The control of quantum transitions using intense laser pulses

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Observation of Chaos-Assisted Tunneling Between Islands of Stability

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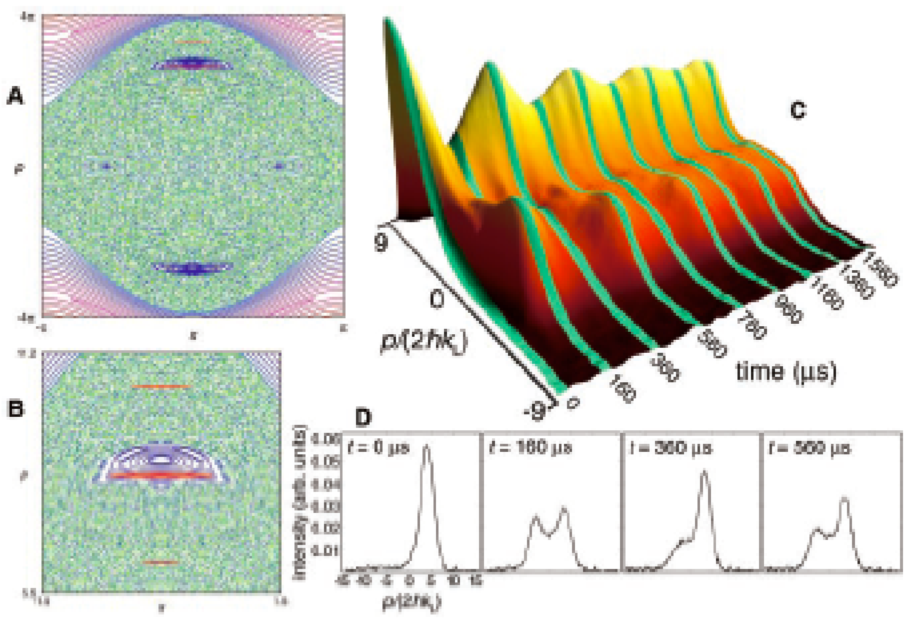
We report the direct observation of quantum dynamical tunneling of atoms between separated momentum regions in phase space. We study how the tunneling oscillations are affected as a quantum symmetry is broken and as the initial atomic state is changed. We also provide evidence that the tunneling rate is greatly enhanced by the presence of chaos in the classical dynamics. This tunneling phenomenon represents a dramatic manifestation of underlying classical chaos in a quantum system.



Our experiment studies the motion of cold cesium atoms in an amplitude-modulated standing wave of light. Because the light is detuned far from the D_2 line (50 GHz, or 10^4 natural linewidths, to the red of the $F = 3 \rightarrow F'$ transition, where F is the atomic hyperfine quantum number), the internal dynamics of the atom can be adiabatically eliminated (*21*, *22*). The atomic center-of-mass Hamiltonian can then be written in scaled units as

$$H = p^2/2 - 2\alpha\cos^2(\pi t)\cos(x) \qquad (1)$$

where x and p are the canonical position and momentum coordinates, respectively, t is time, and α is given by $(8\omega_r T^2/\hbar)\, V_0$ [V_0 is the amplitude of the ac Stark shift corresponding to the time-averaged laser intensity, T is the period of the temporal modulation, $\hbar$ is the reduced Planck constant, and ω_r is the recoil frequency, which has the numerical value $2\pi \times 2.07$ kHz for this experiment];

Dynamical tunnelling of ultracold atoms

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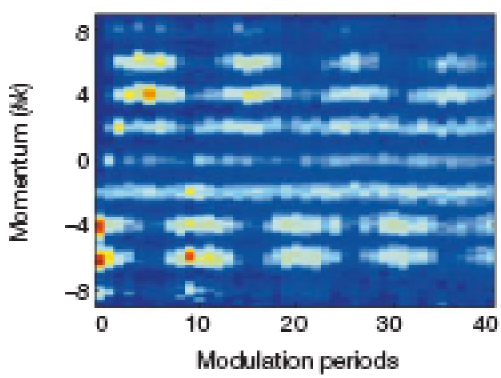
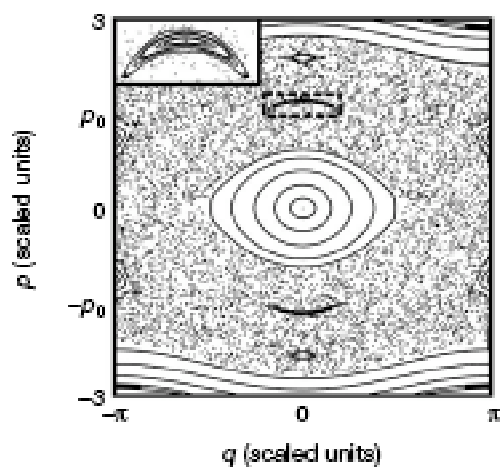
The divergence of quantum and classical descriptions of particle motion is clearly apparent in quantum tunnelling[1,2] between two regions of classically stable motion. An archetype of such non-classical motion is tunnelling through an energy barrier. In the 1980s, a new process, 'dynamical' tunnelling[1–3], was predicted, involving no potential energy barrier; however, a constant of the motion (other than energy) still forbids classically the quantum-allowed motion. This process should occur, for example, in periodically driven, nonlinear hamiltonian systems with one degree of freedom[4–6]. Such systems may be chaotic, consisting of regions in phase space of stable, regular motion embedded in a sea of chaos. Previous studies predicted[4] dynamical tunnelling between these stable regions. Here we observe dynamical tunnelling of ultracold atoms from a Bose–Einstein condensate in an amplitude-modulated optical standing wave. Atoms coherently tunnel back and forth between their initial state of oscillatory motion (corresponding to an island of regular motion) and the state oscillating 180° out of phase with the initial state.



