PHY 392K (551900) Solid State Physics I, spring 2020

Homework #5

Due April 14, 2020

Problem 1

Assume the central force approximation, and show that the change in the interatomic distance is equal to:

$\delta_{i,j} = \frac{1}{\left|R^0_{i,j}\right|} \vec{R}^0_{i,j} \cdot \left(\vec{u}_i - \vec{u}_j\right)$, where $\boldsymbol{R^0_{i,j}}$ is the equilibrium distance between the atoms $\boldsymbol{i}$ and $\boldsymbol{j}$, and $\boldsymbol{u}$'s

Problem 2

Starting with a classical normal coordinate:

$Q\binom{q}{j} = \frac{1}{2}\left[A\binom{q}{j} e^{-i\omega_j(q)t} + A^*\binom{-q}{j} e^{i\omega_j(q)t}\right]$,

and a conjugate "momentum," define the creation and annihilation operators $\boldsymbol{a}^+\binom{-q}{j}$ and $\boldsymbol{a}\binom{q}{j}$.

Show that to have proper commutation relations, you should have the following operator:

$\mathbf{Q}\binom{q}{j} = i\left(\frac{\hbar}{2\omega_j(q)}\right)^{1/2}\left[\boldsymbol{a}\binom{q}{j} + \boldsymbol{a}^+\binom{-q}{j}\right]$.

Calculate the matrix element of this operator between states with n phonons of type s=(q,j) and n phonons of type s'=(q',j').

Problem 3

Show that the lattice specific heat can be found if you know the phonon density of states as:

$$c_v = V\int_o^\infty d\omega\, D(\omega)\frac{\partial}{\partial T}\frac{\hbar\omega}{e^{\beta\hbar\omega} - 1}$$

Problem 4

Assume the crystal is made of N identical oscillators, find the specific heat using the result of problem 3.