1. Assuming the acceleration of gravity at the surface of the earth for the acceleration of the rocket in Figure 6.4 of my notes, how wide does the rocket have to be for the light ray to fall by 1 mm over the course of its transit? Is this a thought experiment that could be realized on the surface of the earth?

2. (a) Transform the line element of special relativity from the usual \((t, x, y, z)\) rectangular coordinates to new coordinates \((t', x', y', z')\) related by

\[
\begin{align*}
  t &= \left( \frac{c}{g} + \frac{x'}{c} \right) \sinh \left( \frac{gt'}{c} \right) \\
  x &= c \left( \frac{c}{g} + \frac{x'}{c} \right) \cosh \left( \frac{gt'}{c} \right) - \frac{c^2}{g} \\
  y &= y' \\
  z &= z'
\end{align*}
\]

for a constant \(g\) with the dimensions of an acceleration.

(b) For \(\frac{gt'}{c^2} \ll 1\), show that this corresponds to a transformation to a uniformly accelerated frame in Newtonian mechanics.

(c) Show that a clock at rest in this frame at \(x' = h\) runs faster compared to a clock at rest at \(x' = 0\) by a factor \((1 + \frac{gh}{c^2})\). How is this related to the equivalence principle?

(d) The rocket also has extent in the \(y'\) and \(z'\) directions of \(h\). Use the line element above to show the height of the laboratory remains constant in time, i.e., the laboratory is rigid. The rocket is made of steel and is now oriented so that the \(y\) directions becomes the top bottom direction. How high is the rocket now and what are its transverse directions?

(e) Compute the invariant acceleration \(a \equiv (g_{\alpha\beta}a^{\alpha}a^{\beta})^{-\frac{1}{2}}\), where \(a^{\alpha} \equiv \frac{dx^{\alpha}}{d\tau}\), and show that it is different for the top and the bottom of the laboratory.

3. A paraboloid of revolution is generated by revolving the parabola \(z = ax^2\) about the \(z\) axis. Find its Gaussian curvature at the general point. Do this extrinsically. Now do it intrinsically. Now do this for the situation in which the length \(x\) varies with angle as \(x' = x(2 + \cos \theta)\) where \(\theta\) is the rotation angle. Now do the intrinsic and extrinsic curvature for the surface of revolution generated by \(z = a\sqrt{x}\).
4. At time $t = 0$, Bernice records Alberto at a distance of $L = 1$ lyr moving toward her with a constant velocity $v = \frac{3}{5} c$. At that moment, Cornelius, who is $L = 1$ lyr away in the opposite direction and at rest with respect to Bernice, begins accelerating toward her. Cornelius gauges his constant acceleration such that he will meet Bernice at the same moment that Alberto does.

(a) Draw a space-time diagram of the events and trajectories.
(b) How long does it take Alberto to meet Bernice, by Bernice’s clock?
(c) How long by Alberto’s clock?
(d) When all three meet, whose clock will have advanced the most and whose the least (assuming all clocks read 0 at Bernice’s $t=0$)?
(e) Find Cornelius’ acceleration.
(f) Find the time until meeting by Cornelius’ clock.
(g) What is the maximum velocity of Cornelius relative to Bernice?
(h) When Cornelius is $\frac{1}{2}$ lightyear from Bernice, he releases a piece of chalk. Draw a spacetime diagram. When does Bernice say that the piece of chalk hits her?
(i) If Cornelius rocket mass was 10 metric tons and the acceleration was generated by a light emitting rocket, what was the mass when he reached Bernice?
(j) What is Cornelius’ maximum velocity relative to Alberto?

5. Three observers with accurate identical indestructible clocks are standing on the surface of the earth. One observer tosses his clock up such that it returns to its original height after a time $T$ by that clock. The second observer merely holds her clock at that height while the first observers clock is in the air. The third observer moves his clock so that it has constant speed as it moves up and down up to the same height as the first observer’s toss and returns to the original height simultaneous with the arrival of the tossed clock. In other words all three clocks start together and end together. Draw a spacetime diagram. Which clock reads the longest time? Which clock reads the shortest time? What is the difference in time recorded by each clock when they return to the original height? To do this last part you may have to make several approximations. Please justify them.

6. Given what you know about atmospheric pressure, what is the height of the atmosphere? Do not look any thing up. From anything else that you know about the atmosphere or the earth does this make sense?