PHY355 Useful Formulae

Relativistic Energy and Momentum:

$$\mathbf{p} = \gamma m \mathbf{v}$$
$$E = \gamma m c^2$$

Energy-momentum invariant:

$$E_0^2 = E^2 - p^2c^2$$

For photons: $E = pc$

Lorentz Transformations:

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \frac{v}{c^2}x) \\ u'_x &= \frac{dx}{dt} = \frac{u_x - v}{1 - vu_x/c^2} \\ u'_y &= \frac{dy}{dt} = \frac{u_y/\gamma}{1 - vu_x/c^2} \end{aligned}$$

where

$$\gamma = \frac{1}{(1 - v^2/c^2)^{1/2}}$$

Length Contraction:

$$l = l_0(1 - v^2/c^2)^{1/2}$$

Time Dilation:

$$\tau = \frac{\tau_0}{(1 - v^2/c^2)^{1/2}}$$

Doppler shift for light:

$$\nu' = \nu \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/2}$$

A space-time invariant:

$$\begin{aligned} s^2 &= (x)^2 + (y)^2 + (z)^2 - (ct)^2 \\ &= (x')^2 + (y')^2 + (z')^2 - (ct')^2 \end{aligned}$$

Bohr formula for transition wavelengths in any one electron atom:

$$\frac{1}{\lambda} = Z^2 R(\frac{1}{n_l^2} - \frac{1}{n_m^2})$$

where R is the Rydberg constant (109677 cm^{-1}) and Z is the atomic number.

Planck Blackbody Spectral Distribution:

$$\frac{dR}{d\lambda} = \frac{2\pi hc^2}{\lambda^5(\exp^{hc/\lambda kT} - 1)}$$

$$\frac{dR}{d\nu} = \frac{2\pi h\nu^3}{c^2(\exp^{h\nu/kT} - 1)}$$

Wein's Law:

$$\lambda_{\mathrm{max}} = \frac{0.00290 \text{ m-K}}{T}$$

Compton scattering :

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

Mosley formula for characteristic x-rays:

$$\frac{1}{\lambda} = \frac{3}{4}R_H(Z-1)^2$$

Rutherford α scattering :

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi}{2}\left(\frac{2Zke^2}{E_k}\right)^2 \frac{1}{(1 - \cos\theta)^2}$$

Rutherford angle-impact parameter relation:

$$b = \frac{Z_1 Z_2 ke^2}{2E_k}\left(\frac{1 + \cos\theta}{1 - \cos\theta}\right)^{1/2}$$

X-ray diffraction equation :

$$n\lambda = 2d\sin\theta$$

de Broglie wave relation: $\lambda = \frac{h}{p}$

Heisenberg unbestittheit principle:

$$\Delta x \Delta p_x \geq \hbar/2 \quad \text{or} \quad \Delta E \Delta t \geq \hbar/2$$

One dimensional time dependent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V\Psi(x,t) = \imath\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

One dimensional time independent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\psi(x) = E\psi(x)$$

Particle in a box wave functions and energies:

$$\Psi_n(x,t) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}\exp(-\imath\frac{\hbar\pi^2 n^2 t}{2mL^2})$$

$$E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$$

Harmonic oscillator (potential $V(x) = \frac{1}{2}\kappa x^2$) first few wave functions and energies:

$$\begin{aligned}
\psi_0(x) &= \exp -\xi^2/2 \\
\psi_1(x) &= 2\xi\exp -\xi^2/2 \\
\psi_2(x) &= (4\xi^2 - 2)\exp -\xi^2/2 \\
\psi_3(x) &= (8\xi^3 - 12\xi)\exp -\xi^2/2
\end{aligned}$$

$$E_n = (n + \frac{1}{2})\hbar\omega$$

where $\xi^2 = \frac{\sqrt{mk}}{\hbar}x^2$, and $\omega = \sqrt{\frac{\kappa}{m}}$.

Transmission through a barrier (tunneling):

$$T = \left(1 + \frac{V_0^2\sinh^2(\kappa L)}{4E(V_0 - E)}\right)^{-1}$$

for κL much greater than 1:

$$T = 16\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)\exp^{-2\kappa L}$$

where $\kappa = \sqrt{2m(V_0 - E)}/\hbar$

Transmission above a barrier:

$$T = \left(1 + \frac{V_0^2\sin^2(kL)}{4E(E - V_0)}\right)^{-1}$$

where $k = \sqrt{2m(E - V_0)}/\hbar$

Hydrogen atom wave functions and energies:

$$\psi_{nlm_l}(r,\theta,\phi) = R_{nl}(r)Y_{lm_l}(\theta,\phi)$$

$$E_n = -\frac{\hbar^2}{2\mu a_0^2}\frac{1}{n^2}$$

where $a_0 = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}$

angular momentum $L = \sqrt{l(l+1)}\hbar$

and projection $L_z = m_l\hbar$

Zeeman energy $V_B = \frac{e\hbar}{2m}m_l B$

Maxwell-Boltzmann Distribution

$$F_{MB}(E) = A\exp(-\frac{E}{kT})$$

Constants and conversions:

Boltzmann constant

$k_b = 1.381 \text{ x } 10^{-23}$ J/K

Speed of light in a vacuum

$c = 2.998 \text{ x } 10^8$ m/s

electron mass

$m_e = 9.11 \text{ x } 10^{31}$ kg $= 0.511$ MeV$/c^2$

proton mass

$m_p = 1.6726 \text{ x } 10^{27}$ kg $= 938.27$ MeV$/c^2$

neutron mass

$m_n = 1.6749 \text{ x } 10^{27}$ kg $= 939.57$ MeV$/c^2$

Stefan-Boltzmann constant

$\sigma = 5.67 \text{ x } 10^{-8}$ W/m^{2}K^4

Planck's constant

$$\begin{aligned}
h &= 6.626x10^{-34}\ \text{J}\cdot\text{s} \\
&= 4.136x10^{-15}\ \text{eV}\cdot\text{s} \\
\hbar &= 1.054x10^{-34}\ \text{J}\cdot\text{s} \\
&= 6.579x10^{-16}\ \text{eV}\cdot\text{s}
\end{aligned}$$

Combinations:

$$\begin{aligned}
hc &= 1.9864x10^{-25}\ \text{J}\cdot\text{nm} \\
&= 1239.8\ \text{eV}\cdot\text{nm} \\
\hbar c &= 3.1615x10^{-26}\ \text{J}\cdot\text{nm} \\
&= 197.33\ \text{eV}\cdot\text{nm}
\end{aligned}$$

$k_b T = 0.0253$ eV at room temperature

1 eV $= 1.602 \text{ x } 10^{-19}$ J