Chapter 1

Introduction

1.1 Purpose of This Course

Today, it is apparent that you cannot function in society without contact with science. Not only are the things we use in our daily lives based on the discoveries of science, but also our attitudes and beliefs are derived from conceptual aspects of science. More importantly, the methods used for the acquisition of scientific knowledge are extraordinary and successful; a better system has not been developed. Every student at The University of Texas, in particular, every student in the Plan II program, must be exposed to the basic concepts and methodology of modern science. In addition, it is important that as a part of that exposure, you learn about the concepts of modern physics. Physics has been the most successful of the sciences, and its fundamental methods, based on experimental verification, reduction, and synthesis, has become a paradigm for all the other sciences.

This course is a part of a sequence of science courses that is required for all Plan II students. It concentrates on the conceptual foundations of modern physics. This course is different from any other that is taught at The University of Texas at Austin or anywhere else that I am aware of for two reasons. These concepts are very important but difficult to understand. The junior Plan II student has had enough other preparatory material and shown a maturity that makes it possible to discuss these issues. In addition, the students in the Plan II sequence have diverse majors and many will take or have taken a physics course at the university level. For these students, it is important that this course offer new ideas. Fortunately, all these other courses deal with the material at a more applied level and do not treat modern physics in the detail required to understand the basic conceptual ideas.
Most other physics courses spend almost all of their time developing the concepts of classical physics. This is because so much of our lives is effected by these ideas and concepts. We live in a world that is dominated by the objects that are dealt with in classical physics. Without the foundation of classical physics, it is impossible to understand the ideas of modern physics. In the case of the Plan II students, we are fortunate to be able to assume a reasonable level of understanding of the ideas of classical physics. It is anticipated that all students taking this course will have had an introductory physics course in high school or at The University of Texas at Austin.

Another feature of most university physics courses is that they serve as a foundation course for subsequent studies of a more specialized nature; therefore, these courses have to cover a certain content. That content is also predominantly based on classical physics. These courses also tend to be dominated by problem solving techniques and preparation for subsequent standardized tests such as the LSAT or MCAT. In this course, in contrast, the primary emphasis will be conceptual. Many of the concepts that will be discussed will not be used in your future work. One of the purposes of this course is to provide an opportunity to understand the basis for our current descriptions of matter, and the universe. These ideas are often contrary to everyday experience and thus require a level of understanding that is generally more abstract and subtle. The skills required for this type of reasoning are valuable in almost any context and are another reason that this course is offered.

We will treat only modern physics. Well, almost. There are some aspects of classical physics that are not treated adequately in most classical physics courses, but I feel they are essential to the understanding of modern physics (field theory, action, and symmetry). In addition, because this course will emphasize conceptual foundations, it will spend little time on the “things” of modern physics. These “things”, such as models of the atom, transistors, or lasers, are covered by most courses with a modern physics component and we will only deal with them as they provide examples for the development of the basic concepts.

The goal of this course is to develop a sense of the processes that are inherent in the articulation of the discoveries of modern physics. Scepticism is inherent in an honest and objective search for truth; in the use of reason to establish a successful model of a subtle and difficult to understand phenomena; and in an analytic approach that reduces phenomena so as to discover its essential minimum. In some sense, the detailed calculations that are required in the homework and the tests are not important; instead, what is important is the process by which they are derived. Rather than just having
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students rewrite or restate the principles, in this course, an understanding of
the basic concepts is developed in the application of theoretical principles to
diverse examples. I hope that the student who successfully completes this
course will appreciate the value of scientific reasoning. The fact that the
universe is knowable by these techniques, and that the successful enterprise
of physics leads to a knowledge system that is more powerful than any other
because it deals with the objective reality that we all share.

This is intended to be a terminal course in physics. There is no follow
up course and the issues that are discussed here are at the limit of our
current knowledge. This course will show you how our modern theories are
developed and what is the basis for our belief in these theories. It will also
show you how to deal with ideas that are outside your usual approach to
understanding. I hope that you are willing to undertake the task and will
enjoy the exercise.

1.2 Physics that you should know

1.2.1 Introduction

This course is intended for people with minimal formal exposure to physics.
Basic ideas and relevant definitions will all be introduced as they are re-
quired. This does not mean that some background information or expe-
rience is not helpful. Prior exposure to physical reasoning and to physics
vocabulary should make the material more accessible to you. Some concepts
that we all use and discuss in our daily lives, such as energy, become more
refined in the context of physics, and they will be treated this way in our
course. More important than a physics background will be experience with
consistent logical reasoning and curiosity about the world.

It is important that you have a quantitative understanding of the phe-
nomena about us. When you discuss things that you see, you should use
specific terms, such as density and speed, and you must understand what
they mean. Whenever possible, you should discuss them in a quantita-
tive manner. Basic mathematical concepts, such as area and volume, are
essential. An extended discussion of the mathematical requirements is in
Section 1.3.1. You should familiarize yourself with all of the items in the
“Things that Everyone Should Know” list, Section 1.4.2. Simple exercises
that we all do, like computing a route for a trip or estimating the cost of a
vacation trip, are important skills; they are quantitative and require com-
plex reasoning. Many of these skills will be essential for Fermi problems,
Section 1.4.1, and are an important part of this course.
The following is a brief outline some of the important ideas from basic physics and skills that any student in this course should know. It is rather terse and, in some places, abstract. You may have to read it carefully to recognize that it is something that you already know but may know in a different way of expressing it. If you do not know them, and you feel that you will have difficulty, you should work with someone to develop a basic understanding at the level described here.

1.2.2 Kinematics

Kinematics is the study of the relationships between the quantities that are involved in the study of motion. You should realize that to describe a place you need to select an origin or reference point from which to measure displacements. Displacements are the separation between the origin and a place. Displacements are measured in lengths. We will discuss the issue of length, Section 2.2 and place, Section 8.1 in great detail later. Again these are actually subtle issues and our attitude toward them has changed in modern physics.

You should know how to identify a place in a three dimensional space. You should realize that this descriptor of place is a vector quantity and that as such it has both a magnitude and a direction. In general, the displacement vector can be thought of as the most accessible of the larger class of objects called vectors and the rules of vector algebra are those of common sense applied to displacements. The displacement can be stated as the triplet of numbers that are the magnitude of the displacements in the three basic coordinate directions or as a magnitude and a direction.

