PHY355 Homework 9 April 25, 2000

"C" Problems

(1) There are two peaks in the radial probability distribution function for the $n = 2$, $l = 0$ state of the hydrogen atom. (The radial *wave function* is given in Problem 3). Find the values of r (in units of Bohr radii a_0) for each peak

(2) (a) List all possible sets of quantum numbers (n, l, m_l) for the $n = 5$ level of atomic hydrogen (you may neglect the electron spin). (b) What is the degeneracy of this level in the absence of a magnetic field? (c) Are there any degenerate levels if there is a magnetic field?

(3) The radial part of the solution for the $n = 2$ state of the hydrogen atom is

$$R_{2,0}(r) = \left(2 - \frac{r}{a_0}\right) \frac{\exp^{-r/2a_0}}{(2a_0)^{3/2}}$$

Show that this function satisfies the radial Schrodinger equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2m}{\hbar^2}(E - \frac{e^2}{4\pi\epsilon_0})R = 0$$

What is the energy E?

"B" Problems

(4) The Lyman-α line in atomic hydrogen is observed to split into three lines, with a spacing of 0.02 nm between two adjacent lines when placed in a magnetic field B. If this spacing is a result of splitting between adjacent m_l states, what is the value of B.

(5) The complete, normalized wave function for the hydrogen 2s state is

$$\psi_{2,0,0}(r, \theta, \phi) = \frac{1}{2\sqrt{\pi}}\left(2 - \frac{r}{a_0}\right) \frac{\exp^{-r/2a_0}}{(2a_0)^{3/2}}$$

For the 2p$_z$ state the wave function is

$$\psi_{2,1,0}(r, \theta, \phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cos\theta \frac{r}{a_0} \frac{\exp^{-r/2a_0}}{\sqrt{3}(2a_0)^{3/2}}$$

Calculate the probability that the electron is inside the nucleus (radius of 1x10^{-15} meters), for these two states.

"A" Problems

(6) In class we showed that a superposition state could have a time dependent expectation value for position. For the particle in a box, the strength of the oscillation of $\langle x \rangle$ was proportional to the integral

$$\int_0^L \sin\frac{n_1\pi x}{L} x \sin\frac{n_2\pi x}{L} dx$$

This is called the *oscillator strength.* Find the relative magnitude of this term for $(n_1, n_2) =$ (1,2), (1,3), (1,4), (1,6) and (1,8). Mathematica can be a big help here.

(7) At the web site, an MS Excel spreadsheet is available for the potential from problem 5 of the homework 8. Download this document and feed it into Excel. To do this, you must hold down the shift key and click on the link; this initiates a download. After you have the file, you need to unzip it and then open it with Excel.

Familiarize yourself with the variables that are presented (all in column B): you can adjust the particle mass, the box width, the height of the potential step, and the particle energy. The calculation forces continuity of the wave function at $x = L/2$. Useful outputs are the graph, and the wave function derivatives at $x = L/2$, and the integrated probabilities for the two halves of the box.

In the raw form, the energy is set to 10 units which is not a valid energy; you can see from the graph of the wave function that the derivative is not continuous at $x = L/2$.

(a) For a barrier height of 5 units, find the ground state energy.

(b) Find a few of the excited state energies, and note the probabilities for the two halves of the box. How do you know what the quantum number is for an energy that results in a derivative match?

(c) Keep the step height at 5 units, set the particle energy to 10 units, and adjust the box width to find the n=10 state. How much probability is in each half of the well?

Try this by hand for a while before attempting the following. Under the "Tools" menu is a choice called "Goal Seek". Use this to minimize the difference in the derivatives by adjusting the energy. This will automatically find an energy value, but only one close to your initial guess. This is typical of minimization problems: the solution is usually not analytic, and often requires a good first guess.

Due: May 2, 2000