Atoms in optical potentials[8–11] provide an ideal test bed to explore quantum nonlinear dynamics. The quantum driven pendulum, a paradigmatic system for the study of 'quantum chaos', is realized by a periodically modulated optical standing wave, which can provide a highly non-dissipative, one-dimensional potential. Using ultracold trapped atoms whose action is of the order of Planck's constant makes quantum effects significant. Atoms in a standing wave, detuned far from an atomic resonance, with sinusoidal amplitude modulation, are described by the centre-of-mass hamiltonian:

$$H(t) = \frac{p^2}{2} + 2\kappa(1 + 2\epsilon \sin\omega t)\sin^2(q/2)$$

where $q = 2kx$ and $p = (2k/m\omega)p_x$ are the scaled position and momentum variables respectively, t is the time, $\kappa = 4U_0\omega_R/\omega^2$ is the scaled well depth, ω is the modulation frequency, $\omega_R = \hbar k^2/2m$ is the atomic recoil frequency, U_0 is the well depth in units of $\hbar\omega_R$, and ϵ is the modulation parameter. The position and momentum along the standing wave are x and p_x, m is the mass, $k = 2\pi/\lambda$ is the wavevector, and λ is the wavelength of the optical standing wave.



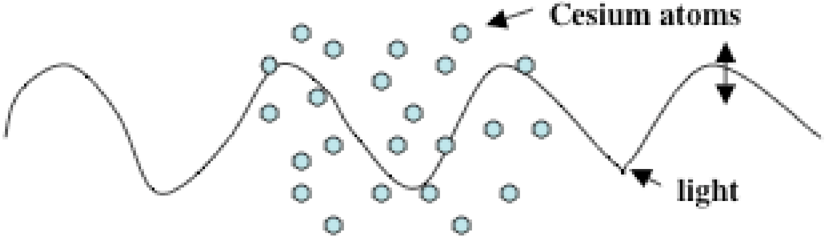
Observation of an Avoided Crossing in Atom Optics Experiment

Steck, Oskay, Raizen, PRL **88** 120406 (2002); Science **293** 274 (2001).

In the Texas cold atom optics experiment, millions of ultra-cold cesium atoms are cooled to 10^{-7} K and interact with a time-periodically modulated standing wave of light.

The atoms underwent coherent periodic oscillations over large regions of the underlying atomic phase space.

The period of the tunneling oscillations was unrelated to the period of the modulation. For a finite range of laser field intensities, the tunneling oscillations had a beat frequency.

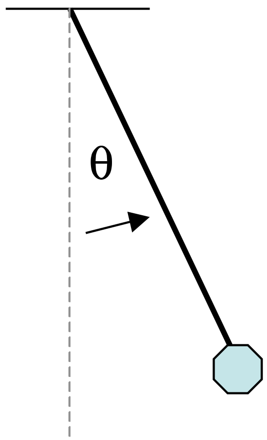


The Hamiltonian for the center of mass motion of the cesium atoms (in dimensionless units)

$$H(t) = p^2 + \kappa\cos(x) + \frac{1}{2}\kappa\left(\cos(x+\omega t) + \cos(x-\omega t)\right)$$

κis determined by the intensity of the laser radiation, p and x are the center of mass momentum κand position, respectively.

