Holographic visualization of laser wakefields

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Holographic visualization of laser wakefields

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Abstract. We report ‘snapshots’ of laser-generated plasma accelerator structures acquired by frequency domain holography (FDH) and frequency domain shadowgraphy (FDS), techniques for visualizing quasi-static objects propagating near the speed of light. FDH captures images of sinusoidal wakes in mm-length plasmas of density $1 < n_e < 5 \times 10^{18}$ cm$^{-3}$ from phase modulations they imprint on co-propagating probe pulses. Changes in the wake structure (such as the curvature of the wavefront), caused by the laser and plasma parameter variations from shot to shot, were observed. FDS visualizes laser-generated electron density bubbles in mm-length plasmas of density $n_e \gtrsim 10^{19}$ cm$^{-3}$ using amplitude modulations they imprint on co-propagating probe pulses. Variations in the spatio-temporal structure of bubbles are inferred from corresponding variations in the shape of ‘bullets’ of probe light trapped inside them and correlated with mono-energetic electron generation. Both FDH and FDS average over structural variations that occur during propagation through the plasma medium. We explore via simulations a generalization of FDH/FDS (termed frequency domain tomography (FDT)) that can potentially record a time sequence of quasi-static snapshots, like the frames of a movie, of the wake structure as it propagates through the plasma. FDT utilizes several

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probe–reference pulse pairs that propagate obliquely to the wake, along with tomographic reconstruction algorithms similar to those used in medical CAT scans.

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1. Introduction

Relativistic interactions of intense laser pulses with underdense plasma underlie applications such as laser–plasma accelerators ([1–3] and references therein), fast ignition of laser fusion [4, 5] and generation of ultrashort x-ray pulses [6, 7]. These interactions create spatio-temporal electron density structures \( n_e(r, \phi, \zeta, z) \) (e.g. Langmuir waves, ‘bubbles’ [8] and ionization fronts) that vary with radial distance \( r \) from, and azimuthal angle \( \phi \) around, the propagation axis, distance \( \zeta \) behind the drive pulse and drive pulse propagation distance \( z \) into the plasma. Because of their microscopic size and luminal velocity, these structures eluded direct single-shot visualization in the laboratory. Consequently, knowledge of such structures derives mostly from computer simulations, which usually approximate and simplify the initial profiles of the laser pulse and plasma and can become prohibitively time-consuming and expensive for long interaction lengths.

In a previous paper [9], we reported single-shot visualization of quasi-static laser wakefield accelerator structures with resolution better than plasma wavelength \( \lambda_p \) using frequency domain holography (FDH) [10, 11]. Pump pulses (30 fs and 10 or 30 TW) propagating in plasma of density \( 1 \lesssim n_e \lesssim 5 \times 10^{18} \text{ cm}^{-3} \) resonantly generated sinusoidal wakes, but produced few relativistic electrons. Individual wake oscillations extending 10–15 periods behind the pump, details of their radial structure and co-propagating ionization fronts were then reconstructed by fast Fourier transformation (FFT) of phase modulations \( \Delta \phi_{pr}(r, \zeta) \) imprinted on a chirped probe pulse that overlapped and co-propagated with the plasma structure, providing nearly real-time feedback for experimental optimization. This represented a significant advance over prior direct measurements of laser-generated plasma structures with sub-\( \lambda_p \) resolution [12–14] using frequency-domain interferometry [15], in which a probe pulse measured local electron density \( n_e(\zeta) \) at only a single time delay \( \zeta/c \) behind the drive pulse for each shot. In this approach, wake
structure was accumulated painstakingly by probing a different $\zeta/c$ on each subsequent shot, resulting in an average over (often substantial) shot-to-shot variations of the laser-generated structure. FDH, by contrast, yielded ‘snapshots’ of the plasma structure created by a single pump pulse.

Here, we present FDH methodology and results in greater detail than was possible in a brief Letter [9]. In addition, we scale previous experiments to denser ($n_e \geq 10^{19}$ cm$^{-3}$) plasma. At this density, pump pulses self-focus and self-steepen [16, 17] to sufficient intensity that they produce plasma ‘bubbles’ [8] capable of capturing and accelerating copious plasma electrons to relativistic energy, often mono-energetically [16]–[26]. We find that bubbles in such dense plasma modulate co-propagating probe light primarily by refraction, producing intense optical ‘bullets’ in the reconstructed probe amplitude $|E_{pr}(r, \zeta)|$ imaged from the gas jet exit, while radially scrambling probe phase $\Delta \phi_{pr}(r, \zeta)$. Since useful information about bubble structure resides in the probe amplitude rather than the phase, we call the modified technique frequency domain shadowgraphy (FDS) [27]. Variations in the structure of the optical bullets (and indirectly of the plasma bubbles) are correlated with the generated electron spectrum. Finally, we explore, through simulations, a generalization of FDH/FDS (that we call frequency domain tomography (FDT)) that can potentially record a time sequence of quasi-static snapshots, like the frames of a movie, of the wake structure as it propagates through the plasma. FDT utilizes several probe–reference pulse pairs that propagate obliquely to the wakefield, along with tomographic reconstruction algorithms similar to those used in medical computer-aided tomography (CAT) scans.

2. Experiments

2.1. Frequency domain holography (FDH)

FDH images quasi-static structures propagating near the speed of light $c$ through a transparent medium from the phase modulation they imprint on a long, wide probe pulse that co-propagates with and illuminates the entire object at once, like the ‘object’ beam of conventional holography (see figure 1). Interference of this probe with a co-propagating, temporally separate ‘reference’ pulse on a detector encodes the object’s phase structure, which is subsequently ‘read’ to reconstruct the object, completing the analogy with conventional holography.

