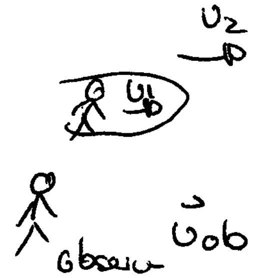
Special Relativity: velocities close to the speed of light c

classical



$\vec{v}_{ob} = \vec{v}_1 + \vec{v}_2$

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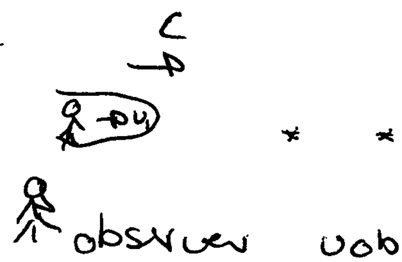
relativistic



$v_{ob} \neq \vec{c} + \vec{v}_1$

$v_{ob} = \vec{c}$

t_p L ob

t L_p ob

$$t = \gamma t_p$$

$$L = \frac{L_p}{\gamma}$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Experiment:

collision CM frame

observer lab frame

CM lab + CM collision

→ ← same → ←

→| not the same → ←

lab CM

Energy and momentum

Relativistic variables

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$\frac{v}{c}$

Total Energy: $E = KE + mc^2 = \gamma mc^2$

Kinetic Energy (KE), rest Energy (mc^2)

$$\Rightarrow KE = \gamma mc^2 - mc^2$$

$$= (\gamma - 1)\, mc^2$$

momentum:

$$p = \gamma m v$$

$$E = \gamma mc^2$$

algebra

$$\Rightarrow E^2 = p^2c^2 + (mc^2)^2$$

$p = \gamma m v$; $\beta = \frac{v}{c}$

$v = \beta c$

$(p = \gamma m \beta c)$

$= \gamma \beta mc$

photon: $E = pc$

$m = 0$ $\quad E = hf$

Units of Energy, momentum, mass

Energy:

$E = mc^2$ Proton mass $= 1.6726 \times 10^{-27}$ kg

$c = 2.9978 \times 10^8$ m/s

$E = (1.6726 \times 10^{-27}\ \text{kg})(2.9978 \times 10^8\ \text{m/s})^2$

$= 1.5032 \times 10^{-11}$ J $\qquad$ 1 eV $= 1.6 \times 10^{-19}$ J

≈ 938 MeV

k $= 10^3$
M $= 10^6$
G $= 10^9$
T $= 10^{12}$

mass: $m = \frac{E}{c^2} = \frac{938\ \text{MeV}}{c^2} = 938\ \text{MeV}/c^2$

$E^2 + p^2c^2 + (mc^2)^2$

momentum: Unit in $\left[\frac{1}{c}\right]$

mass unit $= \left[\frac{1}{c^2}\right]$

Lorentz transformation

Compare different collision systems →← →|

collider fixed target

center of mass Energy → lorentz invariant

$$E^2 = p^2c^2 + (mc^2)^2$$

↑ lorentz invariant Rest energy

$$(mc^2)^2 = E^2 - p^2c^2$$

$E_1\, p_1 \rightarrow \leftarrow E_2\, p_2$

$$mc^2 = \sqrt{E^2 - p^2c^2} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 c^2}$$

$$E_{CM} = \sqrt{s_{NN}} = mc^2$$

↑ per nucleon

LHC Pb + Pb

$\sqrt{s_{NN}} = 2.76$ TeV (per nucleon)