Vectors can be added to produce new vectors, and they have simple addition rules. There are two general rules: To find the sum of two vectors place the tail of the second vector on tip of first. The sum is the displacement produced by going from the tail of the first to the tip of the relocated second. Said another way, two displacements can be combined, and their result is also a displacement. This is a general property of all objects called vectors; there is a rule for addition and the addition of two vectors is also a vector.

The magnitude of a vector is its length. The magnitude of the displacement is the distance (actually the shortest distance of the many possible distances that depend on the path) between the initial and final places. Note that distance is always a positive quantity whereas a displacement can be either positive or negative.

Velocity is the time rate of change of displacement. In this sense it is a difference of two displacements and thus us also a vector. The length
of the velocity vector is the speed. Note that speeds are always positive. Velocities can be added using the same "tip to tail" addition rule that was used for displacement. Note that if you change the displacement in any way, you have a non-zero velocity. Even if you do not change the distance, but change just the direction of the displacement, you have a velocity.

Acceleration is the time rate of change of velocity. It thus basically a difference in velocities and thus is also a vector. Accelerations can be added using the same “tip to tail” rule. If you change the velocity in any way, you have a non-zero acceleration. Even if you do not change the speed, but change just the direction, you have an acceleration. You should understand situations in which acceleration stays constant but velocity changes.

It should be obvious that if you know the position of an object for all times, you know the velocity and the acceleration for all times. You should also realize that if you know the acceleration for all times, the initial position, and the initial velocity, then you know the position for all subsequent times.

Any description of motion depends on a choice of reference frame from which all displacement, velocity, and acceleration measurements are made.

1.2.3 Dynamics

Dynamics is the study of the causes of motion. The motion is the temporal evolution of systems in space. Newtonian physics is based on the idea that space and time are absolute. They are unaffected by what is in it and how it moves.

A primary notion is that there are forces. These forces represent the effect of other bodies on the body whose motion is under study. Your third-grade definition of a force—a push or a pull—is as good as any for a start. In this sense, forces are contact actions of one body on another. To do physics, we need to expand this idea beyond contact forces to action at a distance influences, see Section 4.1. To get a better understanding of forces, consider the world made up of several parts. This system of parts is isolated and thus all influences are from the parts on each other. This is the essence of reductionism, see Section 1.5.2: you can reduce the whole to its parts and the action of any part on a given part does not depend on the remaining other parts. The important point is that a force is the effect of one body on another and is only considered when you replace the body by its force, see Figure 1.1. We are interested in the motion of body one. We talk about the force of body two on body one and the force of body three on body one and so forth. Once we know the forces and use the fact that force in simple cases is a vector quantity and obeys the usual rules for vector addition, we can
get the total force by addition. In a real sense, bodies two and three etc. are replaced by their forces. Later in the semester, we will have to broaden our idea of force so that it becomes separated from the body that is its source and just talk about it as a thing unto itself. For now, all forces are due to other bodies and they have meaning only in the sense that they are there when we want to discuss the effect that one body has on the other.

![Diagram of forces](image)

Figure 1.1: **Adding Forces** A system composed of 5 parts. The forces are there in the sense that $F_{12}$ is the push or pull on body 1 due to body 2. $F_{12}$ can depend only on the relationship between bodies 1 and 2 and $F_{12}$ does not depend on the presence of the other bodies. Similarly $F_{1i}$ is the effect of body $i$ on body 1. Note also there is a set of forces that act on body 2 and so forth.

The rest of basic dynamics is contained in what are generally called Newton’s three laws of motion. The first law states that if a body has no net forces acting on it, it will continue in its present state of motion. This means that the velocity of an unforced body is unchanged; there is no acceleration. Newton took this idea from Galileo. We will look at this law from a different perspective and, in fact, closer to the original spirit of Galileo. An object at rest and subjected to no forces remains at rest. An object with a velocity, $\vec{v}$, subjected to no force will continue to move at a velocity $\vec{v}$. In a sense, there is no difference for an object at rest and an object with a uniform velocity. This is called Galilean invariance and will play a very important role in what we do in this course. This law can be stated in many forms and each way provides new insight into its meaning. One of the more intuitive is that, for any body that is subject to no net forces, there exists a reference frame in which the body is and remains at rest, see Sections 5.4.2, ??, and 7.2. Since by reference frame, we mean an unforced observer, an observer
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that also notes no forces, there may appear to be some circularity in this definition. The important observation is the observers that detect no forces are those that are in uniform motion. Another way of interpreting this result is to say that all force-free motions have constant velocity and that uniform motion, motion with constant velocity, is the same experimentally as no motion. This was a long way around to the statement that all uniformly moving coordinate systems are equivalent and that it is meaningless to say how fast you are going in any absolute sense. You can measure accelerations absolutely but you cannot measure velocities except as relative concepts.

In order to present the second law, we need the concept of mass. For our present purposes, we can take the simple definition of mass: it represents the amount of matter in an object. We will spend considerable time in this course clarifying the idea of mass; it was a difficult concept for Newton and the modern interpretations are also subtle. In its simplest form, Newton’s second law states the a body responds to the presence of an unbalanced force by accelerating. The acceleration is the net force divided by the mass of the body, the famous $\vec{F} = m\vec{a}$. It is important to note that acceleration is a kinematic quantity and is defined once we have a length and a time.

Newton’s third law states that if two bodies exert forces on one another, these forces are equal and opposite. The force of body two in body one is equal to the negative of the force of body one on body two, $\vec{F}_{21} = -\vec{F}_{12}$. This law is also known as the law of action reaction. When this concept of force is a part of the interactions of bodies, this law is always true. In our course though, we will find cases in which it does not hold, see Section 4.3.

It is very important to realize that if you know the forces acting on a body, either as a function of position or time, and you know the initial position and velocity, then you know the subsequent motion, i.e. the position as a function of time, see Section 1.2.2. This is the essence of causality. Given the initial position and velocity, and knowing the forces between all the bodies determines all the subsequent behavior of the bodies. We will find that there is more to the world than just localizable point objects and that our requirements for causality have to increase to account for all the phenomena observed in the universe, see Section 4.1.

You should know several simple examples of forces. There are two types: basic and phenomenological. Basic forces are those that we attribute to the fundamental aspects of matter, such as electric force between charged particles and gravitation between massive bodies. Phenomenological forces are due to very complex involvements of many things but, despite the complications, are simple to describe.

