DIFFUSION IN THE VICINITY OF AN X-POINT

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X-POINT DIFFUSION - P. MORRISON

I

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OVERVIEW

INTRODUCTION

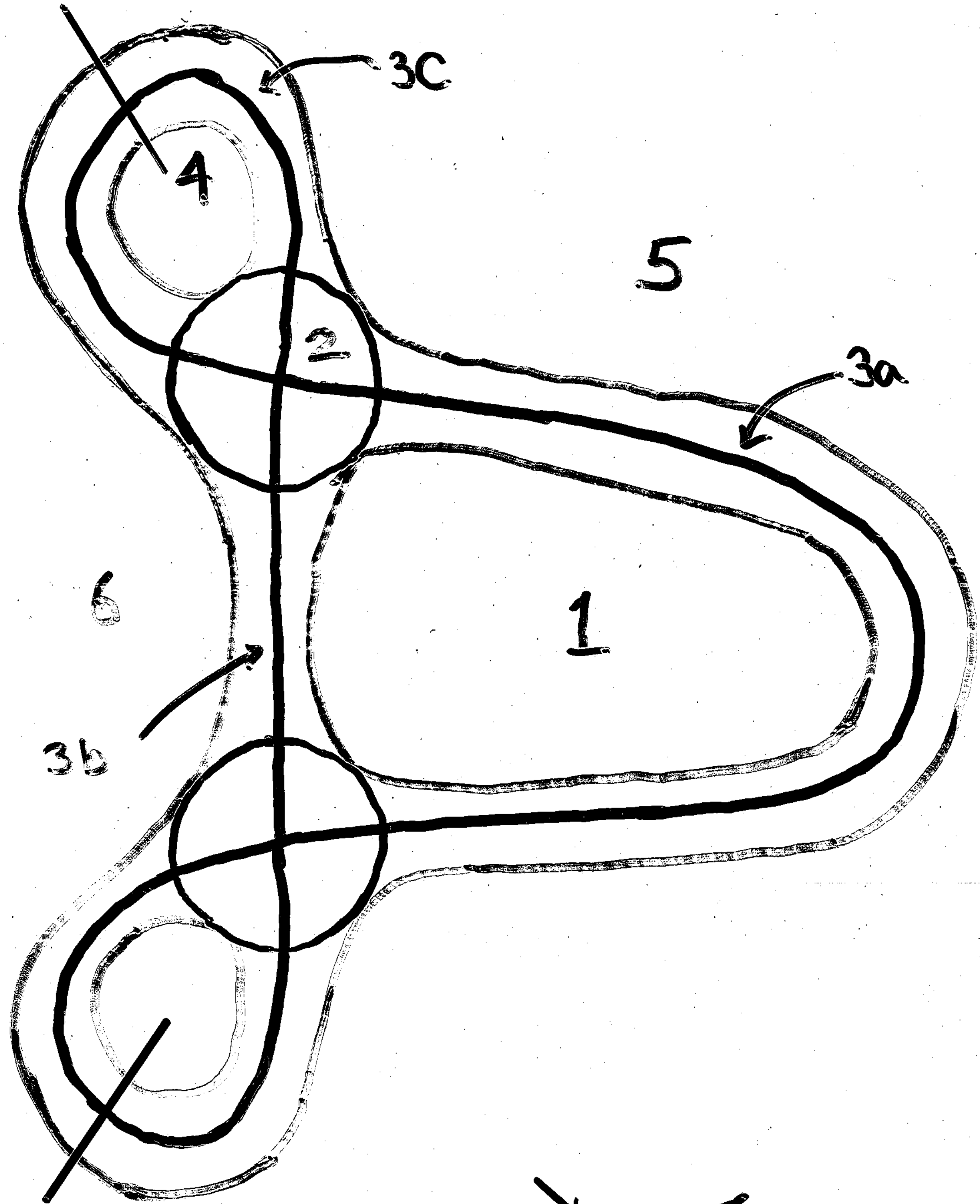
ORDERING

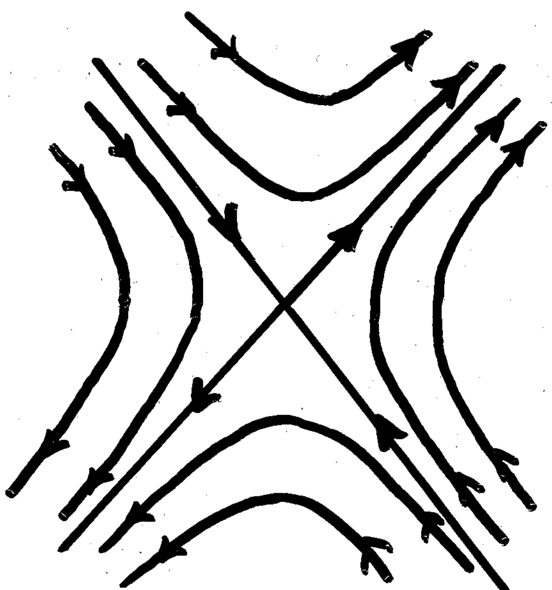
ORGANIZATION OF CALCULATION

PRELIMINARY RESULTS

FUTURE DIRECTIONS

II





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PREVIOUS WORK

STRAIGHT SYSTEMS

APPLICATION TO

MAGNETOSPHERE
MAGNETOTAIL
SOLAR FLARES

SWEET & PARKER

SONNERUP
YEH & AXFORD

PETSCHEK

REVIEW BY

VASYLIUNAS, REV. GEOPHYS. & SPACE SCI., 13, 303, (1975).

MOST RELEVANT HERE

AUERBACH & BOOZER PPL

EFFECTS WE CONSIDER

EXISTENCE OF B_T

TORIDAL EFFECTS

SHEAR

VISCOSITY (INCLUDING GYRO.)

HEAT TRANSPORT

THERMO-ELECTRIC

BRAGINSKII

SYSTEMATIC ORDERING

LEANS ON KINETIC THEORY

INTERNAL CONSISTENCY

ORDERING

FOUR SCALE LENGTHS

