Solution to Problem #1: In a lecture hall of the future.....:

The wall is moving at me at $\frac{1}{3}c$ and it takes 10 seconds for the wall to reach me. Therefore, the width of half the room is 6 ltsec or the whole room is 12 ltsec wide. Of course to the students, I am the one moving toward the wall at $\frac{1}{3}c$ and they say that it takes me $\left(\sqrt{1 - \frac{9}{25}}\right)^{-1} \times 10$ seconds or $\frac{50}{4}$ seconds or that half the room is 30 ltsec wide or the whole room is 60 ltsec wide. My line of simultaneity once I start to move is inclined upward and thus the students behind me have not yet opened their books. The first leg of my trip is ten seconds and the subsequent two are each 20 seconds and thus the total trip is 50 seconds long to me. To the students the time from the start of the lecture to my return to the first wall was $5 \times \frac{50}{4}$ or $\frac{250}{4}$ seconds or 62.5 seconds. Below is the pattern from the frame of my first movement after the start of the lecture.

I am moving toward the students and the farthest is the one that opens his notebook the first. We know that, in the frame of the classroom, the openings were at (1,0), (2,0), and (3,0). Using the Lorentz transformations, in the moving frame these events are at times $-\frac{xx}{\sqrt{1 - \frac{9}{25}}}$. Thus the times are $-\frac{3}{4}$ sec, $-\frac{6}{4}$ sec, and $-\frac{9}{4}$ sec. In the interval between 10 and thirty seconds to me, the wall is moving toward me at a speed of $\frac{3}{5}c$. In this interval, the comover see the nearest student open the books first. The interval is the same as it was before. My new speed relative to my speed during the interval from 0 to 10 seconds is $\frac{\frac{5}{1 + \frac{9}{25}}}{1} = \frac{15}{17}c$. Since I am traveling so fast, almost the speed of light, it takes almost no time.

Solution to Problem #2: For a very heavy particle......
Let's look at the general dink or dank propagator. It's form is
\[
\frac{1}{m_0} \left( \frac{C_1}{m_c} \frac{C_2}{m_c} \cdots \frac{C_n}{m_c} \right)
\]
where the \(C_i\) are either A or B depending on a dink or dank. There are all kinds of patterns of the A's and B's. In fact, the terms of \((A + B)^n\) exhibits these same patterns and these terms are in one to one correspondence with the graphs that are drawn. Thus the sum of all graphs with n dinks or danks is \(\frac{(A+B)^n}{m_0 m_c^n} \) and thus all the graphs can be summed to \(\frac{1}{m_0} \left( \frac{1}{1 + \frac{A + B}{m_0}} \right)\). Thus the effective mass of the huge is \(m = m_o - A - B\).

Solution to Problem #3: Do Problem 5.10...

A) Proper time is the time measured between two events by a clock that is present at both events. Therefore, clocks P and Q measure proper time between A and B.

B) Clock P measures the spacetime interval between A and B, since it is inertial. Clock P's velocity is defined by \(\Delta x\) and \(\Delta t\), which determine the spacetime interval.

C) The largest measurement is given by the R-S clock system, since in general we have \(\Delta t > \Delta \tau\) (see the formula in the book). The proper time measured by P is longer than that measured by Q, since P is a constant velocity path connecting the two events.

Solution to Problem #4: Do Problem #6.7...
A) The space time diagram above shows the events described in problem 6.7. Event A is the origin, where the Klingdon ship passes the Execrable. The ship is travelling at \( v = 3/5 \ c \). At event B the Klingon ship fires its phaser (which travels at c) towards the Execrable. The Execrable is hit at C, and the Klingon ship passes into Klingon territory at D. Note that 1 minute is two squares along the t axis, and 1 light-minute is two squares along the x axis.

B) Reading off the location of Event C from the diagram, we see that the cruiser is hit by the phaser at \( t = 8 \) minutes. The Klingon ship passes into it's own territory at \( x = 6 \) light minutes, \( t = 10 \) minutes. Of course, we could have gotten that from the fact that \( x = v t \), and \( v = 3/5 \ c \). From the Execrable's point of view, it was hit by the phaser before the Klingon ship passes into Klingon territory.

C) What do the Klingon's think? The Klingon ship's line of simultaneity passing through event D is drawn on the diagram above. You can clearly see that the Klingons think that the Execrable was hit after they passed into Klingon territory.

Using the Lorentz transformations to verify this just means plugging the Execrable's coordinates for the two events into the formula for the Klingon's coordinates, and seeing that the Klingon's time coordinate for event D comes before their time coordinate for event C. Those nutty Klingons.

Solution to Problem #5: Do Problem 7.7...