The Pendulum



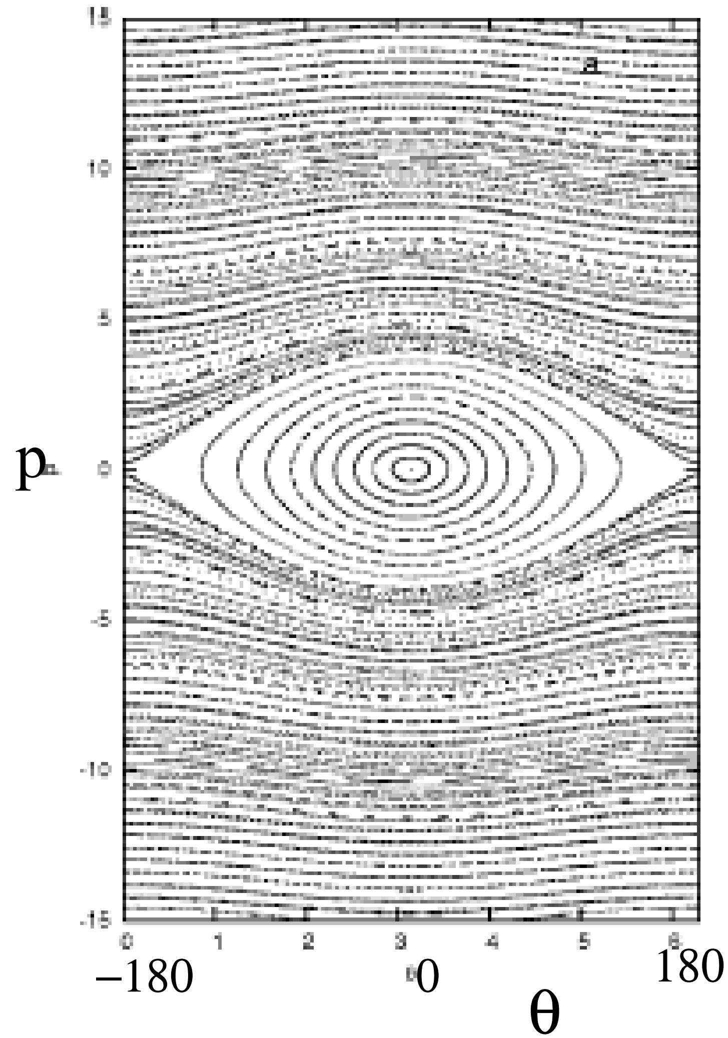
The pendulum is the basic structure leading to a transition to chaos.

The energy of the pendulum has the form

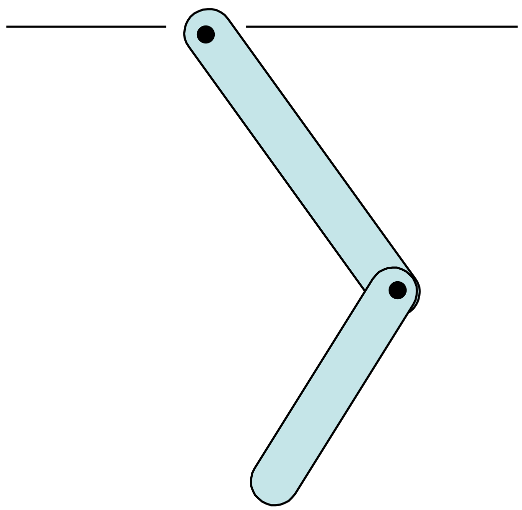
$$H = p^2 + \kappa \cos(\theta) = E$$

where p is the angular momentum and θ is the angular displacement from equilibrium.