2.1.1. The plasma object. An He plasma containing a luminal-velocity plasma wakefield (the ‘object’) was created by focusing a $\sim$1 J, 800 nm, 30 fs pulse from the HERCULES laser system [28] into a supersonic He gas jet with an $f/13$ off-axis parabolic mirror (see figure 2), resulting in a typical pump spot radius of $\sim$25 $\mu$m at the jet entrance. Images of moving plasma objects $n_e(r, \phi, \zeta, z)$ reported here are cross-sections at fixed $\phi$, so we shall drop the argument $\phi$ hereafter. Changes in the object’s shape with $z$ can occur because of longitudinal variations in gas jet density and/or drive pulse intensity, and/or because of inherent instabilities in the propagating plasma object. Since FDH averages over such variations, the sharpest images are recovered for quasi-static objects that experience minimal longitudinal variations.

Phase variations $\Delta \phi_{pr}(r, \zeta)$ that the object imprints on the probe are obtained from its refractive index profile $\eta(r, \zeta, z) = [1 - \omega_p^2(r, \zeta, z)/\omega_{pr}^2]^{1/2}$, where $\omega_p = (n_e e^2 / \gamma e_0 m)^{1/2}$ is the plasma frequency corresponding to local electron density $n_e(r, \zeta, z)$, $\gamma = (1-v^2/c^2)^{1/2}$ is the relativistic Lorentz factor, and $e$, $m$ and $v$ are the electron charge, rest mass and
Figure 1. Schematic FDH configuration for imaging the laser wakefields. Two chirped picosecond pulses (reference and probe) co-propagate with the pump. Phase alterations imposed on the trailing probe by the wake are encoded in an FD interferogram, shown at the bottom with (upper) and without (lower) a pump, recorded by a CCD camera at the detection plane of an imaging spectrometer with $\sim 12$ pixels/fringe. Fourier transformation of these data recovers wake structure.

oscillation velocity, respectively. For a tenuous fully ionized plasma, $\eta^{-1} \approx \omega_p^2 / 2 \omega_{pr}^2$, so $\eta^{-1}$ is proportional to $n_e(r, \zeta, z)$—i.e. $\eta(r, \zeta, z)$ is a replica of the object.

2.1.2. Reference and probe pulses. Approximately 10% of the energy of the driving pulse was split off prior to compression for a diagnostic pulse, which was independently compressed to 30 fs and then subdivided into reference and probe pulses. In their original FDH work, Le Blanc et al [10] produced the latter in a modified Michelson interferometer, which proved sensitive to small misalignments and yielded probe and reference pulses with different temporal and spatial profiles. Matlis et al [9] developed a more robust, compact configuration that resembles a Fabry–Perot interferometer (see figure 2), and produced nearly identical reference and probe pulses. The incident 800 nm diagnostic pulse first up-converted to 400 nm in a $\sim 200 \mu m$ KDP crystal, thin enough to preserve its wide bandwidth. The nearly undepleted 800 and 400 nm pulses then passed through 2–3 cm of fused silica, in which they separated temporally by $\tau \approx 3$ ps by group-velocity (GV) walk-off. The temporally advanced 800 nm pulse then up-converted in an identical KDP crystal, generating a second 400 nm pulse collinear with the first, of identical spatial profile and advanced in time by $\tau \approx 3$ ps. The 400 nm pulses recombined collinearly with the pump through a 2.5 cm thick high reflector for 800 nm with high transmission and group velocity dispersion (GVD) at 400 nm. Both 400 nm pulses chirped to $\tau_{pr} \sim 1$ ps duration upon transmission through this optic, establishing the length $c \tau_{pr}$ of the object that was illuminated for FDH imaging. By inserting additional dispersive glass into the
2.1.3. Recording frequency domain (FD) holograms. By its interacting with the object over length $L$, the probe acquires a phase shift

$$\Delta \phi_{pr}(r, \zeta) = \frac{2\pi}{\lambda_{pr}} \int_0^L [1 - \eta(r, \zeta, z)] dz \approx \frac{e^2 \lambda_{pr}}{mc^2} \int_0^L n_e(r, \zeta, z) \gamma(r, \zeta, z) dz$$

with respect to the reference pulse, thus encoding the object’s structure, where the last expression holds for tenuous plasma. From equation (1), if the object’s instantaneous structure $\eta(r, \zeta, z)$ or $n_e(r, \zeta, z)$ evolves with $z$, the probe phase imprint averages these changes. Even for a non-evolving object, GV walk-off between the 400 nm probe pulse and the wake propagating at the GV of the 800 nm drive pulse causes longitudinal averaging. As a rough criterion, the plasma wakes and probe should walk-off less than $\lambda_p/4$ to avoid blurring the sub-$\lambda_p$ structure, limiting interaction to $L = \lambda_p^3/4\lambda_{pu}\Delta \lambda$, or $L \sim 1$ mm for $n_e \sim 10^{19}$ cm$^{-3}$. When structural
evolution and GV walk-off are negligible, equation (1) simplifies to

$$\Delta \phi_{pr}(r, \zeta) \approx \frac{2\pi[1 - \eta(r, \zeta)]L}{\lambda_{pr}} \approx \frac{e^2\lambda_{pr} L n_e(r, \zeta)}{mc^2 \gamma(r, \zeta)},$$

(2)

where, again, the last expression holds for tenuous plasma. Equation (2) is often adequate for estimating the object’s structure directly from the measured probe phase shift.