An example of a phenomenological force is the normal force that stops
my hand from moving through the table when I lean on it. In this case, the atoms of my hand and the atoms of the table act to produce whatever force is necessary so that my body is supported. Another example is the Hook's Law spring. Here, a complicated structure of coiled metal, when exposed to a force is deformed. If the force is proportional to the stretch of the spring, $\vec{F} = -k\vec{x}$, this is a Hook's Law spring. Many coil springs and lots of other things act like a Hook's Law spring and this is a very useful concept. You should understand the motion of a system that is well described by a Hook's Law spring.

There are four basic forces: strong, weak, electromagnetic, and gravitational. You should know about these forces, along with the simplest forms of the two classical forces: the electrical force between two charges, $Q_1$, and $Q_2$ at locations $\vec{r}_1$, and $\vec{r}_2$: $F_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{|\vec{r}_2 - \vec{r}_1|^2} \times (\vec{r}_2 - \vec{r}_1)$ and the gravitational force between two masses, $m_1$, and $m_2$ at locations $\vec{r}_1$, and $\vec{r}_2$: $F_{12} = -G \frac{m_1m_2}{|\vec{r}_2 - \vec{r}_1|^2} \times (\vec{r}_2 - \vec{r}_1)$. Here $\frac{1}{4\pi\epsilon_0}$ and $G$ are fundamental constants of nature. That means that we have no explanation for why they take on the values that they have and assume, particularly in the case of $G$, that we probably never will. As we will see shortly, Section 2.6, the values of the fundamental constants determine the size of things.

From forces and kinematics almost all of physics can be developed. Certain derived concepts are so important that they take on a fundamental nature. For example: work done by a force which is the force times the distance through which the force acts and kinetic energy which is the energy of motion, and which, for slow moving particles, is $\frac{1}{2}m\vec{v}^2$ where $\vec{v}$ is the velocity. For special cases, there is also an energy of position called the potential energy. For instance, for places not too high above the earth, the potential energy for an object of mass, $m$, at a height $h$ is $mgh$ where $g$ is the acceleration of objects released from places not too high above the earth.

There are two types of momentum: linear which is usually $mv$ and rotational which is usually $mr\omega$ where $r$ is the distance from the axis of rotation and $\omega$ is the angular speed.

You should be aware of the famous conservation laws, such as conservation of energy and momentum. There are two forms for the law of conservation of energy—the equivalence of work and the total energy (both kinetic and potential energy). There is also a related energy conservation law that comes from thermodynamics, the study of heat. In this law, energy is not only mechanical energy, it is also thermal energy and involves concepts like temperature. In this course, we will find a more general definition of energy
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and momentum, see Section 5.1.

1.3 The Role of Mathematics

To most people, there is a big difference between mathematics and physics. This is not the case and, until very recently, all the mathematics that existed had been developed in response to a need for a language that could describe a physical phenomena. The mathematics was not developed and at hand for use. In most cases, the physicist or physicist/mathematician had a problem and invented new mathematics that was needed to provide an appropriate description of the physical system under study; Newton invented the calculus to have a language to describe objects that changed their position; Dirac invented the delta function to describe phenomena in quantum mechanics and this lead to distribution theory.

One of the most important points in this course is to clarify the relationship between mathematics and physics. Mathematics is a carefully articulated set of rules for the manipulation of carefully defined objects. The objects and the manipulations are constructed to have the aspects of interest to the problem at hand. Almost all the mathematics taught at the university level was invented to analyze a physics problem. It is only in the past century that the elements of the mathematics have become rich enough that mathematicians have been able to develop systems that do not have a counterpart in physics. Even in these cases, it is possible that these “mathematical” systems may find a surprising and new application in physics. In this regard, mathematics is a tool for the analysis of phenomena. A very powerful tool since it has had all of its logical elements carefully vetted so that all the manipulations are consistent. It is also an intuitive tool that you could develop if you think about it. For us, mathematics is a language that, because its algorithms are precise and logically consistent, enables anyone to completely understand what is said. It is the objectification of the thinking process. Mathematics is the process for reducing our thought processes to an algorithm. Mathematics is not a substitute for thinking but it provides a framework in which details of the thinking process are codified.

1.3.1 Mathematics and Symbols That You Should Know

Mathematics is both a language for the description of phenomena and a tool for analysis. Both of these aspects are important. Mathematical terms, such as “radian”, “linearity”, “variable”, and “sum” should all be well understood. Techniques of analysis, such as analytic geometry and algebra, are
invaluable in the analysis of complex situations. In addition to understanding and using the vocabulary of mathematics, you must also understand concise notation. In the following sections I will detail the essential mathematical skills that will be required for this course.

**Number Skills**

It may seem trivial but many people do not have a sense of quantity. This is often traced to an inability to appreciate the order of magnitude of a number, see Section 1.4.3 for further comments. A great way to assess magnitude is by using scientific notation; three million is $3 \times 10^6$. Of course, to use scientific notation, you need to understand the use of exponents, $x^a \times x^b = x^{a+b}$. Using these rules, you can perform algebraic and numeric manipulations with large and small quantities.

Regardless of the ease of manipulation, it is important to realize that an increase by a factor of 10 appears in the exponent as an addition of 1. This is a big change. What if you were suddenly ten times taller? Are there people that are ten times taller than you? It is in this sense that people are the order of $10^0$ meters tall, see Section 1.4.3.