(1) r_L - ION GYRO RADIUS

(2) $\mathbb{X}$ - (SLOW) SCALE CHARACTERISTIC OF REGION 1

(3) X - (FAST) SCALE CHARACTERISTIC OF REGION 2 (SIZE OF REGION)

(4) ℓ - MEAN FREE PATH

NEOCLASSICAL $\frac{r_L}{\mathbb{X}} \sim \epsilon$

HERE $\frac{r_L}{X} \sim \epsilon \quad \frac{X}{\mathbb{X}} \sim \epsilon \quad \Rightarrow$

$$\frac{r_L}{\mathbb{X}} \sim \epsilon^2$$

$B_T \sim 1$ $\qquad$ $B_p \sim \epsilon$

$v_\parallel \sim 1$ $\qquad$ $v_\perp \sim \epsilon$

$$\omega_T \sim \frac{v_\theta}{X} \sim \frac{v_\perp}{X}$$

$$\frac{\omega_T}{\Omega} \sim \frac{v_\theta}{X\Omega} \sim \frac{r_L}{X} \sim \epsilon^2$$

$$\Rightarrow \quad e \sim \frac{1}{\epsilon^2}$$

$$\frac{\nu}{\omega_T} \sim \frac{X}{\ell} \sim 1 \quad \text{(FOR NOW)}$$

TO BE CONSISTENT WITH $v_\perp \sim \epsilon$

$$\frac{\Phi}{X B_T^{(0)} v_T^{(0)}} \sim \epsilon^2 \quad \left(\frac{e\Phi}{T} \sim 1 \text{ NEOCLASSICAL} \right)$$