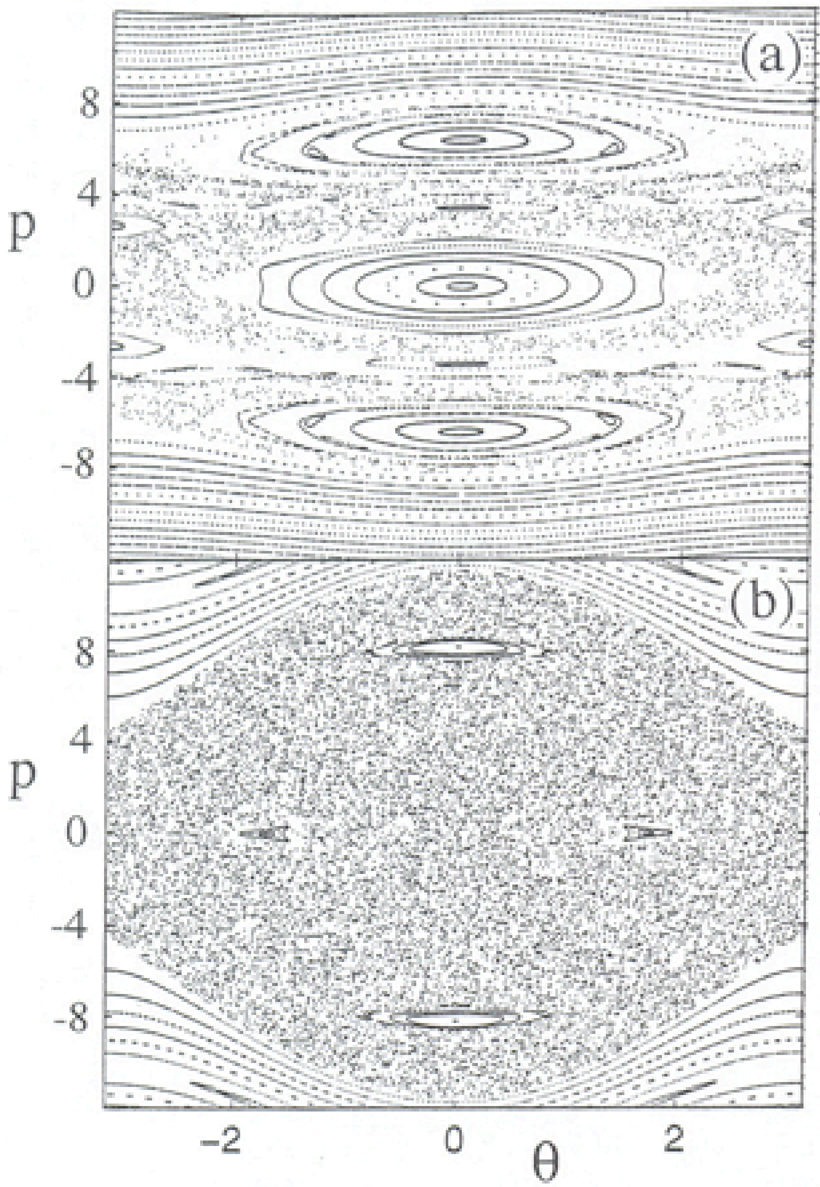
Phase space of a pendulum



Chaos in the Double Pendulum



Atom Optics - Classical Phase Space



Hamiltonian for center of mass motion of cesium atoms

$$H(t) = p^2 + \kappa\cos(x) + \frac{1}{2}\kappa\left(\cos(x+\omega t) + \cos(x-\omega t)\right)$$

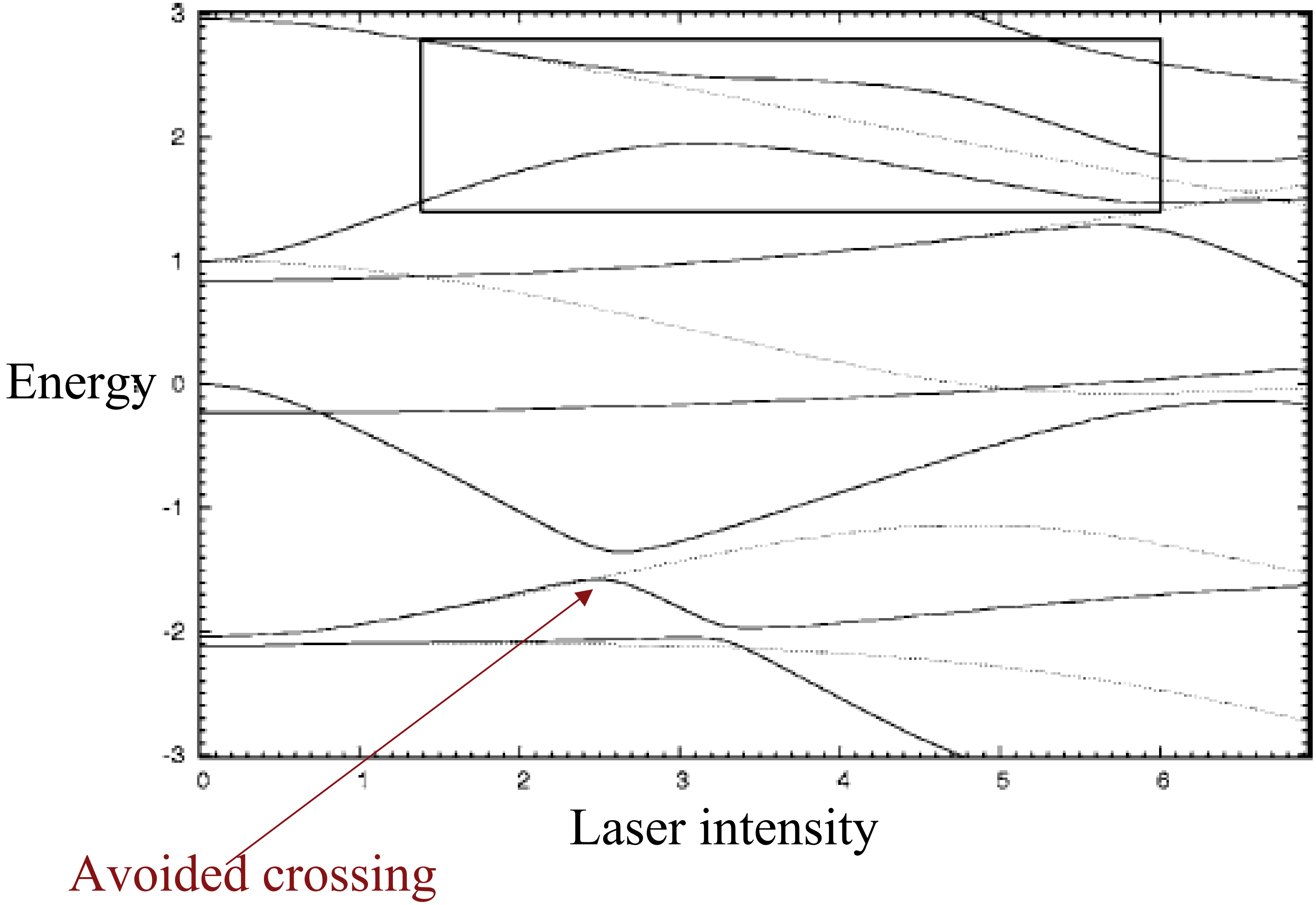
For this experiment the region of phase space strongly affected by the radiation was $-12<p<12$. The laser-atom interaction allows a large number of the atoms to change momentum p in steps of $\Delta p=2$ (in dimensionless units).

