# TOPOLOGY OF BUNDLE DIVERTORS

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# BUNDLE DIVERTOR TOPOLOGY

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## USEFUL PAPERS

STOTT, WILSON & GIBSON, NUC. FUS., 17, 481, (1977).

COWLEY, RADIO SCIENCE, 8, 903, (1973).

## OVERVIEW

(1) GLOBAL DESCRIPTION OF FIELD LINES

(2) BEHAVIOR NEAR NULL

(3) B-D SYMMETRY & CONSEQUENCES

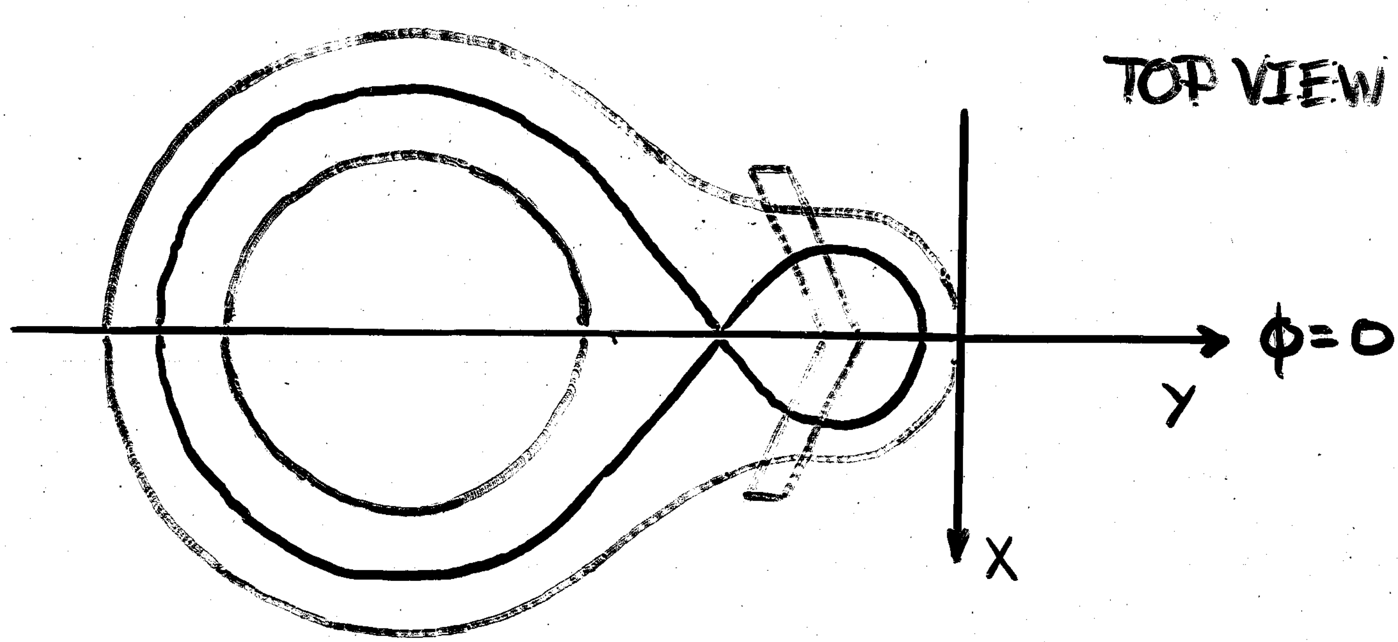
(4) ⊥ WIRE MODEL

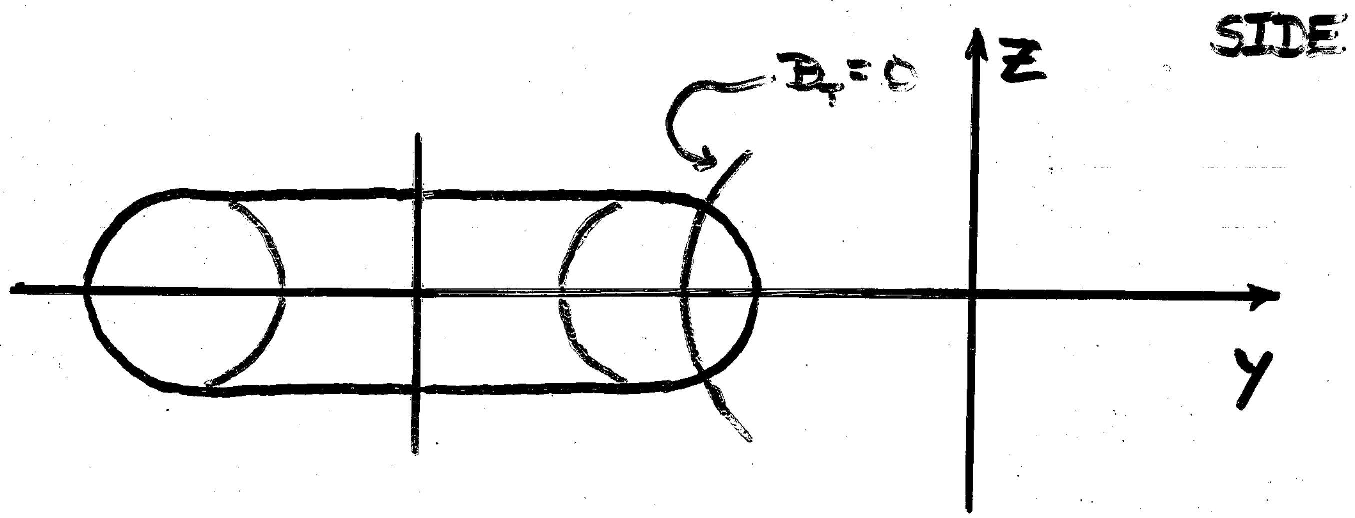