After the interaction region, a dichroic mirror separated the probe and reference pulses from the transmitted drive pulse; then a lens imaged them from the exit plane of the plasma onto the entrance slit of an imaging spectrometer, which selected a constant \( \phi \) cross section (see figure 1). To record an orthogonal cross section for three-dimensional (3D) imaging, a beam splitter could direct half of the probe–reference energy to a second spectrometer with an orthogonal entrance slit. Transverse image resolution was limited by the high intensity of the beam splitter could direct half of the probe–reference energy to a second spectrometer with an orthogonal entrance slit. Transverse image resolution was limited by the high intensity of the transmitted pump, which forces the dichroic and imaging optics to be placed tens of cm from the gas jet to avoid optical damage. For the images presented here, an optic with \( f^* = 8 \) yielding a resolution close to the theoretical limit \( f^* \lambda_{pr} \sim 3 \mu m \) was used.

The signal recorded at the detection plane of the spectrometer has the form \[10, 15\]

$$S(r, \omega) = |E_{pr}(r, \omega)|^2 + |E_{ref}(r, \omega)|^2 + E_{pr}^*(r, \omega) E_{ref}(r, \omega) + E_{pr}(r, \omega) E_{ref}^*(r, \omega)$$

$$= |A_{pr}(r, \omega)|^2 + |A_{ref}(r, \omega)|^2 + 2 A_{pr}(r, \omega) A_{ref}(r, \omega) \cos[\omega_0 \tau + \Delta \phi_{pr}(r, \omega)],$$

(3)

where \( E_{ref}(r, \omega) = A_{ref}(r, \omega) \exp[i \phi_{ref}(r, \omega)] \) and \( E_{pr}(r, \omega) = A_{pr}(r, \omega) \exp[i \phi_{pr}(r, \omega) + i \omega \tau] \) denote the complex electric fields of the reference and probe pulses, respectively, as functions of transverse position \( r \) along the spectrometer slit and frequency \( \omega, \tau \) is the time delay of the probe from the reference, and \( \Delta \phi_{pr}(r, \omega) = \phi_{pr} - \phi_{ref} \) is the phase difference between the probe and the reference caused by interaction of the former with the plasma object. In the absence of a plasma object, \( \Delta \phi_{pr} = 0 \) and hence \( r \)-independent frequency-domain interference fringes \( \cos \omega_0 \tau \) with period \( 2\pi / \tau \) are observed (see the lower interferogram in figure 1). When the plasma object is present, \( \Delta \phi_{pr} \) is given by equations (1) and (2), so distorted fringes that encode the object’s structure are observed (see the upper interferogram in figure 1). CCD pixel density, spectrometer dispersion and \( \tau \) are chosen such that 10–15 pixels record each period of the interferogram, ensuring adequate resolution when recovering images.

2.1.4. Reading FD holograms. Whereas conventional holograms are read by diffracting a laser beam from the exposed recording medium, we read FD holograms electronically by a Fourier transform (FT) procedure. Firstly, the complete probe electric field \( E_{pr}(\omega) = |E_{pr}(\omega)| \exp[i \phi_{pr}(\omega) + i \Delta \phi_{pr}(\omega)] \) is reconstructed in the FD from FDH data at each transverse position \( r_0 \). Secondly, Fourier transformation of \( E_{pr}(\omega) \) yields the time domain field \( E_{pr}(\xi) = |E_{pr}(\xi)| \exp[i \phi_{pr,(\xi)}(\xi) + i \Delta \phi_{pr}(\xi)] \). Finally, the temporal phase perturbation \( \Delta \phi_{pr}(\xi) \) yields electron density profile \( n_e(\xi) \) at each \( r \) via equations (1) and (2). Along with \( \Delta \phi_{pr}(r, \xi) \), FT of \( E_{pr}(r, \omega) \) simultaneously outputs temporal probe amplitude \( |E_{pr}(r, \xi)| \). For weakly refracting plasma, however, \( |E_{pr}(r, \xi)| \) at \( z = L \) hardly changes from its incident profile and thus conveys no information about the plasma structure. In strongly refracting, denser plasma, on the other hand, informative new features can appear in \( |E_{pr}(r, \xi)| \), creating a shadowgram at \( z = L \). This case is discussed in section 2.3.

Reconstruction of \( E_{pr}(r_0, \omega) \) begins with a lineout \( S(r_0, \omega) \) of an FD hologram (see figure 3(a)). Fourier transformation of \( S(r_0, \omega) \) yields \( \tilde{S}(r_0, \tau) \) consisting of a central peak at

Figure 3. Reconstructing the probe electric field. (a) Lineout of recorded intensity $S(r_0, \omega)$ at one transverse position $r_0$. (b) Fourier-transformed intensity $\tilde{S}(r_0, t)$; the dashed box highlights the peak corresponding to $\text{FT} (E_{pr}(\omega)E_{ref}^*(\omega))$, which is subsequently windowed and inverse Fourier-transformed back to FD to isolate the cross term $E_{pr}(\omega)E_{ref}^*(\omega) = |E_{pr}(\omega)||E_{ref}(\omega)|\exp[i\Delta\phi_{pr}(\omega)]$. Normalizing this to the separately measured (c) power spectrum $|E_{ref}(\omega)|$ of the reference pulse and (d) FD interference pattern of the chirped probe with short pump pulse, which measures $\phi_{chirp}(\omega)$, we obtain the complete probe electric field $E_{pr}(\omega) = |E_{pr}(\omega)|\exp[i\Delta\phi_{pr}(\omega) + \phi_{chirp}(\omega)]$.

t = 0 corresponding to the FT of $|E_{pr}(\omega)|^2 + |E_{ref}(\omega)|^2$, and side peaks at $t = \pm \tau$ corresponding to the FT of $E_{pr}^*(\omega)E_{ref}(\omega)$ and $E_{pr}(\omega)E_{ref}^*(\omega)$, respectively (see figure 3(b)). One side peak is windowed (figure 3(b), dashed box) and then inverse Fourier-transformed, yielding $E_{pr}(\omega)E_{ref}^*(\omega) = |E_{pr}(\omega)||E_{ref}(\omega)|\exp[i\Delta\phi_{pr}(\omega)]$. This expression is divided by the separately measured reference power spectrum $|E_{ref}(\omega)|$ (figure 3(c)). Finally, probe chirp $\phi_{chirp}^{(pr)}(\omega)$ is measured independently by a method such as the one shown in figure 3(d), in which the chirped 400 nm probe interferes in the FD with a compressed ($\sim 30$ fs) 400 nm reference pulse. In principle, the measurements in figures 3(c) and (d) should be performed on each shot and at each $r$. In practice, we found $|E_{ref}(\omega)|$ and $\phi_{chirp}^{(pr)}(\omega)$ to be sufficiently stable and uniform that a single, spatially averaged measurement sufficed.