Strobe plot of the classical phase space of cesium atoms for (a) $\kappa=1.5$ and (b) $\kappa=4.5$

Quantum Mechanics

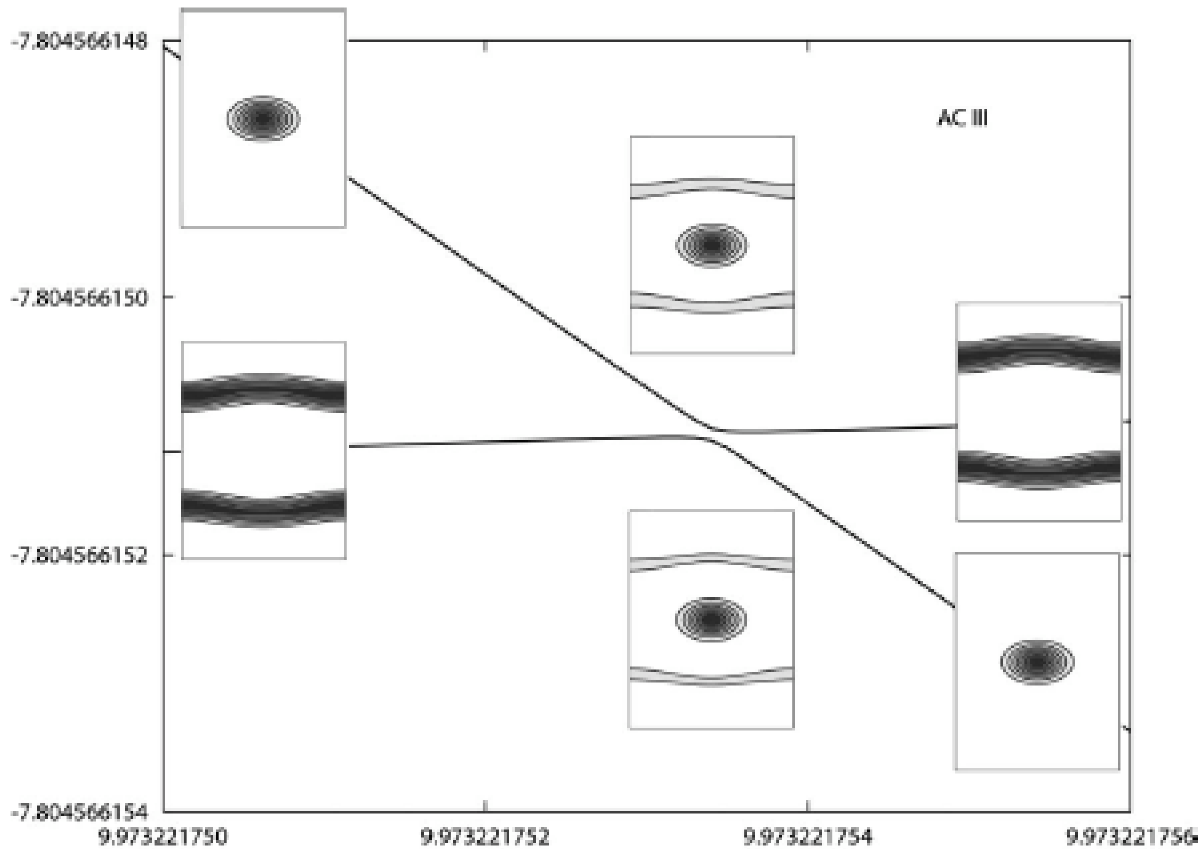
- **Cesium atoms, cooled to a temperature of 0.0000004 Kelvin, behave like WAVES.**
- **The dynamics of the photon-atom “soup” formed by the interacting laser beam/atom system has a symmetry (discrete time translation invariance) that allows <u>conservation of energy</u> (called quasi-energy) <u>of the laser/atom system</u>.**
- **The laser/atom system has 12 quasi-energy states for which the for which the cesium atoms lie in the chaotic region of phase space.**

Energy levels of laser-atom states in the chaotic region.