(5) POLYNOMIAL MODEL

(6) SUMMARY & PROPOSAL

# GLOBAL DESCRIPTION





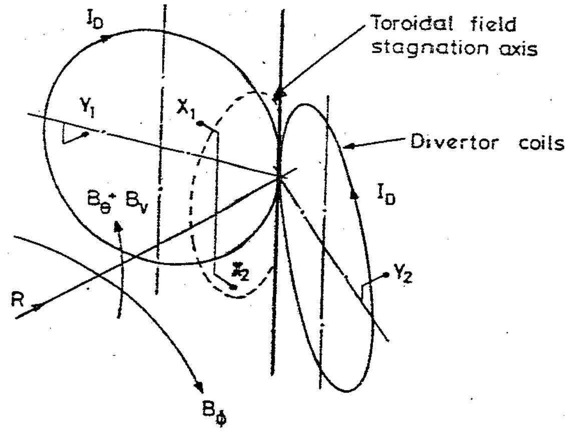




*FIG. 7. Schematic diagram of field zeros in the Bundle Divertor system.*

## (2) NEAR NULL

$$\underline{B}(\underline{x}) = \underline{\underline{A}} \cdot (\underline{x} - \underline{x}_0)$$

$$A_{ij} = \frac{\partial B_i(\underline{x}_0)}{\partial x_j}$$

$$\underline{\nabla} \cdot \underline{B} = 0 \Rightarrow \text{TRACE } \underline{\underline{A}} = 0$$

