Key for Homework #7

**Problem #1:** *Figure #1 shows the power output of warmblooded ...*

This is a log-log plot of the power output versus mass of animals. The data fits a straight line and the slope on a log-log plot is the power law of the variables. The slope of a curve is \( \frac{\log y - \log x}{\log m - \log m_i} \) where \( y_i \) and \( x_i \) are the log of the power and log of the mass values respectively from the plot. In our case the values are logs or the slope \( m = \frac{\log w - \log w_i}{\log m - \log m_i} = \frac{\log \frac{w}{w_i}}{\log \frac{m}{m_i}} \). This turns out to be around 0.75. Therefore \( W = c m^{0.75} \). If you think about it the heat out should be proportional to the amount of biologically active stuff in the animal. Thus I would expect that the result should be \( W = c m^1 \). There are wrong but not bad arguments that the heat should scale as the area or \( m^{0.75} \). Either way, the data does not agree. We need to find an explanation. My guess is that larger animals have a higher ratio of active mass to inactive mass.

**Problem #2:** *In lecture, I talked about the problem...*

A temperature bath is a system that no matter what maintains its temperature when in thermal contact with other bodies. Any really large body can play the role of a temperature bath. The ocean, the atmosphere, the universe are all of this type. The other broad class is systems that because of some processes maintain their temperature regardless of the heat exit or entry. Two examples of this kind are boiling water which holds its temperature at 100°C and the human body whose metabolism and blood flow act to maintain an inner body temperature of about 98°F.

Most of the time the human body is immersed in air which is a reasonable thermal bath. Thus you have two thermal baths and heat is passing from the higher temperature bath, the body, to the lower temperature bath, the air. This is the famous 70 Watts on the list "Things Everyone Should Know". There are several ways that thermal contact is made between the air and the body: convection which is the direct flow of heat through the air, evaporative cooling by transferring water vapor to the air, and radiative cooling by glowing into the cooler air. For humans in air evaporative cooling into air is the principle cooling mechanism.

When you have a pool and the air at the same temperature the pool still feels cooler because the heat flow from your thermal bath is greater in the pool because the thermal convection is much greater in the pool. In an air flow from a fan, clearly the moving air and still air are at the same temperature. The fan moves the moist air away quicker and that's why it feels cooler.

Yes we are in thermal contact with the sun it is just that we are not in complete thermal contact. The sun is trying to heat us up to a temperature of about 5000 K but we are not completely surrounded by the sun. We recieve heat from the side toward the sun and radiate out to the universe which is at a really low temperature, \( \approx 30 \) K. Equilibrium is reached when the heat flow from the sun matches the heat passed to the other thermal bath, the universe.

When you hold a magnifying glass, you form an image of the sun on the page. This is basically a spot at 5000 K. As the part of the paper at the image starts to raise its temperature from the radiant heat transfer, it eventually bursts into flame because the paper is not stable at temperature much above around 500 K to a 1000 K and the molecules start to break up. This is the flame. The temperature can't get much above this temperature since the break up absorbs the heat and also the paper disintegrates. You should realize that a piece of paper held in the sun light is also in thermal contact with the sun. Again it is also in contact with another thermal bath, the air, and the paper transfers heat to the air if its temperature goes above air temperature.
Problem #3: We can model a cereal bowl...

The action for a particle moving above the surface of the earth is the same as our famous chalk toss case, 

\[ S = \sum_{seg} \left( \frac{mv_x^2}{2} + \frac{mv_y^2}{2} - mg h \right) \Delta t, \]

where the height is measured from the bottom of the bowl. For a mass moving on the bottom of the bowl the relationship between the height and the lateral displacement is found by using the fact that a circle centered on the origin has the equation \( R^2 = x^2 + y^2 \). Shifting the origin up the y axis an amount \( R \) and calling \( y \) h, the relationship between \( h \) and \( x \) is \( h = R - \sqrt{R^2 - x^2} \). The velocities \( v_x \) and \( v_y \) are related by \( v_x = v \cos \theta \) and \( v_y = v \sin \theta \) where \( \theta \) is the angle from the point \((x=0, y=R)\) and since this angle is small and we need \( v_y^2 = \theta^2 \) we can neglect the \( y \) velocity. Thus the action in terms of \( x \) is 

\[ S = \sum_{seg} \left( \frac{mv_x^2}{2} - mg \left( R - \sqrt{R^2 - x^2} \right) \right) \Delta t. \]

Since \( R \gg x \), \( R - \sqrt{R^2 - x^2} \approx \frac{x^2}{2R} \). Putting this into the action we have 

\[ S = \sum_{seg} \left( \frac{mv_x^2}{2} - \frac{mgx^2}{2R} \right) \Delta t. \]

This compares to the action for an oscillator \( \sum_{seg} \left( \frac{mv_x^2}{2} - \frac{kx^2}{2} \right) \Delta t \). The the particle in the bowl acts like an oscillator with mass \( m \) and spring constant \( k = \frac{mg}{R} \). Since this is an oscillator, we can use this action to derive an energy and thus write the energy expression immediately as 

\[ E = \frac{mv_x^2}{2} + \frac{1}{2} \left( \frac{mg}{R} \right) x^2. \]

Thus the initial energy is \( E = \frac{mv_x^2}{2} + \frac{1}{2} \left( \frac{mg}{R} \right) x_0^2 \). At the maximum excursion there is no velocity and thus the energy is \( E = \frac{1}{2} \left( \frac{mg}{R} \right) d^2 \). 

Where \( d \) is the maximum \( x \) excursion. This is also to the same approximation \( mg h_{\text{max}} \). The the maximum lateral displacement is 

\[ d = \sqrt{\frac{E}{\frac{mg}{R}}} v_{x_0}^2 + x_0^2 \]

and the maximum height is 

\[ h_{\text{max}} = \frac{1}{2} \left( \frac{v_{x_0}^2}{g} + \frac{x_0^2}{R} \right). \]

Home Experiment #7

When you place one polaroid between your eye and a light, the brightness of the light is reduced. Of course the other polarizer does the same thing. When you stack the polarizers depending on their relative orientation either you see much of the light or little and if you rotate the pair of polarizers nothing changes. If you rotate one of the polarizers, the brightness varies from minimum to maximum every \( \pi \) that you rotate it. It does not matter which polarizer you rotate.

Now starting with two polarizers oriented so that no light goes through, when you add a third one in the stack, as you rotate the last one nothing changes. If you insert the third polarizer between the two and rotate it, the brightness varies and it goes from minimum to maximum every \( \frac{\pi}{2} \) rotation.

I only wanted you to report what you saw. I did not want an "explanation." This was to just an exercise in observing.