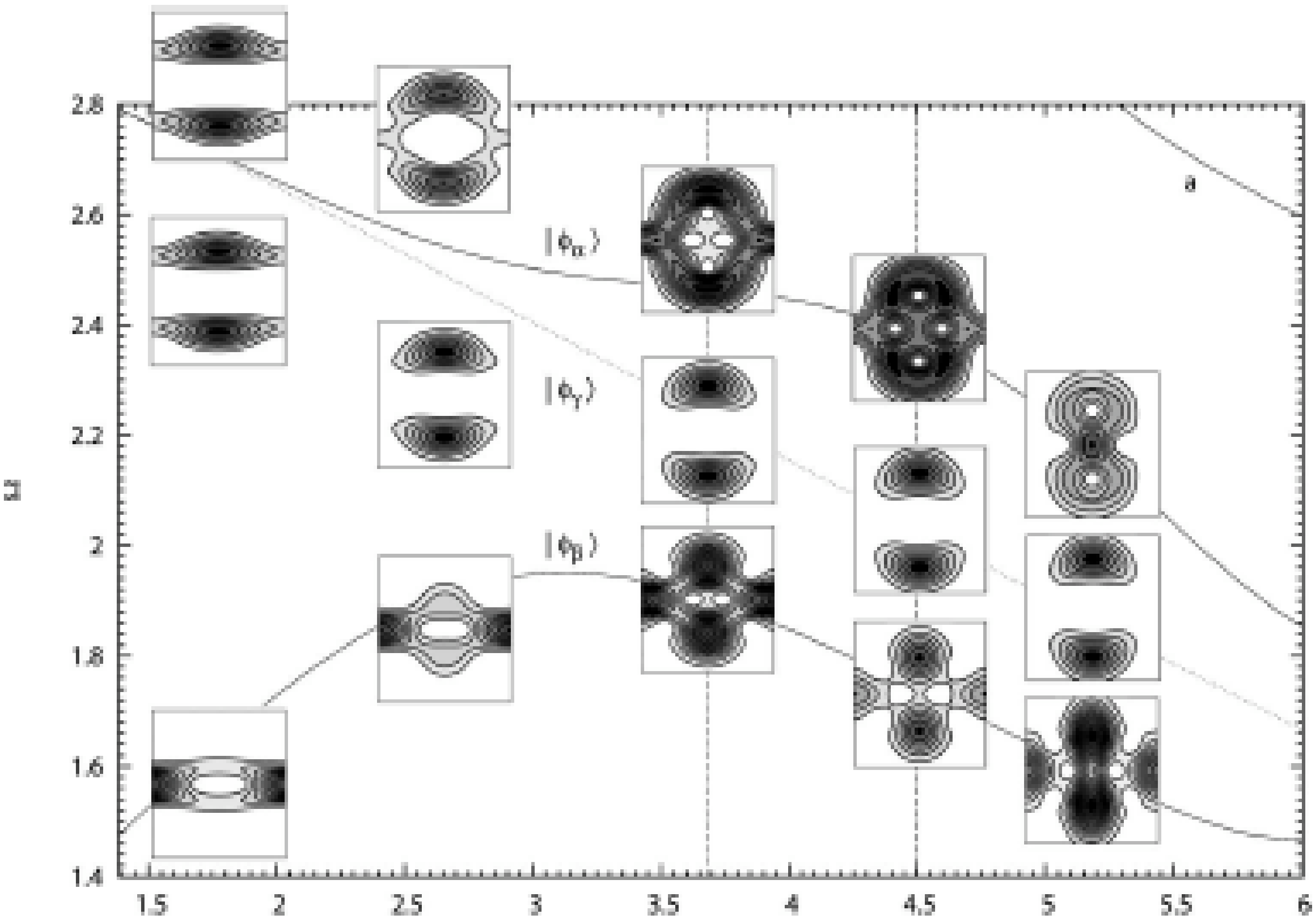


B. Holder and L.E. Reichl, PRA 72 43408 (2005).

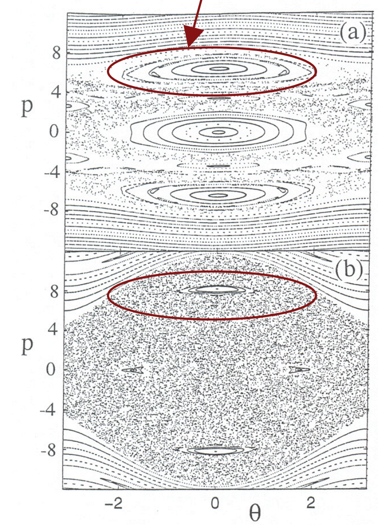
At energy level avoided crossings, states can interchange character



At energy level avoided crossings, states can interchange character



Initial state



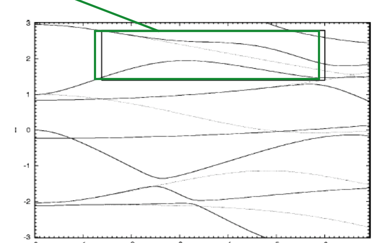
a

b

a

b

(a) κ=1.5, (b) κ=4.5



Equation for time evolution of state of each cesium atom.

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t}=H(t)|\psi(t)\rangle \qquad \text{(Schrodinger equation)}$$

Energy function for each cesium atom in laser field

$$H(t)=p^2+\kappa\cos(x)+\frac{1}{2}\kappa\left(\cos(x+\omega t)+\cos(x-\omega t)\right)$$

Quantum state of each cesium atom in laser field

$$|\psi(t)\rangle=\sum_{\alpha}e^{-iE_{\alpha}t}\langle\chi_\alpha|\psi(0)\rangle|\chi_\alpha\rangle$$

