Consider “Simple Harmonic Motion” (SHM) along the $x$-axis about the origin.

\[ x = A \cos(\omega t + \phi) \]

and

\[ v = -A \omega \sin(\omega t + \phi). \]

At $t = 0$, $x = x_0$, and $v = v_0$, select the correct expression for the amplitude $A$.

A) \( A = x_0. \)

B) \( A = \frac{\omega}{v_0}. \)

C) \( A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}. \)

D) \( A = \sqrt{x_0^2 + \left(\frac{\omega}{v_0}\right)^2}. \)
This is the situation of two equations and two unknown.

Using \( \cos^2 \phi = \sin^2 \phi = 1 \), one may eliminate \( \phi \).
In particular,

\[
( A \cos \phi)^2 + ( A \sin \phi)^2 = x_0^2 + \left( \frac{v_0}{\omega} \right)^2 = A^2,
\]

\[
A = \sqrt{x_0^2 + \left( \frac{v_0}{\omega} \right)^2}.
\]

Answer C.

13.01-01\text{SHM: From } x_0, v_0 \text{ to } A \text{ and } \phi \text{ 2005-4-19}