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Conductivity of Magnetic Metals*

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A Boltzmann equation appropriate for magnetic conductors is presented. The new element in this equation that distinguishes it from the normal case is the contribution from the spatial polarization of the electron wave functions introduced by the spin–orbit coupling. From this Boltzmann equation the wave number and frequency-dependent conductivity $\sigma(q, \omega)$ is calculated for the two limiting cases where the relaxation time is infinite: (a) $q=0$ in the presence of a uniform magnetic induction **B**, and (b) $\mathbf{B}=0$ but q is nonzero. Case (a) is relevant to understanding magneto-optic effects, and case (b) is relevant to understanding the surface impedance in the extreme anomalous skin region (EASR).

I. INTRODUCTION

It is well known that magnetic metals have much larger magneto-optic effects than nonmagnetic metals. The spin–orbit coupling combined with the net spin polarization of the conducting electrons produces these large magneto-optic effects.[1] Yet, at microwave frequencies, magnetic conductors are treated as normal conductors with reasonable results. In this paper we treat the spin–orbit effects consistently for all frequencies but neglect the effects of scattering and obtain expressions explaining why these effects are apparent only in the optical range. We do this by utilizing the fact that all of the spin–orbit effects are included in one physical property, namely the spatial polarization of the electron wave function.[2–4]

II. BOLTZMANN EQUATION

We consider the Boltzmann equation in the limit where the relaxation time of the electrons τ goes to infinity. If $f(\mathbf{k})$ is the distribution function of the electrons which gives the probability that a state $\mathbf{k}$ is occupied, the Boltzmann equation becomes

$$(\partial f/\partial t)+\nabla_r f\cdot\mathbf{v}+\nabla_k f\cdot(d\mathbf{k}/dt)=0. \quad (1)$$

In magnetic conductors

$$\hbar(d\mathbf{k}/dt)=e[\boldsymbol{\varepsilon}+(\mathbf{v}/c)\times\mathbf{B}], \quad (2)$$

where $\boldsymbol{\varepsilon}$ is the electric field and **B** is the magnetic induction.

The feature that distinguishes magnetic conductors from normal conductors is that the electron states, through the spin–orbit interaction, are spatially polarized, i.e., in a unit cell the electron charge from a given state does not coincide in space with the positive charge, producing a dipole moment. Such a dipole moment is exactly cancelled by electrons of the same **k**, but opposite spin, in nonmagnetic conductors. However, in magnetic conductors with a net spin polarization, such is not the case, and effects from the spatial polarization are visible. All of the so-called anomalous properties[5] introduced by the spin–orbit interaction can be described entirely in terms of this spatial polarization.

Because the polarization $\mathbf{P}(\mathbf{k})$ depends also on the spin of the state, namely $\mathbf{P}\uparrow(\mathbf{k})=-\mathbf{P}\downarrow(\mathbf{k})$, we treat each spin state separately and then sum up the effect of spin-up and spin-down. Since spin is no longer an eigenstate, spin-up means that the average value is up. The $\mathbf{P}(\mathbf{k})$ produces two effects: (1) in changing the energy $E(\mathbf{k})$ by interaction with the electric field seen by the electron in its rest frame, and (2) by adding a polarization current to the induced current. In the external $\boldsymbol{\varepsilon}$ and **B** fields

$$E(\mathbf{k})=E_0(\mathbf{k})+\mathbf{P}(\mathbf{k})\cdot\{\boldsymbol{\varepsilon}+[\mathbf{v}(\mathbf{k})/c]\times\mathbf{B}\}, \quad (3)$$

where E_0 is the energy in no external fields. The current contributed by an electron in the state **k** is

$$\mathbf{j}(\mathbf{k})=e\mathbf{v}(\mathbf{k})+[\partial\mathbf{P}(\mathbf{k})/\partial t]=e\mathbf{v}(\mathbf{k})+(e/\hbar)\{\boldsymbol{\varepsilon}+[\mathbf{v}(\mathbf{k})/c]\times\mathbf{B}\}\cdot\nabla_k\mathbf{P}(\mathbf{k}), \quad (4)$$

where

$$\mathbf{v}(\mathbf{k})=(1/\hbar)\nabla_k E(\mathbf{k}), \quad (5)$$

and $E(\mathbf{k})$ is given by (3). We note that $\mathbf{v}(\mathbf{k})$ is affected by the presence of $\mathbf{P}(\mathbf{k})$. Equations (1)–(5) constitute the set of equations to be solved to determine $\sigma(q, \omega)$.