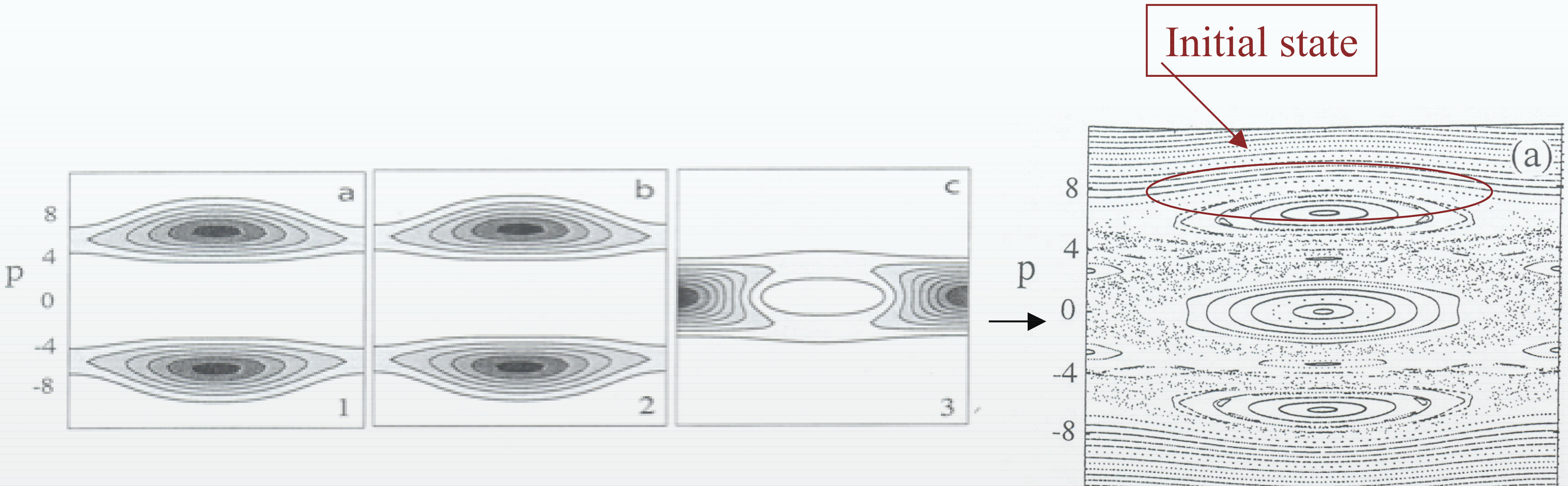
Twelve quasienergy STATES denoted $|\chi_\alpha\rangle$ **with α=1,2,….,12**

Twelve quasienergy VALUES denoted E_α **with α=1,2,….,12**

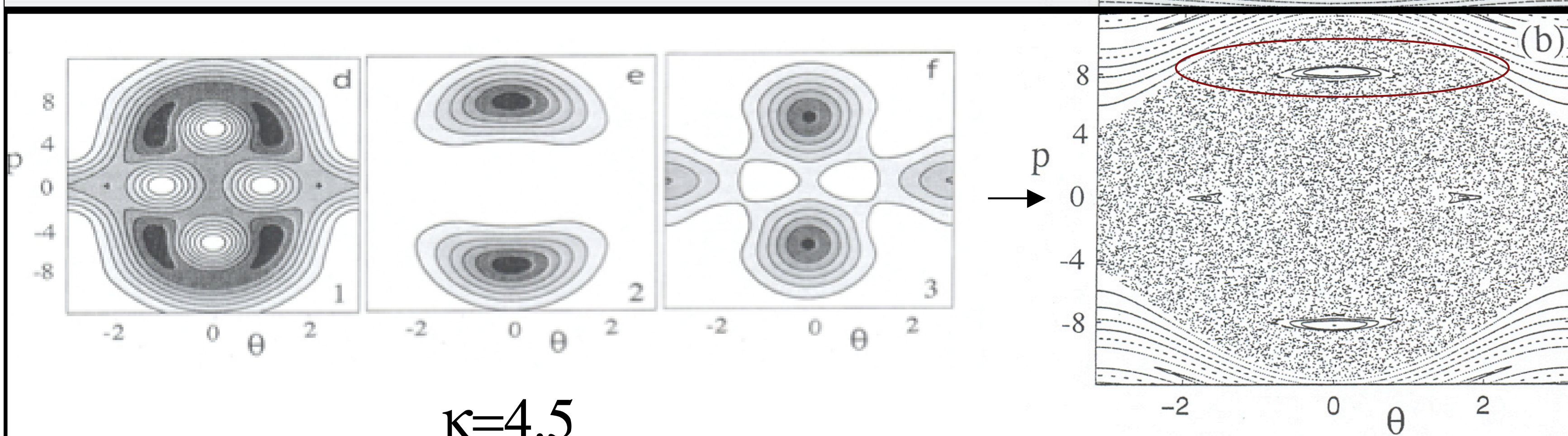
Average momentum (measured in experiment)

$$\langle p\rangle=\sum_{\alpha}\sum_{\beta}e^{-i(E_\alpha-E_\beta)t}\langle\chi_\alpha|p|\chi_\beta\rangle\langle\psi(0)|\chi_\beta\rangle\langle\chi_\alpha|\psi(0)\rangle$$

Only those quasienergy states which overlap in phase space with the initial state of the cesium atoms can participate in the time evolution of the cesium atom cloud.



