Given: Consider a current segment $\overline{CD}$ with current $I$. Let $\vec{B}$ be the magnetic field vector at $P$ due to this segment.

The direction and the magnitude of $\vec{B}$ are

A) into the page, $B = \frac{\mu_0}{4 \pi a} \hat{k}$.

B) into the page, $B = \frac{\sqrt{2} \mu_0}{4 \pi a} \hat{k}$.

C) out of the page, $B = -\frac{\mu_0}{4 \pi a} \hat{k}$.

D) out of the page, $B = -\frac{\sqrt{2} \mu_0}{4 \pi a} \hat{k}$. 

Consider $I\delta y$ at $y$ where $\sin \theta = \frac{a}{r}$, $\frac{\delta y}{r^2} = \frac{\delta \theta}{a}$. Its contribution at $P$ is

$$\delta B = \frac{\mu_0}{4\pi} \frac{I \delta y}{r^2} \sin \theta = \frac{\mu_0}{4\pi} \frac{I \delta \theta}{a} \sin \theta.$$ 

By inspection $I\delta \vec{y} \times r$ gives the direction of $\delta \vec{B}$ to be into the paper. So $\vec{B}$ at $P$ due to $\vec{CD}$ is into the page. Integrating from $C$ to $D$, one obtains at $P$

$$B = \int_{\pi/4}^{3\pi/4} \frac{\mu_0 I}{4\pi} \frac{d \theta}{a} \sin \theta = \frac{\sqrt{2}}{4\pi} \frac{\mu_0 I}{a}.$$ 

Answer B.

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