2.2. Holographic images of laser wakefields

Figure 4(a) shows an FDH image of a wake produced by a pulse of peak power 30 TW and vacuum focused intensity $\sim 3 \times 10^{18}$ W cm$^{-2}$ in a plasma of density $n_e = 2.7 \times 10^{18}$ cm$^{-3}$ at the jet centre, measured independently by transverse interferometry. The image appears both as a 3D false colour plot of $\Delta\phi_{pr}(r, \zeta)$ over the ranges $-60 < r < 60$ mm and $0 < \zeta < 0.4$ ps, and
Figure 4. (a) Single-shot image of a wakefield produced by a 30 TW laser pulse in plasma of electron density $n_e = 2.7 \times 10^{18} \text{ cm}^{-3}$. The colour surface shows a phase change $\Delta \phi_{pr}(r, \zeta)$ of the probe pulse. The grey-scale image is the projection onto a plane. A large index step induced by the ionization front has been subtracted to emphasize the oscillatory wake structure. (b) Electron density snapshot from the WAKE simulation taken near the gas jet centre in the region near the axis (the helium gas is fully ionized there). The peak electron density perturbation in the first three periods is about $2n_e$. The transverse dimension of the images is 120 $\mu$m, while longitudinally they represent a time span of 0.4 ps. The largest $\Delta \phi_{pe}(r, \zeta)$ (white colour in panel (a)) corresponds to 22% refractive index perturbation of the background plasma (yellow in panel (a)) averaged over the interaction length.
oscillate relativistically, causing strongly driven, nonlinear wakes \([31, 32]\). The wavefronts curve because as plasma wave frequency increases, the curvature of the wake fronts from nearly flat profiles immediately behind the pump to electron Langmuir waves. Secondly, the transverse FWHM of the peaks is \(\approx 20 \, \mu\text{m}\), in excellent agreement with the self-focused spot size of the pump pulse. Thirdly, the progressively increasing curvature of the wake fronts from nearly flat profiles immediately behind the pump to concave fronts with radius of curvature \(r_c \approx 60 \, \mu\text{m}\) after six oscillations agrees with simulations of strongly driven, nonlinear wakes \([31, 32]\). The wavefronts curve because as plasma wave amplitude \(\delta_0 \equiv |\delta n_e(r = 0)/n_{e,\text{max}}|\) approaches unity on axis, electrons making up the wave oscillate relativistically, causing \(\omega_p(r = 0)\) to decrease by \(\sqrt{\gamma} \) relative to its off-axis value. For mildly relativistic wakes, theory and simulations \([33]\) suggest that curvature increases with \(\zeta \approx 0.45\zeta(\delta_0/w_0)^2\). Here, \(\delta_0\) refers to the amplitude of the first density maximum behind the pump and \(w_0\) to the transverse radius of the wake. Analysis of wavefront curvature in figure 4 yields \(\delta_0 \approx 0.2\). Fourthly, and finally, increasing phase front curvature is correlated closely with progressive growth in the amplitude of density and probe phase perturbations. In the simulation of figure 4(b), density perturbations increase in a slightly off-axis annulus, whereas on axis their amplitude does not change. In the FDH image of figure 4(a), the dip on axis is not resolved, but the growth in peak wave amplitude agrees closely with the simulation in most other respects.

The enhancement of wake amplitude observed in the experiment and simulation can be understood by viewing the nonlinear plasma wake as a self-consistent plasma channel with a radially dependent plasma frequency. Whereas in the conventional plasma channel this dependence arises from the density gradient, in our case it is caused by the gradient of the relativistic \(\gamma\) of the plasma electrons. Earlier calculations \([34–37]\) have found that the off-axis wake amplitude can grow with distance behind the driver, and can even result in the electron injection into the channel. As the distance back from the driver grows, trajectories of electron fluid elements oscillating at close radial locations approach each other more and more narrowly, and the off-axis density perturbation monotonically increases. At some finite distance the trajectories cross \([13]\), and the wake breaks transversely \([31, 38]\). The simulation of figure 4(b) stops before this moment. Correlated growth in wave curvature and amplitude is thus a precursor of wave breaking and electron injection. Wavefront curvature can also help collimate an accelerated electron beam. FDH renders these important features of laser wakefields visible in the laboratory for the first time.

