Two parallel-plate capacitors are shown below. Both are identical except one has a dielectric slab inserted into the gap between the plates. Both capacitors contain the identical charges on their plates.

\[ E_\kappa = \frac{E}{\kappa}, \quad u = \frac{U}{A d}, \quad u_\kappa = \frac{U_\kappa}{A d}, \quad U = \frac{Q^2}{2 C}, \quad \text{and} \quad U_\kappa = \frac{Q^2}{2 C_\kappa}. \]

Find the ratio of the energy densities \( \frac{u_\kappa}{u} \).

A) \( \frac{u_\kappa}{u} = 1 \)

B) \( \frac{u_\kappa}{u} = \kappa \)

C) \( \frac{u_\kappa}{u} = \frac{1}{\kappa} \)

\[
\begin{align*}
 u &= \frac{U}{A d}, \quad \text{and} \quad U = \frac{Q}{2 C} = \frac{\epsilon_0 E^2}{2} \\
 u_\kappa &= \frac{U_\kappa}{A d}, \quad \text{and} \quad U_\kappa = \frac{Q}{2 \kappa C} = \frac{\epsilon_0 E^2}{2 \kappa} = \frac{\epsilon_0 \kappa^2 E_\kappa^2}{2 \kappa} = \kappa \frac{\epsilon_0 E^2}{2} \\
 \frac{u_\kappa}{u} &= \kappa.
\end{align*}
\]

Answer B.

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