In medium sum rules for vector and axial-vector mesons

Paul Hohler
Cyclotron Institute, Texas A&M University

work in progress with R. Rapp
Chiral symmetry restoration

Lattice calculations clearly show the quark condensate approaching 0 at higher T, indicative of chiral symmetry restoration.

Question: Can chiral symmetry restoration be confirmed experimentally?

Direct measurement of the chiral condensate is not possible.

In medium sum rules for vector and axial-vector mesons may provide indirect means to study chiral symmetry restoration experimentally.
Sum Rules

• Theoretical tool used to study correlation functions.
• Technique is useful because it relates high energy to low energy physics.
  – i.e. relates non-perturbative effects to perturbative effects.
  – Applied to a wide range of topics, from hadron spectroscopy to hydrodynamics.

\[
\Pi^{\mu\nu}(q) = i \int d^4x \ e^{iq \cdot x} \langle T j^\mu(x) j^\nu(0) \rangle
\]

\[
\Pi(q^2) = \Pi(0) + \Pi'(0)q^2 + \frac{q^2}{\pi} \int ds \ \frac{\text{Im} \Pi(s)}{s^2(s-q^2-i\epsilon)}
\]

\[
\Pi(Q^2) = \sum c_n \left( \frac{Q^2}{Q^2} \right)^n
\]

Dispersion relation
Operator product expansion

Sum rules provide a way to determine the low energy operators, or use the low energy operators (determined independently) to gain information about the spectral function.
• QCD sum rules (with Borel transform, in vacuum):
  – Applicable to vector or axial vector channel individually.

\[
\frac{1}{M^2} \int ds \frac{ρ_v(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2} \left( 1 + \frac{α_s}{π} \right) + \frac{m_q⟨\bar{q}q⟩}{M^4} + \frac{1}{24M^4} \left( \frac{α_s}{π} G^{2}_{μν} \right) - \frac{56πα_s}{81M^6} ⟨O^V⟩
\]

\[
\frac{1}{M^2} \int ds \frac{ρ_A(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2} \left( 1 + \frac{α_s}{π} \right) - \frac{m_q⟨\bar{q}q⟩}{M^4} + \frac{1}{24M^4} \left( \frac{α_s}{π} G^{2}_{μν} \right) + \frac{88πα_s}{81M^6} ⟨O^A⟩
\]

Factorization statement (in vacuum): \[⟨O_4⟩ = ⟨O^V_4⟩_0 = ⟨O^A_4⟩_0 = \kappa⟨\bar{q}q⟩^2\]

Condensates are universal and can thus be determined from other processes.

Places constraints on the vector and axial-vector spectral functions separately.
• QCD sum rules (with Borel transform, in medium):
  – Condensates acquire a temperature dependence.
  – Higher twist operators are added
    • related to in medium density of hadrons.

\[
\frac{1}{M^2} \int ds \frac{\rho_V(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) + \frac{m_q \langle \bar{q} q \rangle_T}{M^4} + \frac{1}{24M^4} \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle_T - \frac{56\pi\alpha_s}{81M^6} \langle \mathcal{O}_4^V \rangle_T 
\]

+ twist-2 operators

Same structure for the axial vector QCD sum rule (twist-2 operators are chirally symmetric).

By studying the medium effects of the operators, one can learn something about the (axial) vector spectral function(s).
Weinberg type sum rules (in vacuum):
- Studies the difference between vector and axial meson spectral functions.
- Directly related to chiral symmetry breaking.
- Some can be derived from the QCD sum rules.

\[ \int ds \frac{\rho_V - \rho_A}{s^2} = \frac{1}{3} f_\pi^2 \langle r_\pi^2 \rangle - F_A \]  
Das, Mathur, and Okubo, 1967

\[ \int ds \frac{\rho_V - \rho_A}{s} = f_\pi^2 \]  
Weinberg, 1967

\[ \int ds \rho_V - \rho_A = -4m_q \langle \bar{q}q \rangle = 2f_\pi^2 m_\pi^2 \]  
Kapusta and Shuryak, 1994

\[ \int ds (\rho_V - \rho_A)s = f_\pi^2 m_\pi^4 - 2\pi \alpha_s \left( \frac{16}{9} \langle \mathcal{O}_4^{SB} \rangle \right) \]  
\[ \langle \mathcal{O}_4^{SB} \rangle = \langle \mathcal{O}_4 \rangle = \kappa \langle \bar{q}q \rangle^2 \]

The chiral condensate (and $f_\pi$) can be measured experimentally by detecting both the vector and the axial-vector spectral functions.
Weinberg type sum rules (in medium):