Two features of the \(\Delta \phi_{pr}(r, \zeta)\) data in figure 4 do not agree quantitatively with theoretical expectations for \(n_e(r, \zeta)\). Firstly, the amplitude of normalized \(\Delta \phi_{pr}(r, \zeta)\) oscillations is significantly smaller than the simulated amplitude of normalized \(n_e(r, \zeta)\) oscillations. For example, the first two peaks behind the pump in figure 4(a) have normalized amplitude \(|\Delta \phi_{pr}(r = 0, \zeta)/\Delta \phi_{pr}^{(0)}|_{\text{max}} \sim 0.06\), about three times smaller than the simulated normalized density perturbation \(\delta_0\). Here, \(\Delta \phi_{pr}^{(0)}\) represents the phase shift induced by background plasma. Analysis of \(\Delta \phi_{pr}(r, \zeta)\) using simulations that include a probe pulse, presented elsewhere \([9]\), shows that this discrepancy stems primarily from the non-uniform density of the gas jet.
measured by transverse interferometry, which causes the probe to average longitudinally over wakes of varying frequency and amplitude, as expressed by equation (1). Nevertheless, \( \Delta \phi_{pr}(r, \zeta) \) faithfully reproduces the structure of \( n_e(r, \zeta, z_0) \) near the jet centre \( z_0 \) in most other respects. Secondly, the images in figure 4 show erratic structure near \( \zeta = 0 \) that does not correspond to the expected wake structure \( n_e(r, \zeta) \). One cause of this false structure is interference of the radiation at \( \lambda \sim 400 \) nm, produced by the pump via relativistic second-harmonic generation or white-light continuum generation, with the probe and the reference, resulting in false structure near \( \zeta = 0 \) upon reconstruction [39].

Figures 5(a) and (c) present grey-scale FDH images of wakes generated by pumps of power 10 and 30 TW, respectively, in plasma of lower (higher) density \( (n_e = 1.5 \text{ and } 4.1 \times 10^{18} \text{ cm}^{-3}, \text{respectively, in the doubly ionized He region}) \) compared to figure 4. No background was subtracted, so regions of neutral He gas (black), single-ionized He (diffuse grey halo) and doubly ionized He (white) are clearly visible. The phase shift \( \Delta \phi_{pr} \) in non-oscillatory portions of the doubly ionized region is double that in the singly ionized halo, accurately reflecting the twofold step in \( n_e \). Figures 5(b) and (d) present corresponding false-colour plots of \( \Delta \phi_{pr}(r, \zeta) \) with phase shift from non-oscillating background plasma subtracted. In the doubly ionized regions, nine (figure 5(a)) or 14 (figure 5(c)) wake oscillations are observed in an 800 fs interval, yielding wake periods \( 2\pi/\omega_p = 88 \text{ or } 57 \) fs, respectively, again in excellent agreement with the values expected for the corresponding densities \( n_e \). In figures 5(c) and (d), wavefront curvature (and peak wave amplitude) both increase with \( \zeta \), as in figure 4, whereas in figures 5(a) and (b) wavefronts remain nearly flat (and attenuate slowly in amplitude with \( \zeta \)), a consequence of the lower drive pulse power and oscillation amplitude compared to figures 4 and 5(c) and (d). The maximum amplitude of \( \Delta \phi_{pr}(r, \zeta) \) oscillations is several times smaller than the amplitude of oscillations in \( n_e(r, \zeta) \) expected from simulations. Taken together, figures 4 and 5 illustrate both the strengths (fast, faithful single-shot imaging of most aspects of the wake structure) and limitations (underestimate of plasma oscillation amplitude when the plasma structure evolves significantly, false structure from pump-generated radiation) of FDH imaging.

2.3. Frequency domain shadowgraphy (FDS)

For the experimental parameters described in section 2.1.1, significant relativistic electron yield is observed only for \( n_e \geq 1.5 \times 10^{19} \text{ cm}^{-3} \) [23, 40], considerably higher than the densities at which FDH images described above and in [9] were obtained. Quasi-mono-energetic electron spectra are observed [23, 40] at these densities, suggesting that plasma bubbles form [8]. To visualize bubbles and correlate them with accelerated electrons, it was therefore necessary to extend experiments to densities \( n_e > 10^{19} \text{ cm}^{-3} \).

Several technical difficulties arose in scaling FDH to these densities. Firstly, the short-wavelength tail of the forward white light continuum at \( \lambda \sim 400 \text{ nm} \), generated by the self-phase-modulated pump pulse, became stronger than at lower density. Without filtering, its interference with co-propagating probe and reference light made FD holograms unreadable. A pinhole spatial filter was therefore inserted into the imaging system to suppress the background, while transmitting most probe and reference light, exploiting their contrasting propagation geometries. In addition, probe pulse power was increased nearly by a factor of 10. These two measures improved the signal-to-background ratio sufficiently so that FD holograms became readable, but residual background 400 nm light remained a source of noise in phase reconstructions. Secondly, probe phase shift \( \Delta \phi_{pr}(r, \zeta) \) exceeded \( 2\pi \) at many locations, and
Figure 5. Single-shot images of wakefields produced by (a, b) a 10 TW laser pulse in a plasma of density $n_e = 1.5 \times 10^{18}$ cm$^{-3}$ and (c, d) a 30 TW laser pulse in a plasma of density $n_e = 4.1 \times 10^{18}$ cm$^{-3}$. In the grey-scale images (a, c), the zero of the time scale corresponds to the peak of the pump pulse. Regions of neutral He gas (black), singly ionized He (diffuse grey halo) and doubly ionized He (white) are clearly visible. The $n_e$ values above refer to the doubly ionized He region, in which wake oscillations are visible. Colour images (b, d) are 3D graphs of the corresponding grey-scale data but with background plasma subtracted to highlight the wake, as in figure 4.

reached several times $2\pi$ at some locations, making phase unwrapping extremely difficult and sometimes impossible. Thirdly, the probe amplitude profile $|E_{pr}(r, \zeta)|$ at the gas jet exit plane was strongly altered by refraction. Dark regions in $|E_{pr}(r, \zeta)|$ further complicated phase unwrapping. Moreover, strong focusing and defocusing scrambled phase information radially. As a result, extracted $\Delta \phi_{pr}(r, \zeta)$ profiles were no longer simply or reliably related to $n_e(r, \zeta)$.