κ=1.5

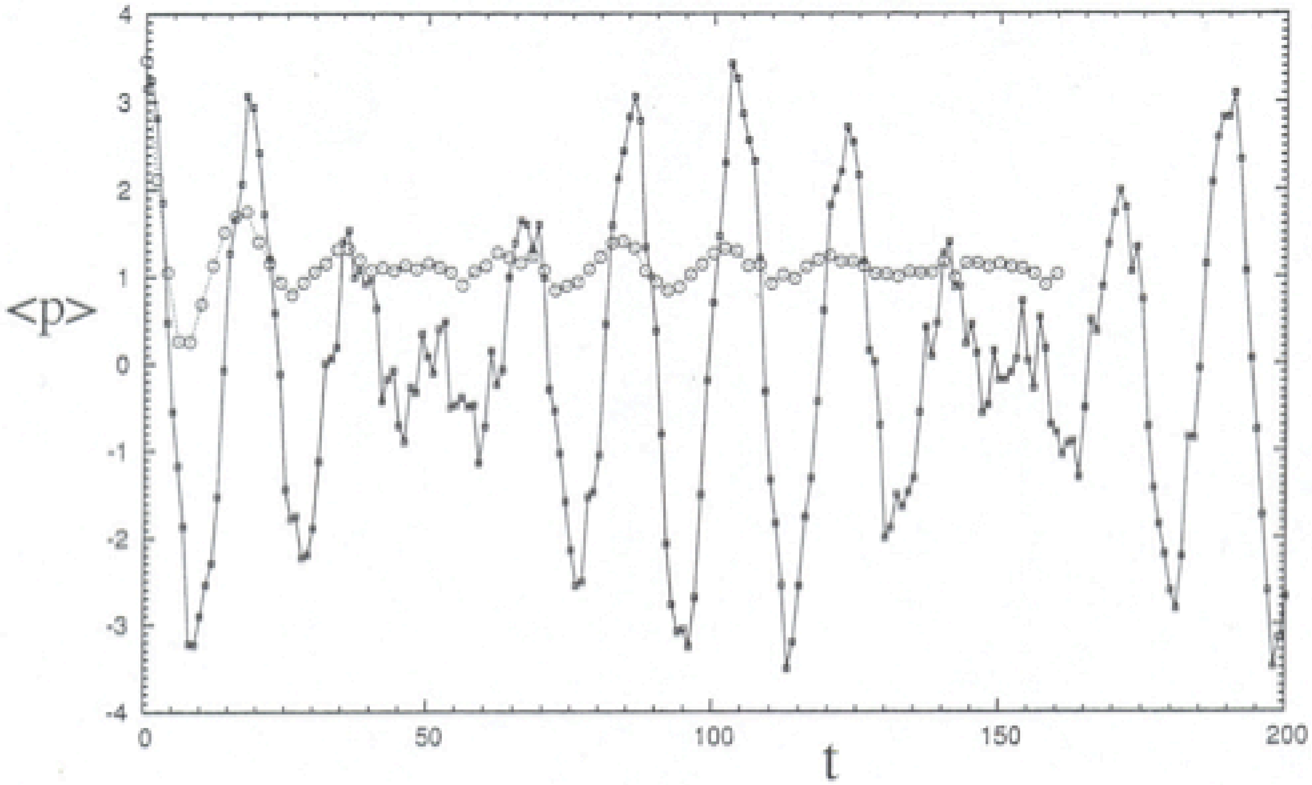


κ=4.5

κ=4.5

$$\langle p\rangle = e^{-i(E_1-E_2)t}\langle\chi_1|p|\chi_2\rangle\langle\psi(0)|\chi_2\rangle\langle\chi_1|\psi(0)\rangle + e^{-i(E_2-E_3)t}\langle\chi_2|p|\chi_1\rangle\langle\psi(0)|\chi_3\rangle\langle\chi_2|\psi(0)\rangle$$

Atom Optics - Average Momentum of Cesium Atoms



○ = Experiment

□ = Numerical

Modulation frequency

f_m=3.14 x 10^5 Hz

Dynamic tunneling frequencies

f_1 = 2.4 x.10^3 Hz

f_2 = 2.9 x 10^3 Hz

$$f_1 = |\,\Omega_2 - \Omega_3\,|$$
$$f_2 = |\,\Omega_2 - \Omega_1\,|$$

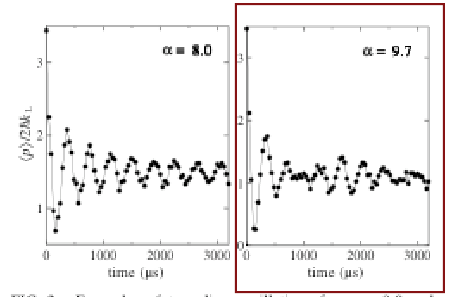


FIG. 2. Examples of tunneling oscillations for $\alpha = 8.0$ and $\alpha = 9.7$, corresponding to two of the measurements in Fig. 1. In the former case, a single frequency persists for the maximum duration of the optical-lattice interaction, while in the latter case the beating of two tunneling frequencies is apparent. The data points are connected by lines for clarity.

The average momentum of the cesium atoms for κ=4.5 and ω=6. The circles obtained from the experiment by averaging over the momenta of all the atoms (not all atoms participate in the dynamic tunneling).

The beat frequency can be seen in the experimental data.

CONCLUSIONS

Particles can behave like waves!

They can undergo wave-like oscillations in phase space similar to those you hear when you tune your guitar.