Key Homework # 12

Solution to Problem #6  Suppose a system of test masses is present in an expanding...

Of course it depends on how you set the test masses out. Using the usual configuration, a central mass surrounded by identical masses with zero relative velocity, the changes will depend on the nature of the universe. The system of test masses is sensitive to the relative acceleration. Thus in closed finite universe, the masses will begin to move together even though the local galaxies are moving apart. This is because the galaxies were comoving at the beginning of the universe. Our set of test masses were just placed out there and have a relative velocity to the nearby galaxies. If you had set the test masses in the same relative motion as the galaxies then they would expand but more slowly over time as the acceleration started to take over. If the universe was flat the set of test masses if started at rest they would remain at rest. If they were placed as comoving then they would continue to drift apart. If the universe was open then they would expand at an increasing rate in either start configuration. You always have to make you treat mass system large enough to sample the large scale structure of the universe. If you make it say solar system size you will be measuring the local gravitation of the sun for instance. Of course, none of this takes into account the recent discovery of a positive acceleration on the largest scales. In this case the test masses would all have always increasing velocity. If the test mass set was started too close so that it was not measuring properties of the Universe, the system would shear since it had no mass inside and the shear might point to a nearby massive body.

Solution to Problem #7:  Consider a galaxy that is ...

First we have to determine how far away a Mps is. It is \(3 \times 10^{16} \text{ m}\). Since Hubble's constant is about \(50 \ \text{km} \text{s}^{-1} \text{Mps}^{-1}\) the speed of our galaxy is \(5000 \ \text{km} \text{s}^{-1} = 5 \times 10^{6} \ \text{m} \text{s}^{-1}\) or that \(\frac{v}{c} = 5 \times 10^{-2}\). The doppler shift is \(\approx 1 - \frac{v}{c}\) so that the frequency is reduced by 1.7%.

Solution to Problem #8:  The universe is expanding...

The escape velocity from the earth will depend on the size of the earth as measured by its radius and little \(g\). It can not depend on the mass of the chalk since in gravity all masses move the same. The only dimensionally consistent form using these is \(v_{\text{esc}} = \sqrt{gR_{e}}\). From the list of "Things Everyone Should Know" we have \(\sqrt{6 \times 10^{5}} \approx 10^{3} \ \text{m} \text{s}^{-1}\). A rather large velocity.

Using us as the center of the sphere of the universe, to get the escape velocity, we have \(G\) and the mass-energy density of the universe and \(R\) the distance to the galaxy. The only dimensionally consistant form using these variables for a velocity is \(v=\sqrt{G\rho R}\). If the current recessional velocity is the escape velocity the universe will collapse. Thus \(HR = v = \sqrt{G\rho R}\) or \(\rho_{\text{crit}} = \frac{H^{2}}{G}\) on dimensional grounds.

Of course our galaxy is not the center of the universe and thus this discussion is questionable. On the other hand because of homogeneity, it should be typical and thus this analysis should be okay.

Solution to Problem #4:  There are three types of universes...

The closed universe has to be positively curved because the positively curved universe has finite geodesics, straight lines. Lorentz invariance requires that if the spatial two planes have finite geodesics, then the space time planes will also have finite geodesics. Thus these universes must collapse back onto themselves.