In view of these complications, we turned to FDS [27], which uses $|E_{pr}(r, \zeta)|$ to infer plasma structure instead of $\Delta \phi_{pr}(r, \zeta)$, for experiments at $n_e > 10^{19}$ cm$^{-3}$. Since amplitude and
Figure 6. (a) FDS snapshot $|E_{pr}(r, \zeta)|$ at $1.2 \times 10^{19}$ cm$^{-3}$; (b) FDS snapshot $|E_{pr}(r, \zeta)|$ at $2.4 \times 10^{19}$ cm$^{-3}$; (c) FDH snapshot $\Delta \phi(r, \zeta)$ at $1.2 \times 10^{19}$ cm$^{-3}$ for the same shot as (a). (d) FDH snapshot $\Delta \phi(r, \zeta)$ at $2.4 \times 10^{19}$ cm$^{-3}$ for the same shot as (b).

Phase are calculated together, no extra data analysis steps were required to obtain $|E_{pr}(r, \zeta)|$. Moreover, amplitude proved simpler, faster and more reliable to recover because it required no phase unwrapping and avoided the additional step of subtracting $\Delta \phi_{chirp}(r, \zeta)$, both of which were sources of noise and uncertainty in reconstructing $\Delta \phi_{pr}(r, \zeta)$.

2.4. Shadowgraphic snapshots of plasma bubbles

Pump pulses of $\sim 28$ TW peak power were used for experiments at $n_e > 10^{19}$ cm$^{-3}$. To enforce bubble formation and relativistic electron production, the pump focal radius was reduced from $\sim 25$ to $\sim 10$ $\mu$m at the jet entrance by introducing a deformable mirror before the $f/13$ focusing mirror. Figure 6(a) shows a shadowgraphic $|E_{pr}(r, \zeta)|$ snapshot at $n_e = 1.2 \times 10^{19}$ cm$^{-3}$, just below the threshold for relativistic electron beam production. Figure 6(b) shows a snapshot at $n_e = 2.4 \times 10^{19}$ cm$^{-3}$, above the threshold, for which $\sim 1$ nC of relativistic electrons was generated. Figures 6(c) and (d), respectively, show corresponding phase reconstructions $\Delta \phi_{pr}(r, \zeta)$ for the same shots. The latter required time-consuming two-dimensional (2D) phase unwrapping procedures to remove large discontinuities from the reconstructed phase profiles. Moreover, significant features of the $\Delta \phi_{pr}(r, \zeta)$ reconstruction depended on the unwrapping algorithm. The corresponding $|E_{pr}(r, \zeta)|$ reconstruction, on the other hand, proved insensitive to these details and thus provided a more robust, reliable signature of plasma structure in this regime.
Figure 7. $\Delta \phi (r, \omega)$ at $r = 0 \mu m$ (left) and $\Delta \phi (r, \omega)$ at $r = 40 \mu m$ (right).

The most prominent and consistent feature of the latter is a bright spot appearing at the front of the probe pulse profile, evident in both figures 6(a) and (b). These bright spots are short ($\sim 30$ fs) in duration and small ($\sim 10 \mu m$) in radius. They were invariably observed in shots at $n_e > 10^{19} \text{ cm}^{-3}$ that produced a strong relativistic electron beam (as in figure 6(b)). In addition, they were often observed in shots slightly below the threshold for relativistic electron production (as in figure 6(a)). Figure 7 compares the spectral phase at the radius $r \approx 0$ at which these bright spots appear (left) with the spectral phase of off-axis parts of the probe pulse (right). The latter retains the parabolic phase dependence of the incident chirped probe pulse. The former, by contrast, has transformed to a linear, unchirped spectral phase, demonstrating that the bright spot has been fully compressed by the plasma bubble. Such short, focused and compressed pulses resemble 3D spatio-temporal solitons in Kerr media, which are called ‘optical bullets’ in the soliton literature [41]. Based on this resemblance, we refer to these features as optical bullets. Spectral broadening (not shown) is observed at the radial location of the optical bullet, although the reconstructed longitudinal size of the optical bullets that appear in figures 6(a) and (b) is limited to $\Delta \xi \sim 30$ fs by the spectral bandwidth of the reference pulse. Similarly, the apparent radial size is limited by the resolution of the probe imaging system.

Analysis of $|E_{pr}(r, \xi)|$ using simulations based on the fully relativistic, quasi-static time-averaged particle-in-cell (PIC) code WAKE [30], to be presented elsewhere [27, 39], indeed demonstrates quantitatively that plasma bubbles reshape co-propagating probe light into such optical bullets. To understand bullet formation qualitatively, we can regard a pump-generated plasma bubble of radius $R_b$ as an optical cavity with an internal transverse refractive index profile that peaks at the centre of the bubble. Such a cavity focuses transversely, and compresses longitudinally, any probe light that began inside the bubble after it formed near the plasma entrance. If we assume a bubble of $R_b \sim 10 \mu m$ with parabolic internal density profile, we can estimate using standard parabolic waveguide formalism that it focuses and compresses probe light within focal length $f \sim R_b \lambda_p / 2 \lambda_{pr} \sim 0.2 \text{ mm}$ at $n_e = 10^{19} \text{ cm}^{-3}$, much less than the length
$L \sim 2$ mm of the jet. Thus a plasma bubble focuses and compresses probe light during jet transit, in agreement with WAKE simulations [27, 39]. Even with limited resolution, optical bullets are a robust signature of bubble formation both below (figure 6(a)) and above (figure 6(b)) the threshold for relativistic electron production.