5

$$\underline{\nabla} \times \underline{B} = 0 \Longrightarrow A_{ij} = A_{ji}$$

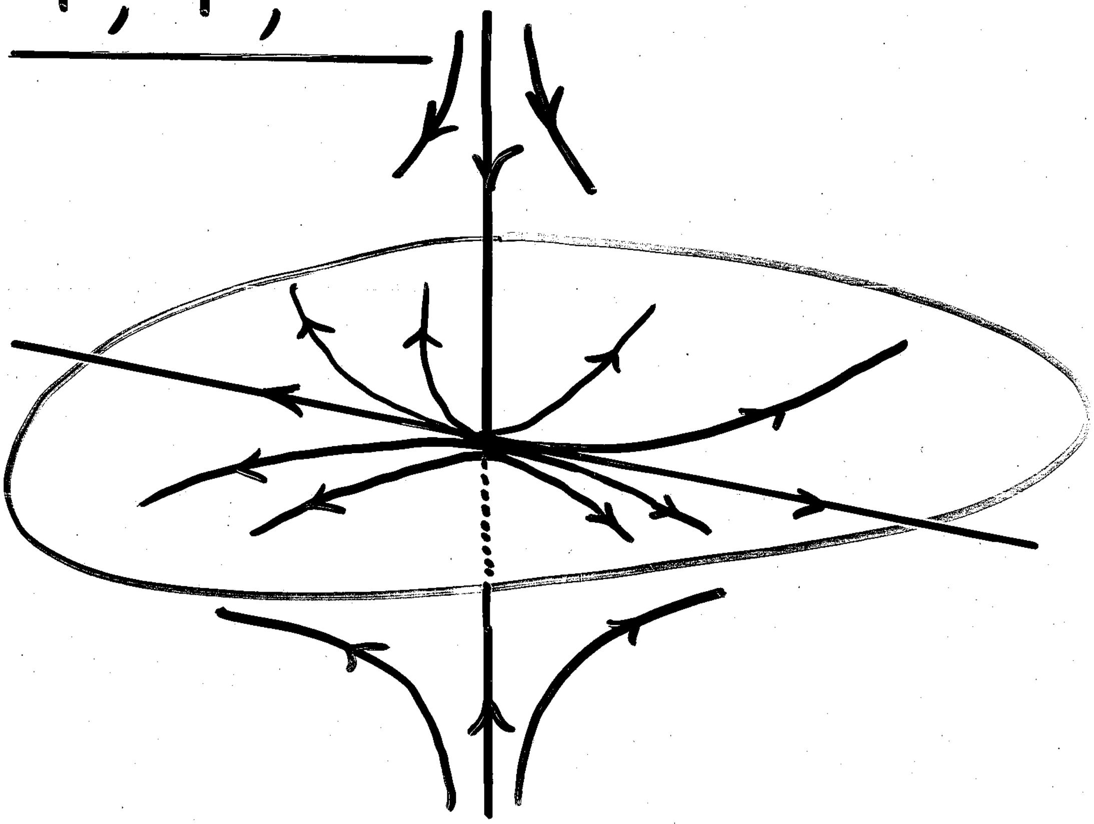
REAL EIGENVALUES

DIAGONALIZABLE

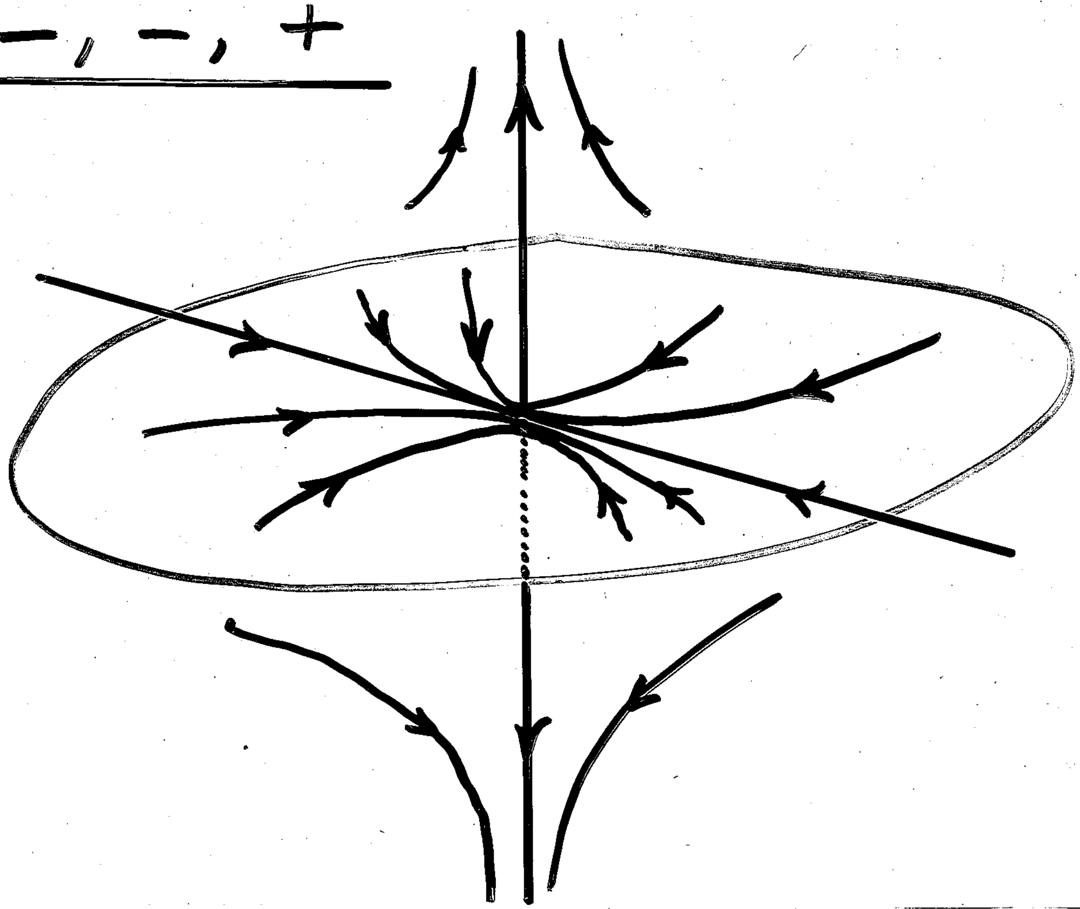
$$\sum \lambda_i = 0 \Longrightarrow$$

$+, +, -$ OR $-, -, +$

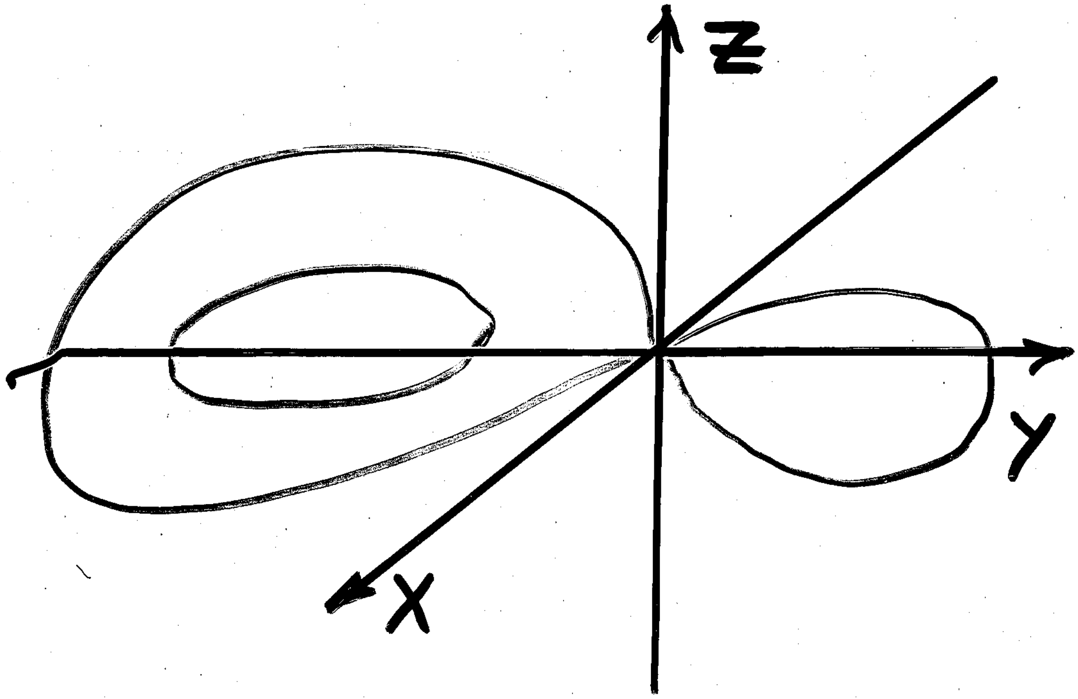
+, +, −



−, −, +



## (3) SYMMETRY




$$B_z(x,y,z) = B_z(-x,y,-z)$$

$$B_x(x,y,z) = B_x(-x,y,-z)$$

$$B_y(x,y,z) = -B_y(-x,y,-z)$$

IF $\underline{X}_0 = (X_0, Y_0, -Z_0)$ IS A NULL SYMMETRY $\Longrightarrow$

$\underline{X}_1 = (-X_0, y_0, Z_0)$ IS, AND

IF NEAR $\underline{X}_0$

$$\underline{B}(\underline{x}) = \underline{\underline{A}} \cdot (\underline{x} - \underline{x}_0) = \begin{bmatrix} \alpha & a & b \\ a & \beta & c \\ b & c & -(\alpha+\beta) \end{bmatrix} \cdot (\underline{x} - \underline{x}_0)$$

