On "Geometrical discretizations in hydrodynamics: from plasma physics to thermal quasi-geostrophy"

by Michael Roop

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Intersections

Mathematical Physics: $\{Mathematics\} \cap \{Physics\}$

Plasma Physics: Physics of hot ionized gasses, with electric and magnetic fields

Geophysical Fluid Dynamics: Physics of oceans and atmospheres with planetary rotation

Computational Physics: In particular *Structure Preserving Algorithms*

Michael Roop's Thesis: $\{Mathematical\ Physics\} \cap \{Plasma\ Physics\} \cap \{GFD\} \cap \{Comp\ Phys\}$

What is a Plasma?

Adding Heat:

Solid $\leadsto$ Liquid $\leadsto$ Gas $\leadsto$ Plasma

97% of visible universe is plasma

Dynamics of Charged Particles $\Leftrightarrow$ Electromagnetic Fields

- Hannes Alfvén 1940-50s ... Nobel Prize in Physics (1970) $\rightarrow$ major effort for fundamental and fusion research

- $\Rightarrow$ Complicated partial differential equations. Magnetofluid models, geometry

What is GFD

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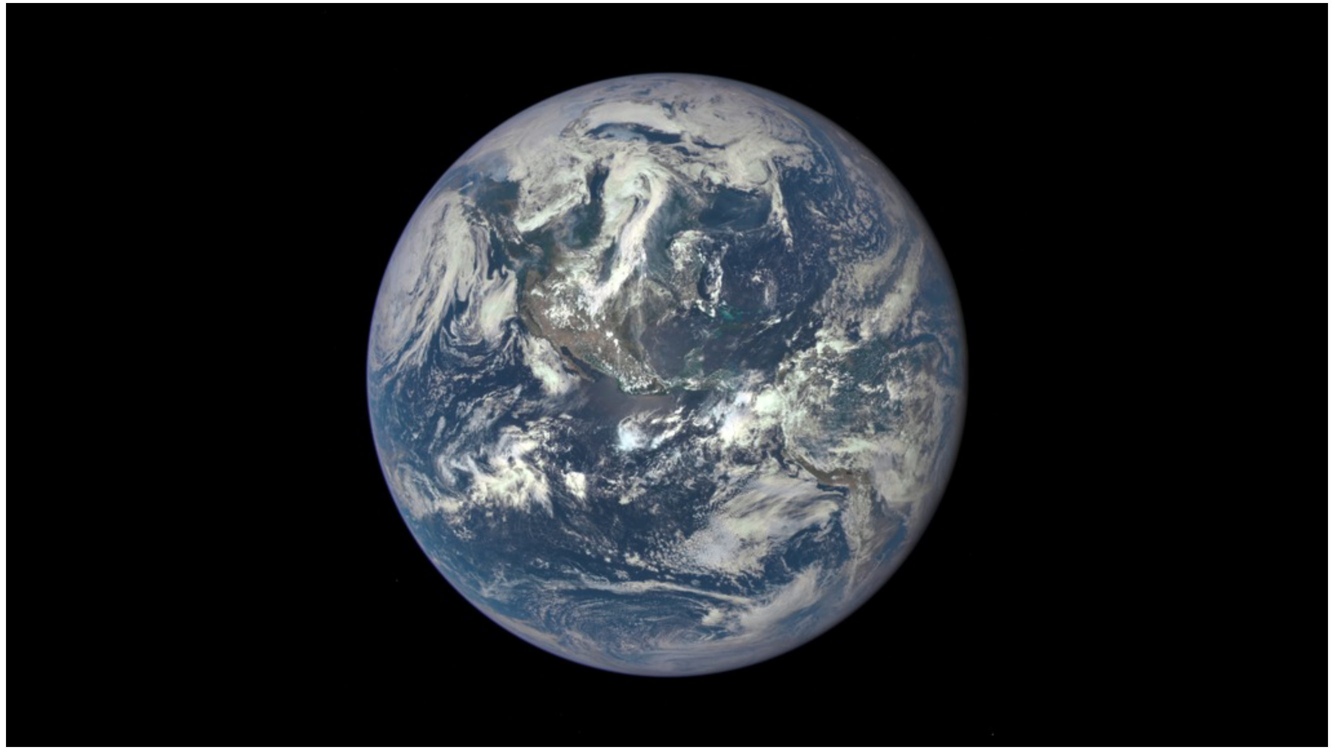

- Major effort for basic climate science
- $\Rightarrow$ Complicated partial differential equations. Coriolis force, stratification, geometry, ...

Complicated PDEs such as extended MHD

Continuity equation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{V}) \, ,$$

Momentum Equation;

$$\rho \left(\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V} \cdot \nabla) \boldsymbol{V} \right) = -\nabla p + \boldsymbol{J} \times \boldsymbol{B} - \frac{m_e}{e} (\boldsymbol{J} \cdot \nabla) \frac{\boldsymbol{J}}{en} \, ,$$

Faraday's Law:

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

Ohm's Law

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \frac{\boldsymbol{J}}{\sigma} + \frac{1}{en} (\boldsymbol{J} \times \boldsymbol{B} - \nabla p_e) + \frac{m_e}{e^2 n} \left[\frac{\partial \boldsymbol{J}}{\partial t} + \nabla \cdot (\boldsymbol{V}\boldsymbol{J} + \boldsymbol{J}\boldsymbol{V}) \right] - \frac{m_e}{e^2 n} (\boldsymbol{J} \cdot \nabla) \frac{\boldsymbol{J}}{en}$$

⇒ Model reductions based on asymptotics, small parameter expansions, 1960s – 1980s, such as reduced MHD, quasigeostrophy, various reduced fluid models.

Noncanonical (Lie-Poisson) Hamiltonian Structure and Preservation

All nondissipative equations have (or should have) Hamiltonian structure:

$$\frac{\partial \Psi}{\partial t} = \{\Psi, H\} \qquad \leftarrow \text{Poisson Bracket}$$

Model Reductions:

Hamiltonian parent $\rightarrow$ Hamiltonian reduced model

Computation:

PDEs $\rightarrow$ ODEs $\rightarrow$ Matrices