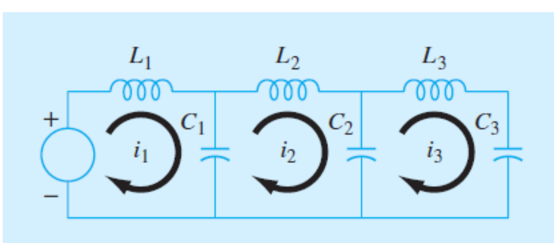

FIGURE P13.6

13.6 As displayed in Fig. P13.6, an *LC* circuit can be modeled by the following system of differential equations:

$$L_1 \frac{d^2 i_1}{dt^2} + \frac{1}{C_1}(i_1 - i_2) = 0$$

$$L_2 \frac{d^2 i_2}{dt^2} + \frac{1}{C_2}(i_2 - i_3) - \frac{1}{C_1}(i_1 - i_2) = 0$$

$$L_3 \frac{d^2 i_3}{dt^2} + \frac{1}{C_3} i_3 - \frac{1}{C_2}(i_2 - i_3) = 0$$

where L = inductance (H), t = time (s), i = current (A), and C = capacitance (F). Assuming that a solution is of the form $i_j = I_j \sin(\omega t)$, determine the eigenvalues and eigenvectors for this system with $L = 1$ H and $C = 0.25$C. Draw the network, illustrating how the currents oscillate in their primary modes.