Consider the mass-spring system shown. The setup is as follows. At equilibrium, the mass is located at the origin, (both spring 1 and 2 are in relaxed states). The forces exerted by the individual springs, and by both springs are respectively given by $F_1 = -k_1 x_1$, $F_2 = -k_2 x_2$, and $F = -k x$.

![Mass-spring system diagram]

$k = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$

The pair of relationships which describe the present set up is given by

A) \( x = x_1 = x_2 \) and \( F_1 - F_2 = F \).

B) \( x = x_1 + x_2 \) and \( F_1 - F_2 = F \).

C) \( x = x_1 = x_2 \) and \( F_1 + F_2 = F \).

D) \( x = x_1 + x_2 \) and \( F_1 + F_2 = F \).
Since there is a common origin, and the 3 axes, \( x_1, x_2 \) and \( x \), coincide.

Therefore the locations of the mass measured in terms of these 3 coordinates are the same; i.e., \( x = x_1 = x_2 \).

Let us check the signs of the forces.

First consider the case: \( x = x_1 = x_2 = a > 0 \).

By inspection, here \( F_1 < 0, F_2 < 0 \) and \( F < 0 \).

Here all 3 forces have the same sign.

From the figure, one sees that the relation: \( F_1 + F_2 = F \) is correct.

The equality \( x = x_1 = x_2 \) is valid throughout oscillations.

In turn the “same-sign feature”, and also the relation \( F = F_1 + F_2 \) are valid throughout oscillations.

Answer C.

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