One square = 10 ns
A) In this problem we have two spacecraft, each is 100 ns long in their own frame, and they are approaching each other at relative speed 3/5 c. Thus, they each see the other as 80 ns long. In the diagram above (drawn in O's rest frame) Event A represents the bow of O and the tail of O' passing one another, and Event B is O firing its tail cannon. For ship O events A and B are simultaneous. This presents an apparent paradox: if ship O thinks it's front end is at the back of ship O', shouldn't ship O' think its front is at the back end of ship O since they each see the other contracted in length? It is this assumption that is problematic. As with the other paradoxes we have addressed this can be explained by noticing that the two ships' relative motion forces them to disagree on the simultaneity of events.

Ship O thinks A and B are simultaneous. But if we draw a line of simultaneity for O' passing through event B (shown above), we see that ship O' does not see the two events as simultaneous. The two wordlines drawn on the diagram represent each end of ship O' in O's frame. So, by looking at where the line of simultaneity through B intersects the worldline representing the front of ship O' (the right line) we see that ship O' thinks that the laser is fired before the front of their ship is at the end ship O.

B) We can get the coordinates for event B in the O' coordinate system using the Lorentz transformations:

\[ x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10 - (3/5)10}{\sqrt{1 - (3/5)^2}} = (5/4)(10) = 12.5 = 125 \text{ ns} \]

\[ t' = \frac{t - \frac{v}{c} x}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{0 - (3/5)10}{\sqrt{1 - (3/5)^2}} = (5/4)(-30/5) = -7.5 = -75 \text{ ns} \]

C) Since ship O' thinks it is 100 ns long, it sees the back of its ship at x = 0 and the front of its ship at x = 10. In part B we got the coordinates for B in the O' frame, which confirmed what the graph said. The laser is fired at x' = 12.5, so O' does not get hit.

Solution to Problem #6: Do Problem 8.5...

Nothing moves faster than the speed of light ... get over it. What's happening in this problem? We have a spotlight which we turn at 100 rotations per second. We project it onto the ceiling, or a cloud and notice that the projection of the spotlight seems to be moving faster than c. Is this a paradox? No. Nothing is moving faster than the speed of light. Suppose you see the light at point A at time = 0, and then at point B at time = 1, and points A and B are far enough apart that the projection seems to be moving faster than the speed of light. Notice that nothing is moving between A and B. Light is coming from point A to your eye, so that you see the light there at t = 0. Shortly thereafter, light left point B, which you then see at time 1. Nothing moved between A and B, but light left each point at almost the same time, close enough to give it the appearance of moving faster than c.

The rate of rotation is called angular velocity, denoted by \( \omega \). In this case:

\[ \omega = 100 \frac{\text{rotations}}{\text{second}} = 2 \pi \times 100 \frac{\text{radians}}{\text{second}} \]

If the spotlight is shining on a surface a distance R away and is being rotated with angular velocity \( \omega \), then the speed of the spot of light on the wall is given by \( v = R \omega \). So if we want the spot of light to move faster than c:

\[ v > c \Rightarrow R \omega > c \]

\[ R > \frac{c}{\omega} \Rightarrow R > \frac{3 \times 10^8 \cdot \frac{\pi}{2}}{200 \pi \frac{\text{m}}{s}} = 477465 \text{ m} \]
So R still has to be pretty large (bigger than about 477 km) in order for the spotlight's projection to appear to be moving faster than c.

**Solution to Problem #7: Do Problem 8.12...**

This is a typical addition of velocities problem. One rocket is moving to the right at 3/5 c, so it has a velocity of +3/5 c. The other rocket is moving to the left at 4/5 c, so it has a velocity of -4/5 c. We plug these into the addition of velocities formula:

\[
\mathbf{v} = \frac{v_1 - v_2}{1 - \frac{v_2}{c}}
\]

\[
\mathbf{v} = \frac{(3/5 \, c) - (-4/5 \, c)}{1 - \frac{(3/5 \, c) (-4/5 \, c)}{c^2}}
\]

\[
= \frac{35 \, c}{37}
\]

So the rockets see each other approaching at a relative speed of 35/37 c.

**Solution to Problem #8: Do Problem # 9.7...**

We have

\[
(\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z) = (18 \, \text{kg}, 9 \, \text{kg}, 15 \, \text{kg}, 1 \, \text{kg}).
\]

Now \(p_t = \frac{E}{c}\), but since all three momenta are given values in kg, we see that \(c = 1\) here. (If \(c\) were not 1, we would have units of kg m s\(^{-1}\) for momentum, which we don't.) Thus \(p_t = E\). Since in general

\[
\mathbf{p} = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

it is clear that

\[
\mathbf{v} = c^2 \frac{\mathbf{p}}{E},
\]

so in our case \(\mathbf{v} = \frac{\mathbf{p}}{E}\) and in particular

\[
\mathbf{v}_x = \frac{9 \, \text{kg}}{18 \, \text{kg}} = \frac{1}{2} \quad \mathbf{v}_y = \frac{15 \, \text{kg}}{18 \, \text{kg}} = \frac{5}{6} \quad \mathbf{v}_z = \frac{1 \, \text{kg}}{18 \, \text{kg}} = \frac{1}{18}.
\]