Scientific notation also allows you to discuss the precision of a quantity. In other words, the notation allows you to report the size and the units but also how precisely determined the value is. In Section 1.4.2, there is a list of things that people should know. These are expressed in scientific notation. Most of the items on that list are measured quantities and as such have a certain precision, see Sections 1.4.3, and 2.3.2. The general rule used in scientific notation is that the precision is the range of values obtained by increasing and decreasing the last digit by one unit. In addition, the number has a certain accuracy. The number is accurate if the “real” value is within the precision. For example, we say the the radius of the earth is $6.4 \times 10^3$ km. The precision of this value is at the level of the second digit. By writing it this way we are indicating that we expect that the “real” value is somewhere in the range of $6.3 \times 10^3$ km and $6.5 \times 10^3$ km. The value $6.4 \times 10^3$ km is accurate if the “real” value is somewhere in the range of the expressed precision. An interesting example in understanding scientific notation and precision and accuracy is the value of g on the list. There is some ambiguity. Does the 10 indicate the power of ten or the front digits. If it is the front digits, it is accurate in the sense that the precision implies that the real value of g is between $11 \text{ m sec}^{-2}$ and $9 \text{ m sec}^{-2}$. This is what is meant by the way that it is written. It is not $1 \times 10^0 \text{ m sec}^{-2}$. Maybe it would have been better to write it as $10 \times 10^0 \text{ m sec}^{-2}$ but that would be overkill.
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There may be several reasons why you do not give the “exact” real value. A very important one may be that it, like most physics quantities, is a measured quantity and thus there is an intrinsic limit to how well it can be known. When you look up the values of quantities in a good textbook they will generally give the value with much more precision than I have shown. In our example of the earth’s radius, you will find numbers like $6.371 \times 10^3$ km which is the value given in the text for Phy 302k [Giancoli]. I am not sure what motivates the author to select that level of precision. It is too much to remember and is more than is necessary for most purposes. If you look it up in a tables book, it will be measured to a much higher precision. In the CRC, a popular table book for physical constants, it is given as $6.378245 \times 10^3$ km. Note though that this is defined as the “mean equatorial radius” of the earth because at this level of precision we have to be very precise about what we are discussing. The distance from the center of the earth to the edge varies by more than this at different places. This is another reason that you may limit the precision of a value: variations in the thing you are measuring. The earth’s radius is a good example. The earth is not a perfect sphere. The earth is an oblate spheroid and the north south radius and the mean equatorial radius differ by approximately 21 km. Even if you discuss the equatorial radius there is a 20 km variation due to mountains and valleys. That is why the table book calls it the “mean” equatorial radius.

How much precision is appropriate? I take a very pragmatic view on this subject. You should only use the precision that you need for the problem at hand, and usually you do not need much. In this age of hand calculators, there is a tendency to use the precision of the calculator. I do not own a calculator and feel strongly that the precision should be set by the problem and not the calculating instrument. Going back to the value of the gravitational acceleration on the list of things that everyone should know, Sec 1.4.2, you will notice that I list the acceleration of gravity as $10 \frac{m}{s^2}$ and not as the famous $9.8 \frac{m}{s^2}$. These values differ by 2 parts in 100. In our day to day observations, we are not measuring lengths and times to that precision. So why insist that the acceleration of gravity be to such a high precision? In fact, for the purposes of this course, you will be able (in most cases) to work to a precision of one significant figure. Sometimes less.

Among your number sense skills you should also have some feel for how probabilities operate. If A and B are independent and A has a probability of occurrence of $p_A$ and B has the probability of occurrence of $p_B$, then the probability of occurrence of A and B is $p_A \times p_B$. The probability of A or B occurring is $p_A + p_B$. 
Number sense manifests itself most significantly in our Fermi problems. In most of these you will be working at a very low level of precision: for example at the 30% level. When that is the case, you can forget about small effects below that level. For example, suppose you want to estimate the total biomass on the earth. You do not have the information that would allow you to make a better estimate than possibly 50% precision. At that level of precision, you do not need to worry about the mass tied up in mammals; it is negligible. A useful assistance for Fermi problems is the book “Innumeracy” by John Allan Paulos [Paulos 1988]. I recommend it very highly.

**Algebra, Trigonometry, and Analytic Geometry**

These are all subjects that you should have studied in high school. You should be able to use ideas such as linearity, the relationship between solvability and number of equations and unknowns, and the role of redundant solutions. In this course, you should expect to encounter situations with simple polynomial equations and linear systems of up to three unknowns. I will spend some time developing the properties of the exponential function, see Section 18.6.1, and its inverse (the logarithm), but these should not be totally new to you.

The trigonometric definitions and relations will all be required. You should be prepared to encounter situations that deal with the simpler identities, simple angle addition, and very simple trigonometric equations. I will always go slowly in these places.

From analytic geometry, you should be able to analyze problems graphically and recognize the shapes of the conic sections: parabola, hyperbola, and ellipse. You should also be able to do the opposite – identify the shape from the equation. You should be able to translate and rotate the simple forms discussed above and solve systems of simultaneous equations.

**Calculus**

All of you have had some introduction to the basic ideas of calculus. You will not be expected to use any calculus, but you must understand the concept of the derivative and its inverse, the integral. Although you will not have to perform significant manipulations using calculus, you must be able to recognize its importance in some of the manipulations that I will perform. In addition, you will be asked to approximate calculus procedures, such as the computation of a slope or an integral, and you should realize what the approximation means. I will use the symbols of calculus: \( \sum_{\text{paths}} \)
for sum over paths, or \( \int_a^b \) to summarize an argument. In addition, I will use the shorthand of \( \Delta \) for difference or change. You will see the symbol for differentiation as \( \frac{d}{dx} \) or \( \frac{\delta}{\delta x} \) and should interpret it the change in something that is produced by a small change in \( x \). Also I will introduce some very sophisticated symbols for some of the manipulations of fields. This is due to the fact that fields generally depend on several variables, see Section 4.1.2. In these cases, the symbols \( \frac{\partial}{\partial x} \) means the change in something for a change in \( x \) with the other variables held constant. Similarly, \( \frac{\partial}{\partial t} \) is the change in something for a change in time with the other variables held constant. These terms will be carefully described in words. Again, these are a shorthand for a numeric computational process, and you should not allow terror to replace reason.

In this course, I will use the language of mathematics where it is appropriate to state relationships. In some cases, this will be a rather sophisticated use of the concepts of Mathematics. This is the most concise and careful way to say things. There will be many algebraic manipulations and a great deal of quantitative manipulation – sorry. I cannot cover the material in any other way.

**Spirit of the Mathematics**

One of the primary goals of this course is to convince you that, regardless of your previous mathematical background, you can do some quantitative analysis of a problem, any problem. Often, this will mean using rather crude analytic tools, such as rectified paths for line integrals, but some analysis is better than none. By the end of this course, I hope that you will feel comfortable working problems until you find a satisfactory answer. I will push you until you overcome the stage in which you feel that you cannot get an answer because you lack some analytic skill. Don’t say that you cannot understand something until you learn some esoteric mathematical skill. All things are understandable without advanced mathematics or, at least, I will try to convince you that is the case.

