Discussion - 3

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September 7, 2010
Outline

1. Motion in 1-D
2. Vectors (in 2D)
3. Motion in 2D
What is Mechanics

**Mechanics:** Describes the motion of objects

**Statics:** How do forces keep things stationary?

**Dynamics:** What is the motion resultant from a force?

**NOTE:** Statics and Dynamics are usually not distinguished.
Definitions

- **Particle:** A simplifying construct. Reduce every object to a point, when not concerned with its size.
- **Distance:** Total distance travelled by an object. **Scalar**
- **Displacement:** Distance from the starting point to the final point. **Vector**
- **(Instantaneous) velocity:** \( \vec{v} = \frac{d\vec{x}}{dt} \) Instantaneous rate of change of position. **Vector**
- **Speed:** \( |\vec{v}| \) Magnitude of velocity. **Scalar**

You should know these well
Definitions

- **Average velocity:** $\langle \vec{v} \rangle = \frac{\text{Overall displacement}}{\text{Time taken}} = \frac{\Delta \vec{x}}{\Delta t}$
  - **Vector**

- **Average speed:** $\langle |\vec{v}| \rangle = \frac{\text{Total distance travelled}}{\text{Time taken}} = \frac{\text{distance}}{\Delta t}$
  - **Scalar**

- **(Instantaneous) acceleration:** Instantaneous rate of change of speed: $\vec{a} = \frac{d\vec{v}}{dt}$
  - **Vector**

- **Average acceleration:**
  $\langle \vec{a} \rangle = \frac{\text{Overall change in velocity}}{\text{Time taken}} = \frac{\Delta \vec{v}}{\Delta t}$
  - **Vector**

An average requires two points (two instants of time) to compute. An instantaneous quantity requires just one.
Good notation helps:

- Initial position: $x_0$
- Final position: $x$
- Initial velocity: $v_0$
- Final velocity: $v$
- Acceleration: $a$
- Time: $t$
- Displacement: $\Delta x \equiv (x - x_0)$
- Average quantities: $\langle \rangle$
Relations

- \( x = x_0 + v_0 t + \frac{1}{2}at^2 \)
- \( v = v_0 + at \)
- \( v^2 = v_0^2 + 2a \Delta x \)

These are for constant acceleration only.
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Why?

- Many quantities come with magnitude and direction.
- **Offer!** Specify both in one package! Contact: Mathemagicians.

Two formats:
- **The intuitive way:** Specify magnitude, direction as a package. ($|\vec{v}|, \angle \vec{v}$)
- **The easier-to-handle way:** Specify distances along “standard”, “independent” directions.
Choosing “standard”, “independent” directions

The North ↔ South direction is independent of the East ↔ West. Why?
Choosing “standard”, “independent” directions

The North ↔ South direction is independent of the East ↔ West. Why?
They are perpendicular to each other!

Jargon: Orthogonal vectors ⟷ Perpendicular to each other.
Choosing “standard”, “independent” directions

- Pick two vectors along the two independent directions.
- Call one of these directions the $x$-axis and the other $y$-axis.
- Call the vectors along these directions as $\hat{x}$ and $\hat{y}$. We want only direction, so we set magnitude to 1.
- To get any vector, mix appropriate amounts of either vector.
- These orthogonal unit vectors are called basis vectors.
Mixing Basis Vectors

Recipe to get any vector we want: Mix basis vectors.

- $\vec{v} = 3\hat{x} + 4\hat{y}$ – a mix of 3 units of $\hat{x}$ and 4 units of $\hat{y}$.

**Extracting Components:**

- Check overlap with $\hat{x}$ and $\hat{y}$ separately. Possible because $\hat{x}$ and $\hat{y}$ don’t mix!
Ways of specifying a vector

- Magnitude + Angle made with some standard direction: $|\vec{v}|$ and $\angle \vec{v}$.
- Component form: As mixtures of $\hat{x}$ and $\hat{y}$: $\vec{v} = v_x \hat{x} + v_y \hat{y}$.

From Components to Polar Form:
- Magnitude: $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$
- Direction: $\theta = \angle \vec{v} = \arctan \left( \frac{v_y}{v_x} \right)$

From Polar to Components:
- $v_x = v \cos(\theta)$
- $v_y = v \sin(\theta)$
We’ll do it the component way

- Addition
- Scalar Multiplication
- Subtraction?
- Dot product – a measure of overlap.
Dot Product

Also known as: Scalar product, Inner product
A machine that takes two vectors and churns out a scalar.

- A measure of overlap:

\[
\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos (\angle \vec{b} - \angle \vec{a}) \quad (1)
\]

\[
\Rightarrow \quad \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = |\vec{b}| \cos (\angle \vec{b} - \angle \vec{a}) = \text{proj}_{\vec{a}} \vec{b} \quad (2)
\]

- Orthogonal vectors \( \Rightarrow \) No overlap. \( \hat{x} \cdot \hat{y} = 0 \)
- Magnitude of a vector \( \vec{v} \): \( |\vec{v}|^2 = \vec{v} \cdot \vec{v} \)
- Another way to calculate:

\[
\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y \quad (3)
\]
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Analyzing motion in 2-D

**Advantage of vector components:** Analyze the motions independently, as if they were two 1-D motions!

\[
\vec{v} = \vec{v}_0 + \vec{a}t
\]

\[
\begin{align*}
v_x &= v_{x0} + a_x t \\
v_y &= v_{y0} + a_y t
\end{align*}
\]

\[
\vec{x} = \vec{x}_0 + \vec{v}t + \frac{1}{2} \vec{a}t^2
\]

\[
\begin{align*}
x &= x_0 + v_{x0} t + \frac{1}{2} a_xt^2 \\
y &= y_0 + v_{y0} t + \frac{1}{2} a_y t^2
\end{align*}
\]
Projectile motion is 2-D motion with:

\[
\begin{align*}
a_x &= 0 \\
a_y &= -g \\
y_0 &= 0 \text{(usually)}
\end{align*}
\]

Trajectories are parabolas.
A great approximation to real projectiles.
When does it break? Does a satellite fall to the earth?