$NA \sim 1$
$A \sim \epsilon$

$$\frac{\sigma^{-1}}{\omega_T X^2} \sim \frac{m\nu}{N e^2 \omega_T X^2} \sim \epsilon^4 \Rightarrow \frac{E_\phi}{v_T^{(0)} B_T^{(0)}} \sim \epsilon^4$$

$$\frac{1}{\omega_T}\frac{\partial}{\partial t} \sim \frac{D\,\nabla_\perp^2}{\omega_T} \sim \frac{D}{\omega_T X^2} \sim \frac{\nu\, r_L^2}{\omega_T X^2} \sim \epsilon^2$$

$$\underline{\hat{\nabla}} \equiv \underline{\nabla} + \frac{1}{\epsilon}\underline{\nabla}$$

$$\underline{B}_p \cdot \underline{\nabla} \sim \underline{v}_\perp \cdot \underline{\nabla} \sim B_T \cdot \underline{\nabla}$$

$$\sim \underline{v}_T \cdot \underline{\nabla}$$

$B_T(\underline{X})$, $\underline{\hat{\Phi}}(\underline{X})$, EVERYTHING ELSE

$f(x, X)$ $\left(\underline{\hat{\Phi}}\cdot\underline{\nabla}\,\underline{\hat{\Phi}} \neq 0\right)$

$$\beta \sim 1$$

O
M
I
T

THERMAL CONDUCTIVITY

$K_{\parallel} \sim 1$

$K_{\perp} \sim \epsilon^4$

$K_{\wedge} \sim \epsilon^2$

VISCOSITY

PARALLEL $\eta_0 \sim 1$

$B \Downarrow 0$

PERP. $\eta_1, \eta_2 \sim \epsilon^4$

GYRO. $\eta_3, \eta_4 \sim \epsilon^2$

FLUID EQUATIONS

$$\underline{M} \equiv \rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) + \hat{\underline{\nabla}} p + \hat{\underline{\nabla}}\cdot\underline{\underline{\pi}} - \underline{J}\times\underline{B} = 0$$

$$\underline{\Omega} \equiv \underline{\underline{\eta}}\cdot\underline{J} - \underline{E} + \hat{\underline{\nabla}}\Phi - \underline{v}\times\underline{B} + \frac{\underline{J}\times\underline{B}}{eN} - \frac{\hat{\underline{\nabla}} p}{2eN} - \frac{\alpha_1}{e}\hat{e}\,\hat{e}\cdot\hat{\underline{\nabla}}T - \frac{\alpha_2}{e}\hat{e}\times\hat{\underline{\nabla}}T = 0$$

$$\frac{\partial p}{\partial t} + \underline{v}\cdot\hat{\underline{\nabla}} p + \gamma p\,\hat{\underline{\nabla}}\cdot\underline{v} = -\hat{\underline{\nabla}}\cdot\underline{q} - \underline{\underline{\pi}}:\hat{\underline{\nabla}}\underline{v} + Q$$

SOLVE $\begin{cases} \hat{\underline{\Phi}}\times\underline{M} & \text{FOR } \underline{J}_P \\ \hat{\underline{\Phi}}\times\underline{\Omega} & \text{FOR } \underline{v}_P \end{cases}$

SCALAR EQS $\begin{cases} \hat{\underline{\Phi}}\cdot\underline{M} \\ \hat{\underline{\Phi}}\cdot\underline{\Omega} \\ \text{HEAT} \\ \text{CONT.} \end{cases}$

$$\underline{B}_T = \underline{B}_T^{(0)} + \epsilon \underline{B}_T^{(1)} + \dots$$

$$\underline{B}_p = \epsilon \underline{B}_p^{(0)} + \epsilon^2 \underline{B}_p^{(1)} + \dots \qquad \underline{J} = \underline{J}_0 + \epsilon \underline{J}_1 + \dots$$

$$\rho = \rho_0 + \epsilon \rho_1 + \dots \qquad N = N_0 + \epsilon N_1 + \dots$$

$$\underline{v} = \underline{v}_0 + \epsilon \underline{v}_1 + \dots \qquad \Phi = \epsilon^2 \Phi_2 + \epsilon^3 \Phi_3 + \dots$$

ORGANIZATION OF CALCULATION

ZEROTH ORDER

ENERGY	$\mathcal{O}(\epsilon^0)$	
CONT.	$\mathcal{O}(\epsilon^0)$	$\mathcal{O}(\epsilon^{-1})$
$\hat{\Phi} \cdot \underline{M}_0$		
$\hat{\Phi} \cdot \underline{\Omega}_2$		$\underline{\Omega}_0$