### 1.4 First Day Handout

The first class day handout lists the general policies and grading procedures for the course. The following items are special aspects of this course that merit special comment:
1.4.1  Fermi Problems

These are all simple reasoning problems that are generally solved by making some very basic and plausible assumptions based on your experience or by using simple facts that you already know, see Section 1.4.2. You actually know more than you think you do, and you can apply this information in many interesting circumstances. These problems also point out the value of having a quantitative perspective and often deal only with order of magnitude estimates, see Section 1.4.3.

Fermi problems are named after the famous Italian-American physicist, Enrico Fermi, who was well-known for setting and solving them. For example, he taught at the University of Chicago and he would ask his class to estimate the number of piano tuners in Chicago. Fermi was associated with the Manhattan project, and there are several stories about him and order of magnitude estimates. In one, as he is being escorted about the laboratory in a jeep on the dusty roads of New Mexico, he asks the driver how thick a layer of dust could accumulate on a car window before falling. Knowing the strength of chemical bonds and the size of atoms etc., he could quickly calculate the adhesion and check his result with the amount of the dust on the windshield. The most famous story is how he estimated the strength of the blast from the first atom bomb test by releasing a sheet of paper and noting its deflection as the shock wave passed.

1.4.2  Things Everyone Should Know

In order to develop your quantitative perspective you have to know some things. Many of these things you can know just by looking around; some have to be put together from other facts. In any case, the world is a knowable place, and you already have many of the instruments you need to know it. The sights that you see, the sounds that you hear, and tactile feel of the world around us, supplemented with simple devices, can all be understood and fit into a pattern that allows for all of us to lead a fuller and more meaningful life. Achieving this requires the willingness to approach the world in a quantitative fashion, along with the willingness to probe the world with simple experimental questions. Although in many cases we can reason out the magnitude of some of these facts, such as the radius of the earth, others, like Avogadro’s number, just have to be remembered. The following is a list of things that I think you should know, and you will be expected to know them.
1.4. FIRST DAY HANDOUT

Some Things That Everyone Should Know

Order of Magnitude

- Gravitational acceleration: \(10 \, \text{m/s}^2\)
- Densities of solids and liquids: \(10^3 \, \text{kg/m}^3\)
- Density of air at sea level: \(1 \, \text{kg/m}^3\)
- Length of day: \(10^5 \, \text{s/day}\)
- Length of the year: \(\pi \times 10^7 \, \text{s/year}\)
- Earth’s radius: \(6.4 \times 10^3 \, \text{km}\)
- Angle of width of finger at arm’s length: \(10^{-2}\) or \(\frac{\pi}{180} \approx 1.7 \times 10^{-2}\)
- Thickness of paper: \(0.1 \, \text{mm}\)
- Mass of a paper clip: \(0.5 \, \text{gm}\)
- Heat output per person: \(10^2 \, \text{W}\)
- Highest mountain, deepest ocean: \(10 \, \text{km}\)
- Earth moon separation: \(3.8 \times 10^5 \, \text{km}\)
- Earth sun separation: \(1.5 \times 10^8 \, \text{km}\)
- Atmospheric pressure: weight of 1 kg/cm² or a 10 m column of water
- Avogadro’s number: \(6 \times 10^{23} \, \text{atoms/gm mol}\)
- \(h\) or Planck’s constant: \(1 \times 10^{-34} \, \text{J s}\) or \(6.6 \times 10^{-22} \, \text{MeV s}\)
- Atomic diameter: \(10^{-10} \, \text{m}\)
- Nuclear diameter: \(10^{-15} \, \text{m}\)
- Atomic masses: \(1.6 \times 10^{-27} - 4 \times 10^{-25} \, \text{kg}\)
- Energy conversion: \(1 \, \text{eV} \approx \frac{3}{2} \times 10^{-19} \, \text{J}\)
- Energy content of a chemical bond: \(2 - 5 \, \text{eV}\)
- Energy content of temperature: \(10^{-4} \, \text{eV/°K} \approx 10^{-23} \, \text{J/°K}\)
- Energy content of food: \(1 \, \text{Cal} = 10^3 \, \text{cal} \text{ and } 1 \, \text{cal} \approx 4 \, \text{J}\)
- Charge of the electron: \(1.6 \times 10^{-19} \, \text{C}\)
- Electron mass: \(10^{-30} \, \text{kg}\)
- Ratio of the electron and proton masses: \(1/2000\)
speed of light ................................................. $3 \times 10^8 \frac{\text{m}}{\text{s}}$

speed of sound ................................................... $10^2 \frac{\text{m}}{\text{s}}$

wavelength of light ............................................ $6 \times 10^{-7} \text{ m}$

population of the US ........................................ $3 \times 10^8 \text{ people}$

population of Austin .......................................... $7 \times 10^5 \text{ people}$

$\pi^2$ ................................................................. 10

$\ln 2$ ................................................................. 0.7

$(1 + x)^n$ ....................................................... $\approx 1 + n \ x$ for $x \ll 1$

$\sin x$ ............................................................. $\approx x - \frac{x^3}{3!}$ for $x \ll 1$

$\cos x$ ............................................................. $\approx 1 - \frac{x^2}{2!}$ for $x \ll 1$

### 1.4.3 Order of Magnitude Estimates

As you can see from the above list, for most purposes it is only important to know “about” how big an effect is. Often because to the crudeness of the measurement, you can only know something to within an order of magnitude. In most cases, this may be to within a power of ten. In some cases, you can know it a little better say to within a percentage, say 10%. The range into which you can know a certain value is the precision. It is important to realize that all measurements and estimates have both an accuracy and a precision. In a reasonable estimate or measurement, the true value should be within the range set by the precision. In other words, when you state a value, you are really giving its value and range of correctness for that value. By consensus, an easy way to express the range of precision is the range allowed by letting the last non-zero digit in the number increase and decrease by one unit. For example, you think that the population of the US is 250,000 people. Absurd, but this is an example. By saying 250,000 you are implying that the population is between 240,000 and 260,000. The actual value falls well outside this range and, thus, this is an accuracy problem. These issues, along with the use of precision, are discussed in Section 1.3.1.