THEN NEAR $\underline{X}_1$

$$\underline{B}(\underline{x}) = \underline{\underline{A}}^* \cdot (\underline{x} - \underline{x}_1) = \begin{bmatrix} -\alpha & a & -b \\ a & -\beta & c \\ -b & c & (\alpha+\beta) \end{bmatrix} \cdot (\underline{x} - \underline{x}_1).$$

OBSERVE

$$\underline{\tilde{M}} \, \underline{\underline{A}}^* \underline{M} = -\underline{\underline{A}} \qquad \text{w/} \quad \underline{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

HENCE

EIGENVALUES OF $\underline{\underline{A}}^*$

$=$ − EIGENVALUES OF $\underline{\underline{A}}$

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IF T DIAGONALIZES $\underline{\underline{A}}$

THEN MT DIAGONALIZES $\underline{\underline{A}}^*$

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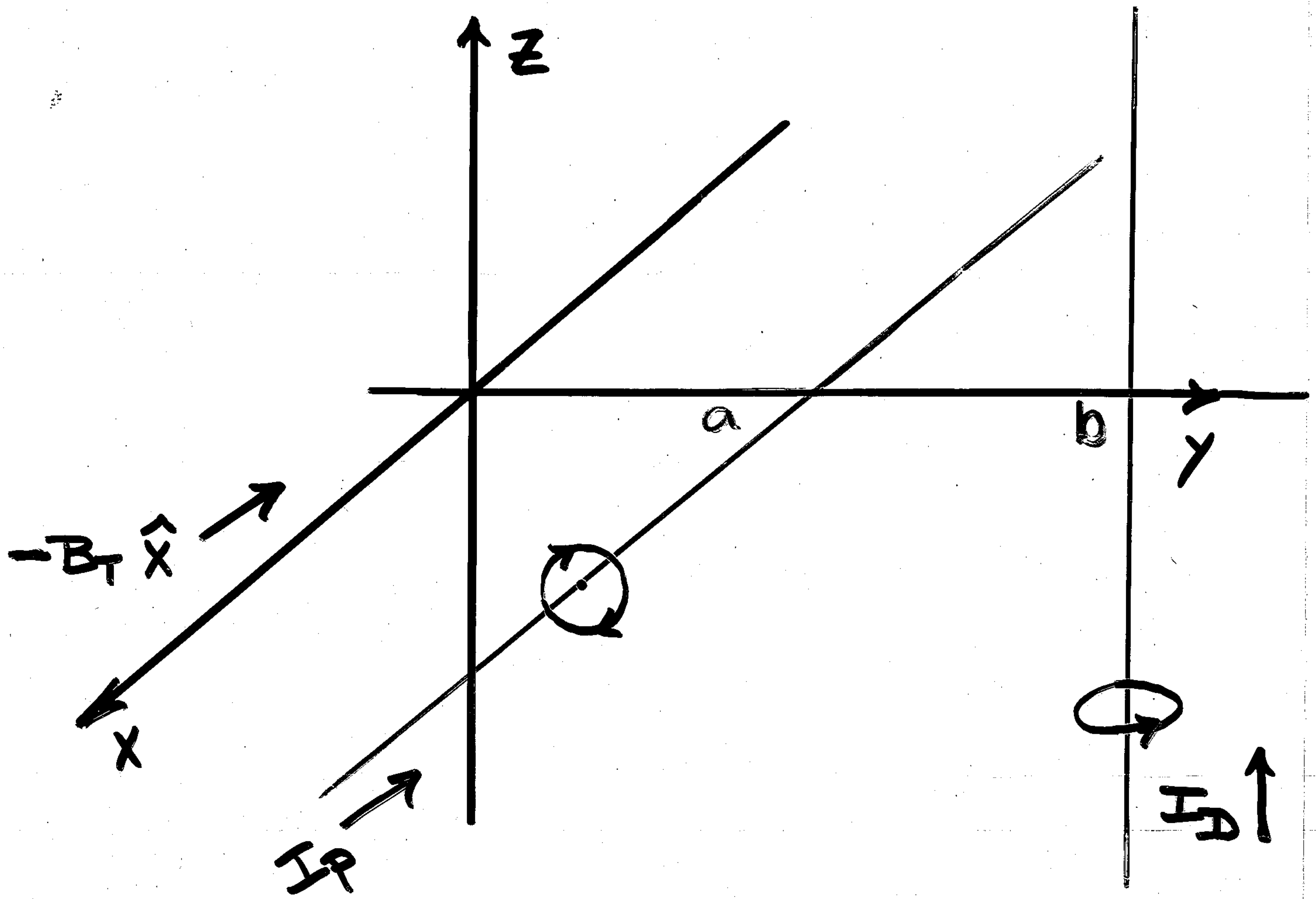
IF $\underline{\xi}_i = (x_i, y_i, z_i)$ IS AN EIGENVECTOR OF $\underline{\underline{A}}$