2.5. Correlation of shadowgraphs with accelerated electrons

In each shot, energy spectra of the relativistic electron beams were measured with a 1 T dipole magnet with a minimum energy cutoff of 30 MeV. Relativistic electrons first appeared at $1.6 \times 10^{19}$ cm$^{-3}$, and were observed at all higher densities. Figure 8 shows electron spectra (left column) and corresponding FDS snapshots (right column). Four acceleration regimes could be identified as the density increased. At $n_e = 1.7 \times 10^{19}$ cm$^{-3}$ (figure 8(a)), electron beams with a low-divergence, Maxwellian energy spectrum with a sharp cutoff around 90 MeV are observed. Total charge is less than 100 pC, but trapped electrons are accelerated to high energy because of the long dephasing length. The FDS snapshot shows a clear optical bullet in the front edge of the probe pulse. At $n_e = 2.4 \times 10^{19}$ cm$^{-3}$ (figure 8(b)), total charge and angular divergence both increased approximately one order of magnitude. The transverse spread is perpendicular to the linear pump polarization and, thus, not driven by the laser electric field. It is possibly caused by out-of-plane betatron oscillations due to asymmetric off-axis injection driven by an asymmetric laser pulse intensity distribution [42]. The corresponding FDS snapshot shows several diffuse optical bullets indicative of filamented laser focus. At densities $n_e = 2.9 \times 10^{19}$ cm$^{-3}$ (figure 8(c)), mono-energetic electron beams with less than 1% energy spread and small transverse size are observed, as reported also by others under similar conditions [43]. An intense optical bullet at the front of the probe pulse profile, the brightest among the four regimes, is invariably correlated with such collimated mono-energetic electrons. Finally, on some shots throughout the density range $1.6 < n_e < 3.2 \times 10^{19}$ cm$^{-3}$, poly-energetic electron beams with several discrete energy peaks were sometimes observed, as displayed in figure 8(d). This might be due to periodic electron injection in the evolving bubble [44] or to trapping and acceleration of electrons in consecutive wake buckets. The FDS snapshot in figure 8(d), however, shows only one intense optical bullet, thus favouring the former explanation.

3. Snapshots to movies

3.1. Frequency Domain Streak Camera (FDSC)

Images produced by FDH and FDS are longitudinally averaged. Thus, if the plasma structure $n_e(r, \xi, z)$ evolves significantly as the drive pulse transits the plasma, the images blur. As a first step toward overcoming this limitation of FDH and FDS, we propose augmenting the collinear probe–reference pulse pair with a probe–reference pulse pair propagating at angle $\alpha$ to the pump. For example, figure 9 shows an oblique probe interacting with a plasma bubble, which as simulations show often evolves substantially during jet transit [17]. The phase ‘streak’ imprinted on the probe (figures 9(b) and (c)) chronicles the evolution of the bubble, which traverses a path across the probe pulse profile in the direction $\xi$ shown in figure 9(b). A change of bubble structure changes the width and depth of the phase streak as a function of $\xi$, which is recovered (figure 9(c)) as in conventional FDH. We therefore call this the FDSC. $\Delta \phi_{pr}(\xi)$ can help interpret a longitudinally averaged conventional FDH snapshot. Moreover, for nonzero $\alpha$,
transmitted pump and forward pump-generated radiations propagate away from the detection system, enabling the use of lower $f$-number, higher resolution imaging optics closer to the object than for conventional FDH and avoiding false structure in recovered images.

We simulated a phase streak for realistic experimental conditions using the PIC code WAKE [30] and the finite element code COMSOL\(^5\). WAKE simulated the axi-symmetric

\(^5\) The code is developed and supported by COMSOL, Inc. (1 New England Executive Park, Ste. 350, Burlington, MA 01803, USA).

Figure 9. Schematic of non-collinear FDH for characterizing the longitudinal evolution of a luminal velocity plasma bubble. (a) The pump (red), trailing bubble (white) and chirped probe overlapping at angle $\alpha$ in a gas jet; (b) the same at a later time, showing how the evolving bubble sweeps out a streak (grey) tilted at angle $\alpha/2$ from the leading edge of the probe; (c) schematic of the reconstructed probe phase shift $\Delta \phi_{pr}(\rho, \tau)$, where $\rho$ denotes distance perpendicular to the propagation axis, and $\tau$ time behind the leading edge, of the probe.

wake structure generated by a pump pulse (peak intensity $9.8 \times 10^{18}$ W cm$^{-2}$, $w_0 = 16 \mu$m and 800 nm) with a Gaussian radial and temporal profile focused at the entrance to a plasma of density $n_e = 1.5 \times 10^{19}$ cm$^{-3}$. Contours $n_e(r, \zeta, z_i)$ were generated at 12 pump propagation distances $z_i (1 < i < 12)$, encompassing various stages of bubble evolution. COMSOL solved Maxwell’s equations for a 400 nm probe pulse propagating obliquely through the plasma object and extracted the phase change $\Delta \phi_{pr}(\rho, \tau)$ induced on its electric field. Here $\rho$ denotes distance perpendicular to the propagation axis, and $\tau$ time behind the leading edge, of the probe pulse. Figure 10(a) shows two of the 12 wake structures: $i = 1$, immediately after the pump entered the plasma and formed a sinusoidal wake (figure 10(a.1)), and $i = 6$, after the pump propagated half way through the plasma, self-focused, self-steepened and formed a bubble (figure 10(a.2)). Figure 10(b) shows corresponding sections of the phase streak: a shallow smooth profile from averaging peaks and valleys of the sinusoidal wake (figure 10(b.1)) and a deeper, more structured profile after the bubble forms (figure 10(b.2)). The contrast between these two sections of the phase streak enables us to identify the onset of bubble formation. Figure 10(c), a lineout of $\Delta \phi(\xi)$ along $\xi$, further illustrates the dramatic change in phase that accompanies bubble formation.