The issue here is that often it is valuable to know something within a very large range called order of magnitude. In this case there are no digits. This is usually appropriate because the range of phenomena is so large. For instance in the “Things” list above the energy content of temperature is given as $10^{-4} \frac{\text{eV}}{\text{R}} \approx 10^{-23} \frac{\text{J}}{\text{R}}$. In fact, these numbers are known to a very high precision but that is not relevant to most usual quotidian applications. For instance, is something hot enough for the chemical bonds to break?
1.4.4 Home Experiments

In each homework assignment, there will be a home experiment. These are simple exercises that require basic materials that will be easily available or provided to you. There are two reasons why we perform home experiments. One: physics is an experimental science. The only route to knowledge is through experiment and, no matter how wonderful your reasoning, if it disagrees with experiment, then the theory has to be replaced. In a course like this one, there is a tendency to see all progress as theoretical when in fact it is the other way around. The problem is that most of the experiments that treat the basic concepts of modern physics are not accessible using a simple home apparatus. Therefore, many of the home experiments are not directly related to the course content. We still set them because they keep us aware of the experimental basis of our knowledge. The second reason is that the world is a knowable place, and it is only by manipulation that we can really test our understanding. This is an important point that differentiates physics from other subjects. In physics, you do not just accept the picture of the world that you are given, you test it, and then you change the conditions to see if it behaves as predicted. This same idea is important to the whole study of physics. You must always seek other ways to test and verify each idea.

1.4.5 Review Syllabus

The syllabus is a general outline of what I would like to cover in this class. There are two different syllabi, one for the fall semester and one for the spring. In both first, I will cover the background of classical physics. This will take a few weeks. My approach to classical physics will be based on principles that will be new to all of you, but which are the techniques used by modern physicists. Next in the fall, we will cover the modern physics of large scale phenomena. This is the theory of space-time called “general relativity”. Before we can cover general relativity, you must have a solid understanding of the “special theory” of relativity. Finally, at the conclusion of general relativity, we will discuss some aspects of cosmology. Then, we will develop the modern theory of light. This is our introduction to quantum phenomena. We will use our study of quantum phenomena to develop our understanding of what things really are. In the spring semester, we will reverse the order of the two general modern physics topics.

As stated above, this syllabus outlines what I would like to cover. It is very aggressive, and in all likelihood we will settle for less. In the fall, it will
be mostly large scale phenomena and, in the spring, microscopic physics. A great deal depends on how the class proceeds.

1.4.6 Text

These notes are the primary text for the course. You should read them carefully before lecture. Another text is “QED” by Richard Feynman [Feynman 1985]. This book is an incredible discussion of microscopic physics, and in our case, it is an introduction to the study of light. Another text that treats these issues is Rae’s book, “Quantum Physics: Illusion or Reality” [Rae 1994]. This is the basis for our discussion of the nature of the material world on the small scale. The auxiliary material on relativity comes from the Space Time Traveler by Moore [Moore 1998]. In addition, the book “Innumeracy” by John Allan Paulos [Paulos 1988] is a great foundation for quantitative reasoning. You should read it immediately. It is an easy and enjoyable read. Readings from the other books will be set at the appropriate times. There will also be some specialized handouts during the semester.

1.5 What is Physics?

Physics is an incredible accomplishment. It sets the tone for all our understanding of all the phenomena of the world around us. Other bodies of knowledge generally aspire to the level of prediction that is required for a physics theory to be accepted. Very few, if any, achieve it. The development of our understanding of the physical world is the greatest accomplishment of mankind. Two thousand years from now, when they write of the significant accomplishments of this century, they will record as the most significant events the discovery of the quantum mechanics and relativity. That is, of course, if they were not discovered earlier on another planet. The great wars of this century will only be noted in passing. Richard Nixon and the collapse of the Soviet Union will hardly merit a footnote.

It is important to realize how the process of physics works. The basic operating procedure of physics is easy to state. It was hard to discover and is often harder still to adhere to in many circumstances but it is the most successful approach to knowledge that has been developed. It is the careful observation of the world followed by the development of idealized objects that reproduce this behavior followed by the extension of these ideas until they either fail of their own accord (because of some intrinsic property) or they are found to no longer agree with experiment. When that happens, you search for a new construction that includes both the successful results of the
first theory and extends to include the new range of phenomena all of which agrees with this new construction. The process is always under continuous development. In this sense, it is hard to envision an end to physics. The phenomena may become more remote from our day to day experience and use but there will always to new questions that emerge as our understanding grows.

Since this approach to the basis of physics may be new to you, it will be worthwhile to give some examples. One set of examples is obvious. The material of this course was selected to illustrate this point. It follows the development of the several modern theories of light. We do not go back to the ancient arabic and greek theories of light but begin with Fermat’s Least Time Theory since it was the first to be based on the experimental methods articulated by Galileo. For an interesting readable account of the ideas that preceded Fermat and even the controversy that surrounded Fermat’s ideas, see the book by Park [Park 1997]. Although developed in about 1660, this idea of the least time of travel for rays of light is still the principle that governs modern lens design. The subsequent development of Fresnel was a consequence of new families of experiments, interference and diffraction, that could not be understood with Fermat’s theory. Note that Fresnel not only had to develop an algorithm for computing optical phenomena associated with interference and diffraction he had to develop a method that contained the results of Fermat in the correct limit. It is in this sense that a new theory supplements an old theory. It does not make it incorrect. As stated above, Fermat’s least time is still used in modern complex lens design. You could also use all the machinery of Fresnel to do lens design but most of what you learn is more than you need to know to make a good lens. Similar statements can be made about how quantum mechanics extends classical mechanics and general relativity extends Newton’s theory of gravitation. Actually in all these cases, since the new theory invariably encompasses a greater range of phenomena, it is better to view the older theory as a special case of the new theory.

In another example, we study the General Theory of Relativity which is the name for the modern theory of gravitation. We all know that Newton developed a very successful theory of gravity. As Einstein tried to develop a new theory of gravity consistent with his ideas from special relativity, it was among the most difficult problems of the new theory to replicate all the successes of Newton’s theory. He also had to find new phenomena that Newton’s theory did not get correct and again these were difficult to come up with. It was for this reason that for many years it was very hard to find confirming experiments for the General Theory. Thus, Einstein’s theory does
not replace Newton’s but includes it as a limiting case. A similar argument can be made for the development of quantum field theory as a replacement for classical mechanics.