THEN $\underline{\xi}_i^* = (x_i, -y_i, z_i)$ IS AN EIGENVECTOR OF $\underline{\underline{A}}^*$

## (4) ⊥ WIRE MODEL




$$\underline{B} = -B_T \hat{X} + \underline{B}_P + \underline{B}_D$$

$$= \hat{X}\left[-B_T + \frac{\frac{2I_D}{c}(b-y)}{(y-b)^2 + z^2}\right] + \hat{y}\left[\frac{\frac{2I_P}{c} z}{(y-a)^2 + z^2}\right.$$

$$\left. + \frac{\frac{2I_D}{c} X}{(y-b)^2 + X^2}\right] + \hat{z}\left[\frac{\frac{2I_P}{c}(a-y)}{(y-a)^2 + z^2}\right]$$

## TWO NULLS

$$Y_0 = a\,,\quad X_0 = \sqrt{(b-a)(2\alpha - b + a)}\,,$$

$$Z_0 = -2\beta\alpha\sqrt{\frac{b-a}{2\alpha - b + a}}$$

AND

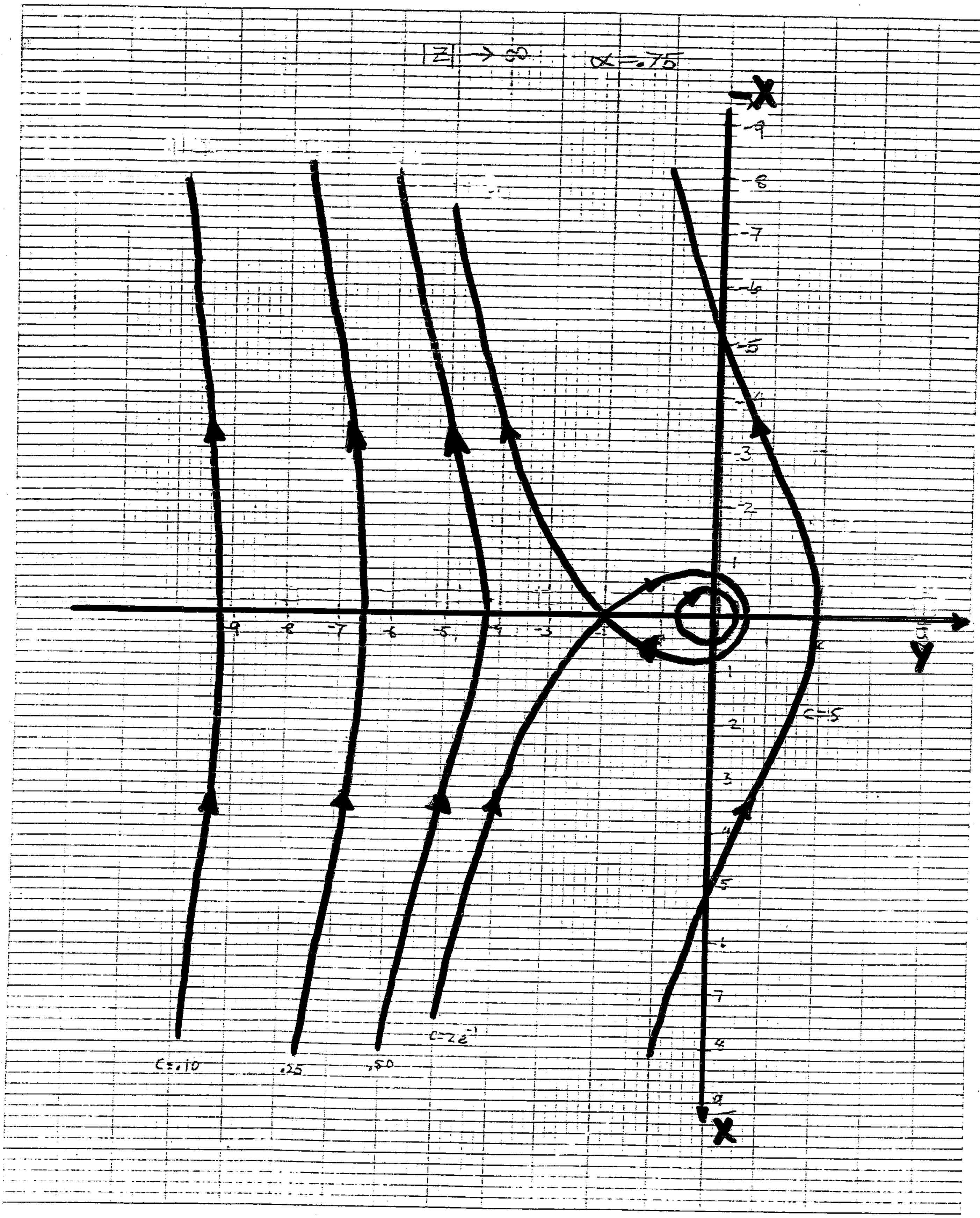
$$Y_p = a\,,\quad X_1 = -\sqrt{(b-a)(2\alpha - b + a)}\,,$$

