PHY 381C, Homework #6, Due December 2, 2019

23.23 As depicted in Fig. P23.23, a double pendulum consists of a pendulum attached to another pendulum. We indicate the upper and lower pendulums by subscripts 1 and 2, respectively, and we place the origin at the pivot point of the upper pendulum with y increasing upward. We further assume that the system oscillates in a vertical plane subject to gravity, that the pendulum rods are massless and rigid, and the pendulum masses are considered to be point masses. Under these assumptions, force balances can be used to derive the following equations of motion

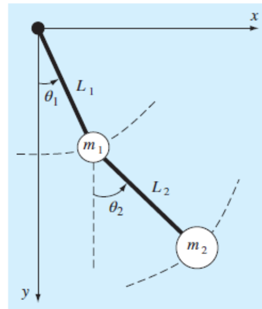

FIGURE P23.23
A double pendulum.

$$\frac{d^2\theta_1}{dt^2} = \frac{-g(2m_1+m_2)\sin\theta_1 - m_2 g\sin(\theta_1 - 2\theta_2) - 2\sin(\theta_1-\theta_2)m_2\left((d\theta_2/dt)^2 L_2 + (d\theta_1/dt)^2 L_1\cos(\theta_1-\theta_2)\right)}{L_1(2m_1+m_2-m_2\cos(2\theta_1-\theta_2))}$$

$$\frac{d^2\theta_2}{dt^2} = \frac{2\sin(\theta_1-\theta_2)\left((d\theta_1/dt)^2 L_1(m_1+m_2) + g(m_1+m_2)\cos(\theta_1) + (d\theta_2/dt)^2 L_2 m_2\cos(\theta_1-\theta_2)\right)}{L_2(2m_1+m_2-m_2\cos(2\theta_1-\theta_2))}$$

where the subscripts 1 and 2 designate the top and bottom pendulum, respectively, θ = angle (radians) with 0 = vertical downward and counter-clockwise positive, t = time (s), g = gravitational acceleration (= 9.81 m/s^2), m = mass (kg), and L = length (m). Note that the x and y coordinates of the masses are functions of the angles as in

$$x_1 = L_1\sin\theta_1 \qquad y_1 = -L_1\cos\theta_1$$

$$x_2 = x_1 + L_2\sin\theta_2 \qquad y_2 = y_1 - L_2\cos\theta_2$$

(a) Use `ode45` to solve for the angles and angular velocities of the masses as a function of time from $t = 0$ to 40 s. Employ `subplot` to create a stacked plot with a time series of the angles in the top panel and a state space plot of θ_2 versus θ_1 in the bottom panel. **(b)** Create an animated plot depicting the motion of the pendulum. Test your code for the following:

Case 1 (small displacement): $L_1 = L_2 = 1$ m, $m_1 = m_2 = 0.25$ kg, with initial conditions: $\theta_1 = 0.5$ m and $\theta_2 = d\theta_1/dt = d\theta_2/dt = 0$.
Case 2 (large displacement): $L_1 = L_2 = 1$ m, $m_1 = 0.5$ kg, $m_2 = 0.25$ kg, with initial conditions: $\theta_1 = 1$ m and $\theta_2 = d\theta_1/dt = d\theta_2/dt = 0$.

You do not have to use ode45, if you have a good solver of your own!

24.1 A steady-state heat balance for a rod can be represented as

$$\frac{d^2T}{dx^2} - 0.15T = 0$$

Obtain a solution for a 10-m rod with $T(0) = 240$ and $T(10) = 150$ **(a)** analytically, **(b)** with the shooting method, and **(c)** using the finite-difference approach with $\Delta x = 1$.

24.14 A biofilm with a thickness L_f (cm), grows on the surface of a solid (Fig. P24.14). After traversing a diffusion layer of thickness L (cm), a chemical compound A diffuses into the biofilm where it is subject to an irreversible first-order reaction that converts it to a product B.

Steady-state mass balances can be used to derive the following ordinary differential equations for compound A:

$$D\frac{d^2c_a}{dx^2} = 0 \qquad 0 \le x < L$$

$$D_f\frac{d^2c_a}{dx^2} - kc_a = 0 \qquad L \le x < L + L_f$$

where D = the diffusion coefficient in the diffusion layer = 0.8 cm²/d, D_f = the diffusion coefficient in the biofilm = 0.64 cm²/d, and k = the first-order rate for the conversion of A to B = 0.1/d. The following boundary conditions hold:

$$c_a = c_{a0} \qquad \text{at } x = 0$$

$$\frac{dc_a}{dx} = 0 \qquad \text{at } x = L + L_f$$

where c_{a0} = the concentration of A in the bulk liquid = 100 mol/L. Use the finite-difference method to compute the steady-state distribution of A from $x = 0$ to $L + L_f$, where $L = 0.008$ cm and $L_f = 0.004$ cm. Employ centered finite differences with $\Delta x = 0.001$ cm.

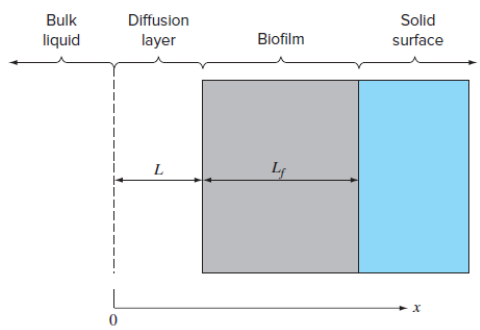

FIGURE P24.14
A biofilm growing on a solid surface.

24.8 An insulated heated rod with a uniform heat source can be modeled with the *Poisson equation:*

$$\frac{d^2T}{dx^2} = -f(x)$$

Given a heat source $f(x) = 25$ °C/m² and the boundary conditions $T(x = 0) = 40$ °C and $T(x = 10) = 200$ °C, solve for the temperature distribution with **(a)** the shooting method and **(b)** the finite-difference method ($\Delta x = 2$).

24.20 Just as Fourier's law and the heat balance can be employed to characterize temperature distribution, analogous relationships are available to model field problems in other areas of engineering. For example, electrical engineers use a similar approach when modeling electrostatic fields. Under a number of simplifying assumptions, an analog of Fourier's law can be represented in one-dimensional form as

$$D = -\varepsilon \frac{dV}{dx}$$

where D is called the electric flux density vector, ε = permittivity of the material, and V = electrostatic potential. Similarly, a Poisson equation (see Prob. 24.8) for electrostatic fields can be represented in one dimension as

$$\frac{d^2V}{dx^2} = -\frac{\rho_v}{\varepsilon}$$

where ρ_v = charge density. Use the finite-difference technique with $\Delta x = 2$ to determine V for a wire where $V(0) = 1000$, $V(20) = 0$, $\varepsilon = 2$, $L = 20$, and $\rho_v = 30$.