$\hat{\Phi} \times \underline{M}_{-1}$

$\hat{\Phi} \times \underline{\Omega}_1$

$\underline{\nabla} \cdot \underline{J}_p^{(0)} = 0$

POLOIDAL VEL. $\mathcal{O}(\epsilon)$

$$\underline{\nabla} \left(N_0, \rho_0, \Phi_2, v_T^{(0)} \right)$$

FIRST ORDER

$$\left\{\begin{array}{ll} \text{ENERGY} & \Theta(\epsilon) \\ \text{CONT.} & \Theta(\epsilon) \\ \hat{\underline{\Phi}}\cdot\underline{M}_1 & \\ \hat{\underline{\Phi}}\cdot\underline{\Omega}_3 & \end{array}\right.$$

$$\left\{\begin{array}{l} \hat{\underline{\Phi}}\times\underline{M}_0 \\ \hat{\underline{\Phi}}\times\underline{\Omega}_2 \\ \underline{\nabla}\cdot\underline{J}_p^{(1)}+\underline{\nabla}\cdot\underline{J}_p^{(0)}=0 \end{array}\right.$$

$$\underline{\nabla}\,(N_1,\ P_1\ ,\ Q_3\ ,\ \underline{v}_T^{(1)})$$

$$\underline{\nabla}\,(v_T^{(0)},\ P_0,\ Q_2,\ N_0)$$

$$\underline{v}_p^{(2)}$$

SECOND ORDER

$\frac{\partial}{\partial t}$ ENTERS

PARABOLIC EQS.

PRELIMINARY RESULTS

ZEROTH ORDER

$$\underline{v}_0 = \hat{\Phi}\, v_T^{(0)} \qquad \underline{\nabla} P_0 = 0$$

$$\underline{v}_p^{(1)} = \frac{\hat{\Phi} \times \underline{\nabla} \Phi_2}{B_T^{(0)}} + \frac{v_T^{(0)}\, \underline{B}_p^{(0)}}{B_T^{(0)}}$$

$$\underline{\nabla}\left(\rho_0\, v_T^{(0)2}\right) = 0$$

$$\underline{B}_p^{(0)} \cdot \underline{\nabla} \begin{bmatrix} \Phi_2 \\ N_0 \\ v_T^{(0)} \\ P_0 \end{bmatrix} = 0$$

FIRST ORDER

$$\underline{\underline{I}}\cdot\left\{\underline{B}_p^{(0)}\cdot\underline{\nabla}\begin{bmatrix}\Phi_3\\ N_1\\ v_T^{(1)}\\ P_1\end{bmatrix}\right\} = -\,\underline{\underline{I}}\cdot\left\{\underline{B}_p^{(0)}\cdot\underline{\nabla}\begin{bmatrix}\Phi_2\\ N_0\\ v_T^{(0)}\\ P_0\end{bmatrix}\right\}$$

$$+\ \underline{\underline{A}}\cdot\underline{\nabla}\ln B_T^{(0)} + \underline{\underline{\underline{B}}}\cdot\hat{\underline{\Phi}}\cdot\underline{\nabla}\hat{\underline{\Phi}}$$

$$\underline{\nabla}P_1 = 0$$

$$\underline{\nabla}\left(P_0 + \frac{B_T^{(0)2}}{2}\right) + \hat{\underline{\Phi}}\cdot\underline{\nabla}\hat{\underline{\Phi}}\left(\rho_0 v_T^{(0)2} - B_T^{(0)2}\right) = 0$$

FUTURE DIRECTIONS

BECAUSE SECOND ORDER IS SO COMPLICATED DO PILOT PROBLEMS

(1) NEGLECT ALL NON-IDEAL TERMS EXCEPT σ^{-1}

ALSO NEGLECT $\hat{\underline{\Phi}} \cdot \underline{\nabla} \hat{\underline{\Phi}}$, BUT KEEP SHEAR

(2) ADD CROSS FIELD VISCOSITY (// TOO)

(3)

(4)

⋮

$\left.\right\}$ "SYSTEMATIC" INCLUSION OF REMAINING TERMS