collider: $p_1 + p_2 = 0 \Rightarrow E_{CM} = E_1 + E_2$

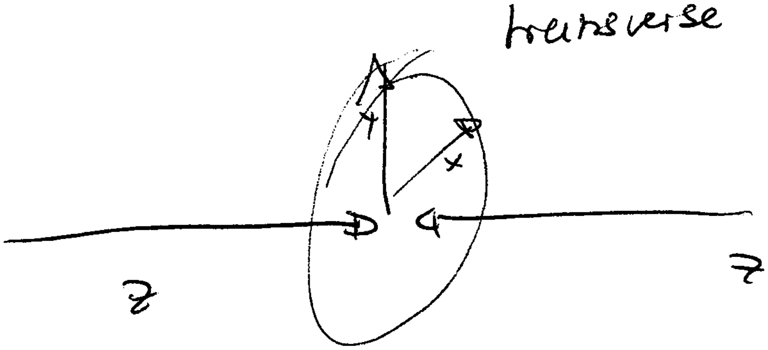
(sometimes $E + p$ $(E + pc)$

short version
$E = E$
$p = pc$
$m = mc^2$)

fixed target experiment

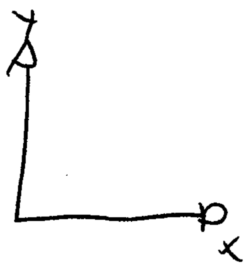
$$E_{CM} = [m_1^2 + m_2^2 + 2E_{1\,lab}\, m_2]^{1/2} \quad E_2 \text{ in Rest}$$

Coordinate system in collision



longitudinal

Beam direction



transverse direction

Momentum:

transverse direction : (Lorentz invariant)

transverse momentum : $p_t = \sqrt{p_x^2 + p_y^2}$

transverse mass : $m_t = \sqrt{p_t^2 + m_0^2}$ m_0 = Rest mass

$(x_t = p_t / p_t^{max})$

longitudinal direction : not Lorentz invariant

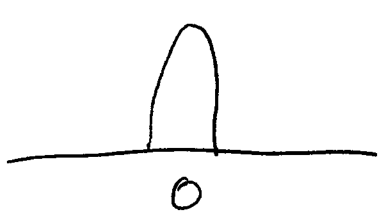
shape is the same

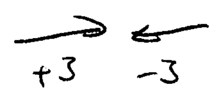
Rapidity : $y = \frac{1}{2} \ln\left(\frac{E + p_z}{E - p_z}\right) = \ln\left(\frac{E + p_z}{m_t}\right)$

$= \left(\tanh^{-1}\left(\frac{p_z}{E}\right)\right)$

before

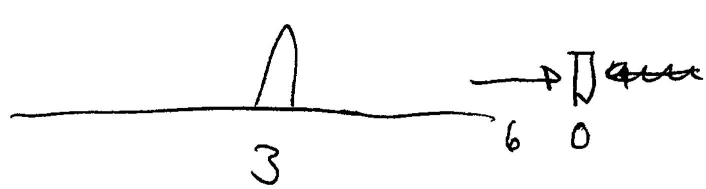
after





before

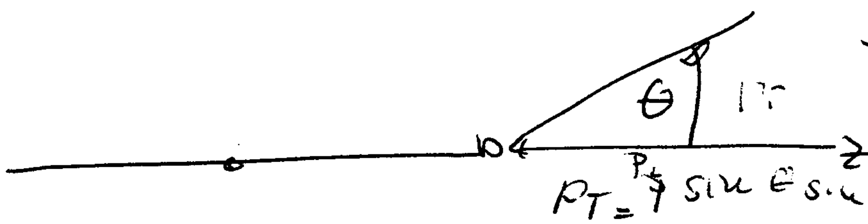
after



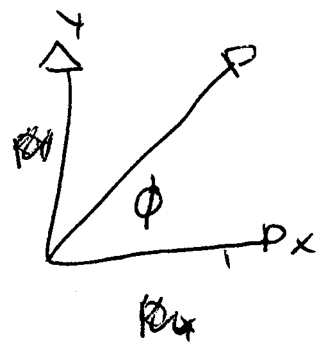
Rapidity has mass dependence

→ Pseudorapidity → no mass dependence
only angle

$$\eta = \frac{1}{2}\ln\left(\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}\right) = -\ln\left(\tan\frac{\theta}{2}\right)$$



Theta - angle



phi - angle
transverse angle
azimuthal angle

$$P_T = P\sin(\theta_{cm})$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{p_y}{p_x}$$