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Mass Difference of $\Sigma^\pm$ and Their Anomalous Magnetic Moments*

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THE idea that the various hyperons constitute definite isotopic multiplets is finding increasing use in the explanation of the interactions of strange particles. One would then expect that the relatively small mass difference between components of the same multiplet is due to interactions which are electromagnetic in origin. In another connection Feynman and Speisman[1] and Peterman[2] have shown that the mass difference between neutron and proton can be understood in terms of electromagnetic self-energies, if their anomalous moments are taken into account. Unlike the proton-neutron case, it will turn out that, since $\Sigma^\pm$ are both charged, a much larger mass difference is possible for comparable values of the anomalous moments.

The mass measurements on the Σ hyperon[3] indicate that

$$m(\Sigma^-)-m(\Sigma^+)=(16.2\pm5.5) \text{ electron masses.}$$

If we assume that the Σ is a Dirac particle, we find that the observed mass difference requires that the sum of the magnetic moments of the Σ^+ and Σ^- is positive and of the order of 3 to 4 nucleon magnetons.

The self-energy contribution to the mass of a fermion of charge e and anomalous moment μ is given by (for notation see reference 1)

$$\Delta m=\frac{e^2}{(2\pi)^4 i}\frac{m}{E(p)}\bar{u}(p)\int d^4k \times\left\{\gamma_\nu-\frac{\mu}{4m}(\gamma_\nu k-k\gamma_\nu)G(k)\right\} \times\{p-k-m\}^{-1}\left\{\gamma_\nu+\frac{\mu}{4m} \times(\gamma_\nu k-k\gamma_\nu)G(k)\right\}\frac{C(k)}{k^2}u(p), \quad (1)$$

where $G(k)$ and $C(k)$ are invariant cut-off factors for the divergent integrals. Performing the indicated integration leads to an expression of the form

$$\Delta m/m=(\alpha/4\pi)\{2I_0\mp3\mu I_1+\tfrac{3}{4}\mu^2 I_2\}, \quad (2)$$

where I_0, I_1, and I_2 are positive functions of the mass of the particle and of the cutoffs; the $\mp$ signs correspond to positively or negatively charged particles, respectively. We have chosen two typical forms of cutoff:

Type	$C(k)$	$G(k)$
(A)	$\Lambda^2/(\Lambda^2-k^2)$	$\lambda^2/(\lambda^2-k^2)$
(B)	$\Lambda^4/(\Lambda^2-k^2)^2$	$\lambda^2/(\lambda^2-k^2)$.

From (2), the mass difference of the charged Σ hyperons, can be written in the form

$$m(\Sigma^-)-m(\Sigma^+) =(\alpha m/4\pi)\{3I_1-\tfrac{3}{4}I_2(\mu^+-\mu^-)\}(\mu^++\mu^-). \quad (3)$$

We note that the sum of the anomalous moments has to be positive in order to explain the observed mass difference. Typical numerical values are given in Table I; the cut-off parameters Λ^2, λ^2 were chosen so as to reproduce the observed neutron-proton mass difference.

The anomalous moments in Table I indicate a significant contribution from the virtual emission and reabsorption of mesons. To judge the qualitative implications of these anomalous moments, we have performed a second-order calculation of the anomalous moments due to the virtual intermediate states in which the hyperon dissociates into a baryon and a meson (and still conserves the strangeness quantum number).[4] Thus, the anomalous moments may be due to the following virtual interaction schemes:

(1) $\Sigma^+\to p+\bar{K}^0$, $\Sigma^-\to n+\bar{K}^-$,
(2) $\Sigma^+\to\Xi^0+K^+$, $\Sigma^-\to\Xi^-+K^0$,
(3) $\Sigma^\pm\to\Sigma^0+\pi^\pm$, $\Sigma^\pm\to\Sigma^\pm+\pi^0$.
(4) $\Sigma^\pm\to\Lambda^0+\pi^\pm$,