$$Z_1 = 2\beta\alpha\sqrt{\frac{b-a}{2\alpha - b + a}}$$

FOR $\quad 2\alpha - b + a > 0\,.$

$$\beta \equiv \frac{I_P}{I_D} \qquad\qquad \alpha \equiv \frac{I_D}{c\,B_T}$$

# $|Z| \to \infty$



# (5) POLYNOMIAL MODEL

$$B_i(\underline{x}) = C_{ijk}\,(\underline{x}-\underline{x}_0)_j\,(\underline{x}-\underline{x}_1)_k$$

$$\underline{x}_0 = (X_0, 0, -Z_0)\ ,\quad \underline{x}_1 = (-X_0, 0, Z_0)$$

NEAR NULLS

$$A_{ij} = 2\left[C_{ij1}X_0 - C_{ij3}Z_0\right]$$

$$A^*_{ik} = -2\left[C_{i1k}X_0 - C_{i3k}Z_0\right]$$

BUNDLE DIVERTOR SYMMETRY $\Rightarrow$

$$C_{131} = C_{113}\ ,\quad C_{331} = C_{313}\ ,\quad C_{213} = -C_{231}$$

$$C_{121} = -C_{112}\ ,\quad C_{312} = -C_{321}\ ,\quad C_{221} = C_{212}$$

$$C_{132} = -C_{123}\ ,\quad C_{332} = -C_{323}\ ,\quad C_{232} = C_{223}$$

9 RELATIONS

$\underline{\nabla} \cdot \underline{B} = 0 \Rightarrow$

$$C_{111} + C_{221} + C_{331} = 0$$

$$C_{222} = 0 \qquad 3$$

$$C_{113} + C_{223} + C_{333} = 0$$

$\underline{\nabla} \times \underline{B} = 0 \Rightarrow$

$$C_{233} = C_{132} = C_{211} = 0$$

$$C_{311} = C_{131} \qquad C_{313} = C_{133}$$

$$C_{232} = C_{322} \qquad C_{221} = C_{122} \qquad 9$$

$$(C_{231} - C_{321}) X_0 + C_{323} Z_0 = 0$$

$$Z_0 C_{231} + X_0 C_{112} = 0$$

$$27 - 21 = 6$$

# SUMMARY

(1) GLOBAL DESCRIPTION
(2) BEHAVIOR NEAR NULL
(3) B-D SYMMETRY & CONSEQUENCES
(4) ⊥ WIRE MODEL
(5) POLYNOMIAL MODEL

# PROPOSAL

(1) UNDERSTAND FIELD LINE TOPOLOGY & CONSTRUCT A LOCAL MODEL

(2) STUDY EFFECT OF PERTUBATION

(3) ADD PLASMA - DO LOCAL MHD NEAR NULLS AND ASSOCIATED "PECULIAR" FIELD LINE GEOMETRY.