This is such an important subject that it bears repeating. Physicists invent idealized forms and endow them with properties that are seen in nature. The manipulation of the idealized forms leads to behavior that mimics those seen experimentally. Once a set of forms provide a complete set of descriptors for some class of phenomena further manipulation of the forms may lead to behaviors that have not yet been manifested experimentally. This is the essence of prediction from theory. If the forms fail to behave like the phenomena being modeled, the theory fails, [Popper 2002], and the idealized form is extended or in some extreme cases replaced until it produces behavior that is observed including all the behavior that had been described successfully before. In this sense, the great theoretical achievements such as Maxwell’s equations, see Section 4.3, are a catalogue of the results of thousands of experimental observations described though a concise organization of idealized forms.

This same perspective on theory construction is also helpful in understanding the role of mathematics in physics. Historically, mathematicians have studied and extended the properties of the idealized forms that have emerged from the observations of nature. In more recent times, because of the richness of the ever growing set of forms, they have been able develop new forms and codify a greater range of phenomena even those needed for descriptions of natural phenomena. In some cases, physicists have found that a set of forms that was developed by mathematicians only for their abstract beauty have application in nature and, in some cases, physicists have had to ignore the constraints that the mathematics have imposed only to then open a new realm of mathematical investigation, a healthy give and take.

1.5.1 Range of Phenomena

“Powers of Ten”

This is a film that starts by looking at a man on a blanket at a beach along Lake Michigan in Chicago. It then expands the point of view until it covers the known universe. Then it focuses in until it is looking at a scale so small that you can see the quarks in a proton.

When students view this film there are many reactions. There are 40 powers of ten between the largest scale phenomena and the small scale ob-
1.5. **WHAT IS PHYSICS?**

Observations. This is truly a fantastic range. No other field of knowledge can come close to a basis of explanation with that range. Generally, we feel that we have some handle on all the observed physical phenomena that occur in this interval. The real ends of our knowledge come at the peripheries. On the large scale, gravity dominates and we require the use of the General Theory of Relativity. Currently, we have difficulty with the origins of space-time or, in the vernacular, the origin of the universe and the union of gravity with microphysics. On the small scale, we have troubles with the basic constituents of matter. Both of these subjects are important and are the two fundamental themes of this course. The Theory of Relativity is our look into space-time and quantum mechanics has given us framework for describing the fundamental constituents of matter. The problem of combining quantum mechanics with the General Theory is among the most pressing problems of current physics.

In the film “Powers of Ten”, people are always impressed by the contrasting periods of activity and inactivity as the scale of length is changed. This pattern indicates the separation of phenomena with differing length scales. Atoms come in only one range of sizes, and the same holds true for galaxies. Stars and biological systems occur in a range of sizes. Sizes are set by the same laws of physics that govern the behaviors of the matter. In the case of atoms, it is the mass of the electron and Planck’s constant that determine size and behavior. We will have a great deal more to say about Planck’s constant in the course of this semester.

**Plot of Masses and Lengths**

*See Insert*

The attached insert is a scatter plot of lengths versus masses. In a scatter plot, you pick two independent variables, in this case length and mass, and for each element of the group under study put at point on the plot at the coordinates associated with that element. For instance, we could be studying GPA versus height. Then on the graph with the height as the ordinate and the GPA as the abscissa, each student with their GPA and height is represented as a point on the plot. These are called scatter plots because, if the variables are unrelated, they will not fall in a pattern on the plot. You would expect that the scatter plot of points of height versus GPA would be all over the allowed ranges of the variable. In the insert, we are scatter plotting the length of a thing against the mass.

The first feature of this scatter plot is that for our case the range of
values for length and mass is extraordinary. This was also pointed out in the movie “Powers of Ten,” see Section 1.5.1. For this reason, to fit all the phenomena on a single piece of paper, we use as the ordinate and abscissa the log of the length and the log of the mass. This is thus a log log scatter plot. As stated earlier, no discipline can claim a quantitative understanding of phenomena in such a range, approximately forty powers of ten in each variable. The next issue is that we want a scatter plot of things of interest. We want a point on the plot for people. What is the mass for a people? There is a range of masses for adult males that can vary by about 40%. This is a small range of variation and is within any size point that can be drawn on the plot. Thus for mass we can chose the generic value of 60 kg and not worry about men or women or any other variation. What do you do for length? There are several candidate lengths for a person – height, ear lob length, ... What do you pick? Generally, once you are talking about people all the other length choices scale as the height, i.e. the ratio of the index finger length to the height is a universal constant that is the same for all people. Thus we can chose the height or largest dimension to set a standard. These are examples of scaling laws which are discussed more in Section 2.5.2. In this general sense, we can now ascribe a length scale to most objects and be reasonably consistent. On the huge range of scales that we are dealing with the variations within the category that we are interested in is negligible.

Expanding out from humans, we want to put on other biological systems. Again, it turns out that, on the range of scales we have here, the different biological systems such as bacteria and ducks have a definite mass and length range and can be represented reasonably well by points. Note that all biological systems fall within a small region in the center of the diagram. It is no accident that this stuff is central but notice that it is also a small part of the total plot. It is central because we got to pick the scale of length and mass, see Section 2.3, and we are biological systems. It is a rather small region of the plot because biological systems are complicated and cannot be to small and still have all the parts that it takes to operate. They cannot be too large for it to be a coherent whole. There was a report some years ago of a mold in Wisconsin that was several kilometers across. Even here, we could debate whether this constituted one living system.

Also note, the patterns of phenomena on the plot. The points are not scattered on the plot but fall on a straight line. What is the implication of the straight line? A straight line on a log-log plot implies a power law
relationship between the variables:

$$\log M = a \log L + b \Rightarrow M = e^b L^a = c L^a$$  \hspace{1cm} (1.1)

where $a$ is the slope of the line and $b$ is the intercept and $c \equiv e^b$. In our case, $a$ is three and this linear relationship is just another reflection on the item in Things That Everyone Should Know, Section 1.4.2 that most solids and liquids have the same density of about $10^3 \text{ kg m}^{-3}$. This is really no surprise. All biological systems have about the same density as water.