Nonvanishing contributions come from two different types of processes, which may be called the baryon current term (denoted by B_1 below) and the meson current term (denoted by B_2 below). An examination of the symmetry properties of the hyperon and pion multiplets shows that, in all orders, the contributions of the reactions (3) and (4) to the anomalous moments of Σ^+ and Σ^- are equal and opposite. There are two further contributions to the moments, from a fermion (boson) current from reaction (1) and from a boson (fermion) current from reaction (2) for the anomalous moment of $\Sigma^+(\Sigma^-)$. The evaluation of the various terms is straightforward[5] if we assume that the K meson is a spin-zero particle; we find

$$\mu^+ = g^2(\Sigma N\bar{K})B_1(\Sigma N\bar{K}) - \tfrac{1}{2}g^2(\Sigma\Xi K)B_2(\Sigma\Xi K) + g^2(\Sigma\Sigma\pi)\{B_1(\Sigma\Sigma\pi) - \tfrac{1}{2}B_2(\Sigma\Sigma\pi)\} - \tfrac{1}{2}g^2(\Sigma\Lambda\pi)B_2(\Sigma\Lambda\pi),$$

$$\mu^- = \tfrac{1}{2}g^2(\Sigma N\bar{K})B_2(\Sigma N\bar{K}) - g^2(\Sigma\Xi K)B_1(\Sigma\Xi K) - g^2(\Sigma\Sigma\pi)\{B_1(\Sigma\Sigma\pi) - \tfrac{1}{2}B_2(\Sigma\Sigma\pi)\} + \tfrac{1}{2}g^2(\Sigma\Lambda\pi)B_2(\Sigma\Lambda\pi),$$

where $g^2 = (G^2/4\pi\hbar c)$ and the functions B_1 and B_2 of the masses have the numerical values given in Table II; the $\pm$ signs are to be taken consistently according as the coupling chosen is scalar or pseudoscalar, i.e., the K meson is scalar (pseudoscalar) or pseudoscalar (scalar), respectively, assuming the parity of the Σ hyperon to be the same as (opposite to) the parity of the nucleon.

Apart from the coupling constants which are still arbitrary, we notice that the scalar and pseudoscalar coupling give similar results, except for a scale factor and a change of sign. This change of sign was already noted by Case[5] in his calculations of the nucleon anomalous moments and is connected with the parity difference of the virtual bosons.

The sum of the anomalous moments of the charged hyperons is given by

TABLE I. Electromagnetic self-energy differences of the charged hyperons.

Cut-off type	Λ^2	λ^2	μ^+	μ^-	$\mu^++\mu^-$	$m^- - m^+$
(A)	$2m^2$	$2m^2$	1.5	1.5	3.0	16.9
(A)	$2m^2$	$2m^2$	3.0	1.0	4.0	17.5
(A)	m^2	$4m^2$	1.5	1.5	3.0	15.1
(A)	m^2	$4m^2$	3.0	1.0	4.0	15.1
(B)	$4m^2$	$4m^2$	1.5	1.5	3.0	16.3
(B)	$4m^2$	$4m^2$	3.0	1.0	4.0	15.0
(B)	$1.5m^2$	∞	1.5	1.5	3.0	12.0
(B)	$1.5m^2$	∞	3.0	1.0	4.0	7.0

TABLE II. Baryon and boson current contributions to the anomalous moments.

Interaction	B_1	B_2
$\Sigma N\bar{K}$	0.45±0.97	0.31±0.42
$\Sigma\Xi K$	0.16±0.54	0.07±0.16
$\Sigma\Sigma\pi$	0.38±0.81	0.52±0.71
$\Sigma\Lambda\pi$	0.43±1.24	0.90±1.04

$$\mu^+ + \mu^- = g^2(\Sigma NK)\{0.76\pm1.39\} - g^2(\Sigma\Xi K)\{0.23\pm0.71\}.$$

Combining this with the earlier quoted results of the value of the same quantity estimated from the mass difference, we obtain an estimate of the coupling constant,

$$g^2 \sim 2.4 \text{ to } 3.2,$$

for scalar coupling, assuming that

$$g(\Sigma N\bar{K}) \simeq g(\Sigma\Xi K).$$

Pseudoscalar coupling gives the wrong sign of the anomalous moments so that Σ^+ would be heavier than Σ^-. It is obvious that the value of the estimated coupling constant is too large for a second-order perturbation calculation to be reliable. However, calculations in the analogous case of the strongly coupled pion-nucleon system reproduce the correct signs of the proton and neutron anomalous moments. We would like to believe that the signs of the anomalous moments are significant in the present case also.

It is clear from experiment that, if the K meson possesses spin 0, both parities are present. If this is so, both types of coupling (scalar and pseudoscalar) must occur for the K meson. From the foregoing calculations, we would conclude that the observed sign of the mass difference of $\Sigma^\pm$ is an argument for the presence of strong scalar coupling of the K meson to the baryons.

Similar calculations have been done for the anomalous moments and the mass spectrum of the other hyperons and the contribution of the virtual emission of K mesons to the nucleon anomalous moments. These results will be published in a separate note.

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