In three dimensions we find \(\mathbf{v}\) from \(\mathbf{v}_x, \mathbf{v}_y,\) and \(\mathbf{v}_z\) by using
\[ v^2 = v_x^2 + v_y^2 + v_z^2 \]
\[ = \left( \frac{1}{2} \right)^2 + \left( \frac{5}{6} \right)^2 + \left( \frac{1}{18} \right)^2 \]
\[ = \frac{307}{324} \]
\[ v = \sqrt{\frac{307}{18}} = 0.97 \]

For the mass, we use
\[ m^2 = E^2 - p^2 = E^2 - p_x^2 - p_y^2 - p_z^2 \]
\[ = (18 \text{ kg})^2 - (9 \text{ kg})^2 - (15 \text{ kg})^2 - (1 \text{ kg})^2 \]
\[ = 17 \text{ kg}^2 \]
\[ m = \sqrt{17} \text{ kg} = 4.1 \text{ kg} \]

The momentum can be calculated two different ways:
\[ p^2 = p_x^2 + p_y^2 + p_z^2 \]
\[ = (9 \text{ kg})^2 + (15 \text{ kg})^2 + (1 \text{ kg})^2 \]
\[ = 307 \text{ kg}^2 \]
\[ p = \sqrt{307} \text{ kg} = 17.5 \text{ kg} \]

or
\[ p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(\sqrt{17} \text{ kg}) \left( \sqrt{\frac{307}{18}} \right)}{\sqrt{1 - \left( \frac{307}{324} \right)^2}} = \sqrt{307} \text{ kg}. \]

Amazingly, the two methods yield the same result. The kinetic energy is found using the relation
\[ KE = E - m = 18 \text{ kg} - \sqrt{17} \text{ kg} = 13.9 \text{ kg}. \]

**Solution to Problem #9: Do Problem #9.10...**

The letter has a mass of 0.025 kg and is shot off at a whopping 0.999c. Since the problem gives us the cost of energy in dollars per million Joules, we are working in units in which \( c \) is not equal to 1, but \( c = 3 \times 10^8 \frac{m}{s} \). In such a case,
\[ KE = E - mc^2 = mc^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \]
\[ = (0.025 \text{ kg}) \left( 3 \times 10^8 \frac{m}{s} \right)^2 \left( \frac{1}{\sqrt{1 - (0.999)^2}} - 1 \right) \]
\[ = 4.8 \times 10^{16} \text{ J}. \]
If the cost of energy is \( \frac{0.03}{10^6} J \), then the money spent to send this letter is approximately

\[
\left( \frac{0.03}{10^6} J \right) (4.8 \times 10^{16} J) = 1.4 \times 10^9 = 1.4 \text{ trillion}.
\]

Let's see that compete with UPS. By the way, if your average nuke dishes out about \( 4 \times 10^{14} J \), then if the letter veers off course and hits something, it will unload the energy of about 120 nukes. Talk about a letter bomb.

**Solution to Problem #10: Do Problem #10.6...**

Let the mass of the whole shebang (rocket plus fuel) before takeoff equal \( M \), let the mass of the rocket by itself equal \( m \), and let the velocity of the rocket once it's used up its fuel equal \( v \). We are assuming that the entire mass of fuel is converted into light, which is shot out the back. Let's write down the equations for energy and momentum conservation for this system. These equations will indicate the following: before heading off the rocket has energy \( Mc^2 \) and zero momentum (it's just sitting there), and after exhausting its fuel the rocket has the energy and momentum appropriate for an object of mass \( m \) moving at velocity \( v \) while the light has energy and momentum related by the expression \( E = pc \). Letting \( p_{light} \) be the momentum of the light, our equations are

\[
Mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} + p_{light} c
\]

\[
0 = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} - p_{light} c
\]

Let's multiply the second equation by \( c \) on both sides, so the second terms of the two equations will be equal and opposite and I can cancel them by adding.

\[
Mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} + p_{light} c
\]

\[
0 = \frac{mvc}{\sqrt{1 - \frac{v^2}{c^2}}} - p_{light} c
\]

Adding the two equations gives me

\[
Mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{mvc}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

\[
= mc^2 \left( \frac{1 + \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = mc^2 \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v^2}{c^2}}}
\]

\[
M = m \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v^2}{c^2}}}
\]

This is the formula for the general case. Now we consider the particular case \( m = 25,000 \) kg \( v = 0.95c \), so

\[
M = (25,000 \text{ kg}) \sqrt{\frac{1 + 0.95}{1 - 0.95}} = 160,000 \text{ kg}.
\]
This means that the rocket needs about 135,000 kg of fuel, or almost 5$\frac{1}{2}$ times its own mass. And this is in the best-case scenario; any type of rocket we could build could not possibly convert the mass of its fuel into a form usable for thrust with 100% efficiency. You can forget about Star Trek altogether.