1st \[ \int ds \frac{\rho_V - \bar{\rho}_A}{s} = 0 \]

2nd \[ \int ds \rho_V - \bar{\rho}_A = -2m_q \langle \bar{q}q \rangle_T \]

3rd \[ \int ds (\rho_V - \bar{\rho}_A)s = -2\pi\alpha_s \left( \frac{16}{9} \langle \mathcal{O}_4^{SB} \rangle_T \right) \]

Chiral symmetry restoration can be observed by experimentally measuring the in-medium spectral functions for both the vector and axial vector mesons.
• Goal: To use both the QCD sum rules and the Weinberg sum rules in conjunction with data, to constrain the possible in medium effects of the vector and axial-vector spectral functions.

In vacuum: ALEPH data

Model (V & A)

Weinberg sum rules

Vacuum $\rho_V$ spectral function

Vacuum $\rho_A$ spectral function

In medium:

Model (V)

QCD sum rules

$\rho_V$ spectral function

Check Weinberg sum rule

Gain insight into the mechanism of chiral symmetry restoration!!

Model (A)

$\rho_A$ spectral function
• We are not the first to look into this:

• All showed a reduction of the continuum threshold.
• Most showed a reduction of the rho mass and a broadening of its width.
Vacuum

• Consider a phenomenological model of the spectral functions for both the vector and axial-vector mesons.
  – Constrain the parameters by the ALEPH data (τ decay) and the Weinberg sum rules (0-2).

• Key and novel features:
  – The rho spectral function comes from a calculation of a microscopic theory. This is left unchanged. Rapp et al (1999)
  – The continuum contribution is chosen to be the same from both the vector and axial-vector channels.
    • A continuous continuum is considered.
    • Pushes the continuum “threshold” to a higher energy.
  – Included the ρ’ resonance with a Breit-Wigner structure.
  – Agreement with Weinberg sum rules requires an excited axial vector resonance state.
Spectral functions in vacuum

Data from ALEPH (Barate et al. 1998)

Some parameters of interest

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho'$ mass</td>
<td>1.56 GeV</td>
</tr>
<tr>
<td>$\rho'$ width</td>
<td>0.32 GeV</td>
</tr>
<tr>
<td>$a_1$ mass</td>
<td>1.24 GeV</td>
</tr>
<tr>
<td>$a_1$ width</td>
<td>0.61 GeV</td>
</tr>
<tr>
<td>$a_1'$ mass</td>
<td>1.74 GeV</td>
</tr>
<tr>
<td>$a_1'$ width</td>
<td>0.20 GeV</td>
</tr>
</tbody>
</table>
In medium vector spectral function

- Values of the condensates change with medium effects.

- **Chiral condensate:**
  - Approximate lattice data with a Hadron Resonance Gas fit.

- **Four quark condensate:**
  - Assumed chiral restoration occurred at same temperature as chiral condensate.
  - Insured that the vector and axial-vector condensates were consistent with low temp expectations.

The in medium effects increase the OPE side of the sum rule.
In medium vector spectral function

- In medium effects on the $\rho$ spectral peak determined from microscopic calculation.  
  Rapp et al (1999)

- The continuum acquires no additional in medium effects.

- All other effects are implemented by shifting the mass and expanding the width of the $\rho'$. 

![Graphs showing spectral function behaviour](image)
In medium vector spectral function

The mass drop and width broadening of the $\rho'$ peak has same effect as the reduction of the continuum threshold!!

Higher resonances “melt” causing the appearance of continuum being extended to lower energies!!
In medium axial vector spectral function

• No calculation from a microscopic model axial vector channel.
  – Fewer constraints as compared with vector channel.

• Continuum acquires no medium effect.
• \(a_1'\) acquires a mass shift and width broadening.
• Medium effects of \(a_1\) modeled in different ways:
  – Simple mass shift and width broadening.
  – Two component model.

• QCD sum rules can constrain parameters, while Weinberg sum rules will select the preferred model.

Stay Tuned!
Conclusion and Summary

• Showed sum rules are a useful tool to study vacuum and in medium spectral functions.

• Constructed vector and axial-vector spectral functions in vacuum consistent with data and QCD and Weinberg sum rules.
  – Postulated the presence of an excited axial-vector resonant state.

• Examined the in medium vector spectral function using QCD sum rules
  – Showed that the $\rho'$ mass decreases and the width broadens.
  – Consistent with idea that the continuum threshold decreases.

• In medium axial-vector function will be examined to understand chiral symmetry restoration mechanism.