3.2. Frequency domain tomography (FDT)

FDSC by itself integrates phase along $\xi$. Thus the bubble structure at each $z_i$ remains unknown. To overcome this drawback, we propose FDT. In CAT, projections of a stationary 3D object are recorded on 2D image surfaces at multiple viewing angles. From these data, cross-sectional images of the 3D object in various planes are then reconstructed using filtered back projection algorithms developed in the 1970s [45]. We can achieve an analogous goal—reconstruction of cross-sectional images of a moving, evolving plasma object at various locations $z_i$—by employing FDSC at multiple viewing angles.

FDT and CAT differ in that the former records accumulated phase of a moving object and the latter of a stationary object. To connect FDT and CAT, consider a reference line $\xi_\perp$.
Figure 10. Simulation of the frequency domain streak camera. (a) Wake profiles \(n_e(r, \zeta, z_i)\) at two pump propagation distances \(z_1\) and \(z_6\) into plasma, simulated by the PIC code WAKE: (a.1) sinusoidal wake at \(z_1\) immediately after the pump enters the plasma; (a.2) fully formed bubble at \(z_6\), half way through plasma. (b) Corresponding ‘frames’ of phase streak imposed by structures \(n_e(r, \zeta, z_{1,2})\) on a probe pulse after propagating through them at right angles to the pump. These ‘frames’ show clear contrast between the wakefield and bubble stages. (c) A lineout of phase change \(\Delta \phi_{pe}(\xi)\) imposed on the probe along the axis \(\xi\) of the phase streak, garnered from 12 consecutive frames corresponding to pump propagation distances \(z_1\) through \(z_{12}\). A large change in \(\Delta \phi_{pe}(\xi)\) in frame 5 identifies the point of bubble formation.
Figure 11. Simulation of FDT. (a) Bubble profile $n_e(r, \zeta, z_9)$ in frame $z_9$ of the simulation in figure 7. The colour scale is labelled in units of $10^{19}$ cm$^{-3}$. The remaining frames show reconstructions of this profile using probes at multiple angles $\alpha$ and filtered back projection algorithm: (b) five probes ($0^\circ \leq \alpha \leq 80^\circ$, $20^\circ$ increments); (c) ten probes ($0^\circ \leq \alpha \leq 90^\circ$, $10^\circ$ increments); (d) 20 probes ($0^\circ \leq \alpha \leq 90^\circ$, $5^\circ$ increments).

perpendicular to $\xi$ in the plane of figure 9(b) that remains stationary in the reference frame of the probe. A bubble of radius $r_b$ crosses this line in time $\tau_{\text{transit}} = r_b/c \sin(\alpha/2)$. For example, the bubble in our simulation ($r_b = 20 \mu$m) crosses it in $\tau_{\text{transit}} = 94$ fs for $\alpha = 90^\circ$. Since the bubble’s total propagation time across a 1 mm gas jet is 3.3 ps, it is reasonable to assume that its structure remains quasi-static during $\tau_{\text{transit}}$. With this mild assumption, the accumulated phase profile $\Delta \phi(\xi_\bot)$ along the reference line $\xi_\bot$ is identical to the phase profile that would be accumulated by the same probe propagating across an identical stationary bubble at angle $\pi/2-\alpha/2$ with respect to its front-back axis. The existence of this equivalent problem establishes the connection between FDT and CAT and enables the use of reconstruction techniques already developed for CAT scans [45] with little alteration.

To complete space–time reconstruction of the bubble, additional reference lines parallel to $\xi_\bot$ are constructed along the phase streak at spacings $\geq \tau_{\text{transit}}$. Phase profiles along these lines represent 1D projections of quasi-static bubble structure at different stages of evolution. We then acquire an equivalent family of phase profiles from streaks recorded at different intersection angles $\alpha$ and apply the clinical CAT algorithm [45]. Figure 11 shows a simulated tomographic reconstruction of the bubble of figure 10 at $z_9$ (figure 11(a)) using different numbers of probe pulses in the same plane. Three probe beams are enough to detect the head and tail of the plasma bubble, where $n_e$ is high (figure 11(b)). With ten probe beams the whole bubble shape is discernible (figure 11(c)), and sharpens with additional probes (figure 11(d)). Similar reconstructions are obtained for other $z_i$. Strung together they form a movie. Oblique angle probes provide the only way to image wakes inside of preformed plasma channels [46], since probe pulses do not propagate freely along the channel axis.
4. Conclusion

FDH faithfully images most structural features of luminal-velocity, quasi-static plasma wakes in plasmas of density–length product \( n_eL < 10^{18} \text{ cm}^{-2} \) with \( \mu \text{m} \) resolution. Future applications of FDH include imaging of wakes driven by electron bunches \([47]\), positron bunches \([48]\) and petawatt laser pulses \([49]\). At higher \( n_eL \), refraction and trapping of probe light become increasingly important, phase profiles \( \Delta \phi(r, \zeta) \) at the plasma exit scramble radially, and phase shifts exceeding \( 2\pi \) become increasingly common. Under these conditions, FDS snapshots of probe amplitude profiles \( |E_{pr}(r, \zeta)| \) at the plasma exit provide more robust pictures of plasma structure. In particular, we showed that laser-driven plasma bubbles reshape co-propagating probe light into 3D spatio-temporal optical bullets that become most intense when mono-energetic electrons are produced. When wake structure evolves significantly, however, both FDH and FDS longitudinally average the changing plasma structure. Wakes propagating in plasma channels cannot be imaged at all. Finally, pump-generated radiation at the probe wavelength can introduce false structure into reconstructed images. To overcome these limitations, we propose a generalization of FDH—FDT—that uses multiple obliquely incident probe–reference pulse pairs, together with reconstruction algorithms developed for medical CAT scans, to visualize evolving plasma structures.

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