III. THE CONDUCTIVITY

We now use the Boltzmann equation in the previous section to calculate the transport properties of magnetized solids. We chose a model of a magnetic conductor which has cubic symmetry, a spherical Fermi surface, and in which all spins are polarized in the same direction. In this simple model, the polarization of the state **k** will be in a direction perpendicular to both the magnetization **M** and **k**. Thus, $P(\mathbf{k})$ can be written as

$$\mathbf{P}(\mathbf{k})=A\mathbf{k}\times\mathbf{M}, \quad (6)$$

where A is a constant.

The two cases which we consider are: first, one in which $\mathbf{B}=0$, and $q\neq0$; and second, one in which $\mathbf{B}\neq0$ and $q=0$. The magnetization **M** is chosen to be in the positive z direction, and we investigate the effect of applying an electric field in the metal of the form

$$\boldsymbol{\varepsilon}(q,\omega)=\epsilon_0 \exp[i(q\cdot y-\omega t)]\hat{\mathbf{x}}, \quad (7)$$

where $\hat{x}$ is a unit vector in the x direction and q is the wavenumber. The conductivity is calculated using standard considerations.[6]

In the first case ($B=0$, $q\neq 0$), the nonvanishing elements of the conductivity tensor to first order in $P(\mathbf{k})$ are

$$\sigma_{ii}=\sigma_{ii}^0(q,\omega), \qquad \sigma_{xy}=2neP_0/\hbar k_0, \quad (8)$$

and in the second case ($B\neq 0$, $q=0$), they are

$$\sigma_{xx}=\sigma_{yy}=i\omega_p^2\omega/4\pi(\omega^2-\omega_c^2)(1+5BP_0/\hbar ck_0),$$

$$\sigma_{zz}=i\omega_p^2/4\pi\omega,$$

and

$$\sigma_{xy}=-\omega_c\omega_p^2/4\pi(\omega^2-\omega_c^2)(1+5BP_0/\hbar ck_0)+2neP_0/\hbar k_0. \quad (9)$$

In the above equations, $\sigma_{ii}^0(q,\omega)$ are the diagonal components of the conductivity tensor which result when $P=0$, ω_p is the plasma frequency, ω_c is the cyclotron frequency, P_0 is the maximum value of $\mathbf{P}(\mathbf{k})$, n is the number of electrons per unit volume, and k_0 is the Fermi wavenumber.

IV. DISCUSSION

The Boltzmann equation presented here has the same limitations as always, namely that the frequency ω be much less than that of interband frequencies. In that case, $\mathbf{P}(\mathbf{k})$ is independent of ω. At frequencies $\omega\rightarrow\infty$, it is known[7] that $\mathbf{P}(\mathbf{k})\rightarrow 0$. In general, one can show that when interband effects are present, $\mathbf{P}(\mathbf{k})$ becomes frequency dependent if we desire to use the form of $\sigma(q,\omega)$ derived in the previous section.

We note that the polarization contribution to $\sigma_{xy}(0,\omega)$ in Eq. (9) dominates only at high frequencies, explaining why only the magneto-optic effects are large. From Eq. (8), and the standard results in the extreme anomalous region,[6,8] we note that the ratio becomes

$$\sigma_{xy}(q,\omega)/\sigma_{xx}(q,\omega)\approx P_0 q/e. \quad (10)$$

Only in the impractical limit of q^{-1} of the order of interatomic spacings will this ratio be of the order of one. Under the usual conditions of ferromagnetic resonance, $q^{-1}\approx 100$ A and the ratio is of the order of 1%. We thus do not expect, off hand, that polarization effects will be important in microwave experiments in the EASR, but a more careful analysis is necessary to verify this.

It is important to emphasize that the discussion in this paper neglects collision effects. At microwave frequencies collision effects are important, except for pure materials and low temperatures where the mean free path of the electrons becomes much greater than the penetration depth—the EASR. The discussion in this paper is applicable under those conditions.

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