Consider the compound mass-spring system shown (where (1) and (2) are the auxiliary diagrams). The setup for the compound system is as follows. At equilibrium, the mass is located at the origin (both spring 1 and 2 are in relaxed states). The forces exerted by the individual springs, and by both springs are respectively given by \( F_1 = -k_1 x_1 \), \( F_2 = -k_2 x_2 \), and \( F = -k x \). Assume the masses of the springs are negligible.

\[
\begin{align*}
\text{(1)} & \quad k = k_1 + k_2 \\
\end{align*}
\]

Determine the relationship among \( x_1 \), \( x_2 \) and \( x \) for the compound system.

A) \( x = x_1 - x_2 \) and \( F_1 = F_2 = F \).

B) \( x = x_1 + x_2 \) and \( F_1 = F_2 = F \).

C) \( x = x_1 - x_2 \) and \( F_1 + F_2 = F \).

D) \( x = x_1 + x_2 \) and \( F_1 + F_2 = F \).
The origin of the spring $k_2$ is displaced by $x_1$, so the position of the mass is the sum $x_1 + x_2$.

Therefore the locations of the mass measured in terms of these 3 coordinates are the same; i.e., $x = x_1 + x_2$.

Let us check the signs of the forces.

First consider the case: $x = x_1 + x_2 = a > 0$.

By inspection, here $F_1 < 0$, $F_2 < 0$ and $F < 0$.

Here all 3 forces have the same sign.

From the figure, one sees that the relation: $F_1 + F_2 = F$ is correct.

The equality $x = x_1 + x_2$ is valid throughout oscillations.

In turn the “same-sign feature”, and also the relation $F = F_1 + F_2$ are valid throughout oscillations.

Answer D.

13.02-05 Compound Springs II 2007-4-10