This feature of the straight line carries on as objects like battleships and pyramids are added except that we note that it is not the same line but one displaced a bit. Again we are seeing that all the heavy things also have about the same density. It is just slightly higher and really justifies our statement that all things have nearly the same density including the heavy things. What is the meaning of this near equality of all densities? The first thing to note is that although we had said that we would make a scatter plot of all things we really have not. We have put on this plot only things that are composed of atoms touching each other or what are called condensed matter systems as opposed to gases for example. We did not put on things like the atmosphere. You notice this deviation as you look to the top of the diagram. As we add the planets, they are still pretty much on the line but objects like galaxies and pulsars are far from the line and not even a point on the plot. We can roughly conclude that all objects made out of atoms that are touching are of comparable density. Well, we know that this is not strictly true. Solid lead has a density that is 10 times that of water. Since the atomic weight of lead and water are also in the ratio of about 10 to one, we conclude that the size of lead atom is about the same as oxygen, the dominant mass in water. In other words, all atoms are about the same size. This is a striking fact. A lead atom has 82 electrons and is still about the size of a hydrogen atom which has only one. In other words, the scaling law for the size of an atom with mass is not the usual one but is instead $M^0$. This is a consequence of the competition between the attraction of the Coulomb force and the Pauli exclusion principle. There is certainly a tremendous amount of information on this diagram. This will hopefully make more sense as the semester develops.

There are several other features of the plot that will make sense as the semester develops. Black holes are a consequence of the theory of general relativity and these forbid mass and length relations in a large part of the diagram. Protons and neutrons also obey the Pauli principle but since they have a different mass than the electron, objects made of them – nuclei and
pulsars – have a different density line. Again, this will all become clear as the semester develops.

It is worthwhile to point out that the same clustering of phenomena that you saw in the film “Powers of Ten” is present here. This separation of phenomena into groupings is one of the great accidents of physics and a very fortunate one. When you are dealing with atoms, you do not need to worry about gravitation. The masses in atoms are small enough and the gravitational force small enough that you only have to consider the electromagnetic force. Also the velocities are small enough that you can neglect the effects of special relativity. In the nucleus again you can neglect gravity. In a galaxy you have to worry about gravitation but, since most of the matter is electrical neutral, you can neglect the electromagnetic effects. It seems unlikely that the great success of modern physics could have been achieved without the ability to categorize and separate the phenomena that we deal with.

1.5.2 Reductionism and General Principles

Much of the success of physics stems from the ability to reduce the whole into smaller parts, to understand the small parts, and to reconstruct the whole from our understanding of these smaller pieces. The recent developments of many other sciences, such as biology, can be attributed to the ability to use these reductionist techniques. Although there has been some speculation that, with our current theories of matter, we may be approaching the limit of this technique and that a final theory may be on the horizon, this is at best speculation. For now there is no reason to believe that the successes of reductionism have all been identified.

Another aspect of the reductionist argument that most people fail to understand is that the elements that constitute the whole do not have to act like the whole. The only requirement is that when the whole is reconstituted, it must behave as observed. For example, a building is made of bricks. A building is not like the brick it is made of, and we all seem to be able to accept that. Similarly, a brick is not like a building, even though bricks are the primary constituents of the building, and again we accept that. When discussing the elementary constituents of matter, the tendency is to require that the constituents have properties like the whole. This is at the heart of the problem of understanding “wave particle duality” problems of modern quantum mechanics. There is no requirement that the elements that are at the basis of the reductionist process must be like the objects that they constitute. This prejudice is not only apparently not required of the theory,
but it does not hold for the elementary constituents of matter.

Although most of the successes of physics are attributed to reductionism, the great global principles of physics have also played an important role. In fact, the idea of understanding is usually based on some underlying global principal such as the concept of a mechanical system. We understand something when we can reduce its operation to that of a mechanical system. This was the accepted approach during the later part of the 19\textsuperscript{th} century. As you will learn in this course, we now have a different criteria. Today, we understand something when we can describe its operations from an action principle. Another example is the idea of force used in Section 1.2.3. In that case, you realize that you can separate the effects from individual sources, see Figure 1.1, and that these operate the same way independently of the presence of other sources. This separability is at the heart of reductionism. In addition, the level and the object that you add as independent sources are generally derived from a global principal. The identification of an objective reality and the articulation of causality are important global concepts that do not exist in all cultures. The assumption of their role in the material world is at the heart of modern physics.

1.6 Problems

1. How many people die in Austin each year? Indicate your reasoning and pick one of the following: $1 \times 10^2$, $\pi \times 10^2$, $8 \times 10^2$, $1 \times 10^3$, $\pi \times 10^3$, $8 \times 10^3$, $1 \times 10^4$, $\pi \times 10^4$, $8 \times 10^4$, $1 \times 10^5$, $\pi \times 10^5$, $8 \times 10^5$.

2. Estimate the order of magnitude of the mass of a speck of dust, a grain of salt, a mouse, an elephant, the water that is equivalent to 1 inch of water over 1 mi\textsuperscript{2} of rainfall, a small hill, and Mount Everest.

3. How tall is Jester Dormitory? (Say, Jester West) Find some way to measure the height to within an accuracy of 5%. If it is to 5%, do we need to specify the tower of Jester?

4. Pick a tree. (Any tree! Well, any live deciduous tree with more than 10 leaves!) How many leaves are on it? Try to get a fairly accurate count by counting the number of leaves in some small volume that you can measure accurately and then roughly measuring the leaf-bearing volume of the tree.

5. (a) What is the height of the National Debt in pennies stacked on top of each other.
(b) Suppose these pennies were distributed uniformly across the land area of the contiguous 48 states. What distance would separate each penny from its nearest neighbor?

(c) How many tons of copper would be required to make these pennies?

(d) Suppose they were distributed by dropping them from the sky. If you were standing outside, how many pennies would hit your head, on average?

(e) If you stuck your finger straight up, what is the probability that a penny would land on it? and stick?

Home Experiment 1. You were given two pieces of paper and a string. Using the string as your unit of length, measure the perimeter of the two pieces of paper and the distance between the two corners indicated as A and B in the figure. Do this with a precision of at least 10%. For each sheet of paper, take the ratio of the length of the perimeter to the distance between A and B. Discuss these results in terms of dimensional variables and scaling. Discuss the use of your string as a candidate for the international standard of length. Consider the area of the two pieces of paper. How does the area scale with perimeter?

![Figure 1.2: Figure of Home Experiment # 1](image-url)