Physics 375R
Spring 2005
Homework #2
Due - February 4, 2005

Reading: Rindler: Chapters 3 & 4
My Notes: Chapter 3

1. Problem #2.4 from Rindler
2. Problem #2.10 from Rindler
3. Problem #2.13 from Rindler

4. It may be of interest to you to know how long our sun is going to last (in its current glowing form). If we assume that the energy of our sun comes purely from gravitational energy, we can make a rough (and wrong) calculation of the sun’s lifetime. To start, use dimensional analysis to estimate the gravitational energy of a star. Remember to first consider which variables (physical attributes of the star and fundamental physical constants) it can possibly depend on. Now, assuming that the constant of proportionality is close to one, estimate the Sun’s gravitational energy. In order to do this, you will need to estimate several things.

(a) Newton’s Constant, G How can you get it from little g? Do not look it up.
(b) We know that there are solar eclipses. Can we find the radius of the sun if we know the radius of the moon?
(c) Now we need the radius of the moon Everybody knows that the gravitational acceleration on the moon is 1/6 that of earth. That’s why the astronauts can jump so high!

Other useful facts are that the density of the Sun is 6 times that of the earth. Finally, we must calculate the rate at which the sun is losing energy. Since we know a lot about lightbulbs, we can use their calibration to get a pretty good estimate of this rate.
So, what is the lifetime of the Sun?

5. An infinite string of mass per unit length $\rho$ and tension $T$ at time $t = 0$ is given a transverse impulse such that the transverse velocity field is $v_0 e^{-\frac{x^2}{\sigma^2}}$ at $t = 0$. What is the subsequent displacement of the string for all time.
6. At \( t = 0 \), there is a small region of width \( \approx l_0 \) that contains a fairly uniform electric field of magnitude \( \vec{E}_0 \) along the \( z \) axis. The system is symmetric in the \( x \) and \( y \) directions. There are no other fields present. What are the electric and magnetic fields at times after \( t = 0 \)?

7. There are three coasting rocket ships converging on each other. From the middle ship, the other two ships are equidistant, moving with the same speed but opposite velocities, and the middle ship measures that distance between the end ships at \( t=0 \) to be two lightyears. The middle ship calculates the time of convergence to be two and one half years.

(a) How fast is the middle ship moving?
(b) How fast are the other two ships moving relative to the middle ship?
(c) Draw all of this on a spacetime diagram.
(d) At middle ship \( t =0 \) and at the end ship, how far away does the end ship say the middle ship is and how fast is it moving? Why.
(e) How fast is the other end ship moving relative to the end ship discussed above?
(f) How long does it take to converge to the end ships?
(g) All three ships have blinking running lights that blink red and green alternately and change color at time intervals every \( \frac{1}{5} \) of a year. At middle ship \( t=0 \), all three ships lights are red.
(h) At this instant, what color light does the end ship see of the middle ship’s light?
(i) What color is the light of the other end ship?

8. A cornerstone of the theory of special relativity is the constancy of the speed of light in all reference frames. Also we require Galilean invariance and that empty space is homogeneous and isotropic. Consider two mirrors positioned opposite from each other and a photon reflecting to and fro between the two mirrors. An observer at rest with regard to these mirrors sees the photon bouncing up and down. This setup can be used as a clock; you count the number of times the photon performs one cycle of its motion, i.e. leaves mirror A, hits mirror B, reflects back to mirror A, hits mirror A. Let’s us this as our definition of time. Now another Galilean related observer makes an identical clock. To do so he aligns his mirrors with yours. Note that this is only possible if the shortest line between the mirrors is transverse to the direction of relative motion. To that observer his clock runs the same as yours but you see this ‘clock’ pass you with constant velocity. For you, it seems as though the photon is moving along a zigzag path (this is required so that it return to the same place on the mirror that is moving at constant velocity). Taking the separation between the mirrors to be \( L \), calculate the length of the path the photon travels to complete one cycle in the two cases. In the second case, you will need to make use of the fact that the photon velocity is the same constant \( c \) in every frame to calculate this time. Your answer will also depend on the velocity \( v \) of the mirrors relative to you. Note that the ratio of the times is independent of \( L \) and thus compare the time interval on the moving clock with your
clock. Now our moving friend turns his clock sideways, along the direction of relative motion. He of course concludes that his clock runs the same, his space is isotropic. Requiring that the slowdown in the running of relatively moving clocks is independent of their orientation, show that for this new arrangement that you no longer think that the mirrors are a length $L$ apart and compute the ratio of the distances that you say is between the mirrors.

9. To get the coordinates of an event for any observer, she needs to measure two times on her clock, labeled $\tau_1$ and $\tau_2$. In terms of the coordinates of the event labeled 1 in Sally’s frame, $(x_s, t_s)$ find Harry’s coordinates, $(x_h, t_h)$. This will be the Lorentz transformations. Do so by finding the value of $\tau_1'$ and $\tau_2'$ in terms of $(x_s, t_s)$. Now find Sally’s coordinates for the events labeled $\tau_1'$ and $\tau_2'$. Now find the values of $\tau_1'$ and $\tau_2'$. Then use the definition to get the coordinates for Harry of event 1 to get $(x_h, t_h)$. The relationship between Harry’s coordinates for event 1 and Sally’s is the Lorentz Transformation.

![Figure 1: The space time diagram for the process by which Sally and Harry develop their respective coordinates for the same event.](image)

$$t_s = \frac{\tau_1 + \tau_2}{2} \quad \tau_1 = \frac{\tau_1' + \tau_2'}{2} \quad x_s = c\left(\tau_2 - \tau_1\right)/2 \quad x_h = c\left(\tau_2' - \tau_1'\right)/2$$