The asymptotic form of K depends on what happens to φ for large σ^2. If $\varphi \to 0$ at infinity, $K(\Delta^2) \to 1$; if φ approaches a positive constant, $K \to 0$. Within the framework of our model as expressed by (B.21), if there is any absorption whatsoever, φ is less than $\pi/2$. This may be seen by writing $\delta_0 = \xi + i\eta$ and noting that

$$\tan\varphi = \frac{e^{-2\eta}\sin 2\xi}{1+e^{-2\eta}\cos 2\xi}. \tag{B.23}$$

It may be argued that since in (B.4) we dropped reference to all states other than that involving a pair, we have no right to contemplate complex δ_0's, for it is just those states which lead to the complexity of the phase. It is our feeling, however, that there is sense to our procedure, since what we require for the validity of the approximation made is confined specifically to (B.4): The other terms may be small because of $\langle 0|J_i|s\rangle$ being small irrespective of the structure of the other factor. This point is also discussed in reference 11 where in addition the structure of K is examined for some simple models.

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Interaction Current in Strangeness-Violating Decays*

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The consequences of the hypothesis that the current of strongly interacting particles contributing to strangeness-violating decays has the transformation properties of an isospinor are investigated. The six processes $K^+ \to \pi^0+\mu^++\nu$, $K^- \to \pi^0+\mu^-+\bar{\nu}$, $K_1{}^0 \to \pi^++\mu^-+\bar{\nu}$, $K_1{}^0 \to \pi^-+\mu^++\nu$, $K_2{}^0 \to \pi^++\mu^-+\bar{\nu}$, $K_2{}^0 \to \pi^- +\mu^++\nu$ would then have the same rates, angular correlations, spectra, etc., and likewise for electron modes. This prediction is compared with available experimental data. The evidence from the nonlepton decays is briefly examined.

1. INTRODUCTION

RECENT experimental and theoretical developments[1] lend some support to a rather specific form of the interaction responsible for weak decays which conserve strangeness. This interaction can be considered as the self coupling of a chiral current, the current itself being constructed additively from baryon and lepton parts. The structure of the lepton current is quite well determined (provided the leptons are assumed to have no strong couplings) but the same cannot be said of the baryon currents in view of the complications introduced by the strong interactions. Nevertheless the present experimental position in β decay is consistent with a chiral baryon current with the transformation properties of an isotopic vector.[2]

In the decays of strange particles, one has two general groups of decay modes according as whether there are leptons in the final state or not. For those cases in which there are no leptons, the expression of the weak interaction as the coupling of a strangeness-conserving current with the strangeness-nonconserving current is consistent with all experimental data. One notices that while in principle the transition matrix element depends on the interaction in a definite manner, lack of adequate methods of studying such a system of strongly interacting particles makes this information on the currents practically inaccessible. In particular cases quantitative estimates of the interaction can be made and the decay of the Λ hyperon is such a case.[3] But in general, one must look to the lepton decay modes for direct information on the strangeness-violating current of strongly interacting particles, since here the transition matrix element is simply expressed in terms of the currents $\mathcal{J}_B{}^\lambda$ [see Eq. (1)].

In the case of the strangeness-conserving decays, the current coupled to the leptons has transformation properties similar to the positive chiral part of the current to which pseudoscalar mesons are pseudovector coupled. Arguing from analogy, one is thus led to postulate that the strangeness-violating current coupled to leptons has isotopic spin transformation properties similar to the positive chiral part of the current to which pseudoscalar K mesons are pseudovector coupled. We shall not discuss here the consequences of the

* Some of these results were presented at the 1958 International Conference on High Energy Physics at CERN.

[1] E. C. G. Sudarshan and R. E. Marshak, Proceedings of the Padua-Venice Conference, September, 1957 (to be published); Phys. Rev. **109**, 1860 (1958). R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] See M. Gell-Mann (to be published).

[3] Okubo, Marshak, and Sudarshan (to be published).

explicit hypothesis that the weak interaction currents are identical (apart from a scale factor) to the positive chiral parts of the strong interaction currents to which the π and K mesons are coupled; instead, we restrict ourselves to an examination of the consequences of the weaker hypothesis that the currents have the same isotopic spin transformation properties. That a $\Delta I=\frac{1}{2}$ part of the current is present follows from the existence of the (dominant) $K_{\mu 2}$ mode.

2. CONSEQUENCES FOR $K_{\mu 3}$ AND K_{e3} DECAYS

We wish to investigate the consequences of the hypothesis that the strangeness-nonconserving weak interaction current leading to the 3-body lepton modes has the same isotopic transformation properties as the current to which the K meson is coupled in strong interactions.* In other words, we make the following assumptions:

(i) The interaction leading to the K_{e3}, $K_{\mu 3}$ modes has the form[4]

$$H_w=\sum_\lambda g_\lambda \mathcal{J}_B{}^\lambda j_\lambda{}^L+\text{H.c.},\quad \lambda=S,V,T,A,P. \tag{1}$$

(We need not assume that the $\mathcal{J}_B{}^\lambda$ and g_λ are the same for muon-neutrino and electron-neutrino vertices.)

(ii) The baryon current transforms like an isotopic spinor ($I=\frac{1}{2}$).

(iii) $\mathcal{J}_B{}^\lambda$ are charged ($\Delta Q=\pm 1$) and violates one unit of strangeness ($\Delta S=\pm 1$) and $\Delta Q/\Delta S=+1$. The last condition follows from the isotopic spinor character of $\mathcal{J}_B{}^\lambda$ since, for any baryon or meson,

$$Q=I_3+\tfrac{1}{2}(N+S)$$

(where N is the baryon number), and hence

$$\Delta Q=\Delta I_3+\tfrac{1}{2}\Delta S.$$

Together with the restrictions $|\Delta Q|=|\Delta S|=1$, $|\Delta I|=\frac{1}{2}$, it follows that the only possibilities are

$$\begin{aligned}&\Delta Q=+1,\quad \Delta I_3=+\tfrac{1}{2},\quad \Delta S=+1;\\ &\Delta Q=-1,\quad \Delta I_3=-\tfrac{1}{2},\quad \Delta S=-1.\end{aligned} \tag{2}$$

which proves our assertion.

Now, under these conditions, let us study the matrix element for the decay

$$K^+\rightarrow\pi^0+\mu^++\nu.$$

Then, the transition matrix element is given by

$$\begin{aligned}M^+&=M(K^+\rightarrow\pi^0+\mu^++\nu)\\ &=\sum_\lambda g_\lambda\langle\pi^0|\mathcal{J}_B{}^\lambda|K^+\rangle\bar{u}_\nu O_\lambda v_\mu.\end{aligned} \tag{3}$$

Similarly the amplitudes for $K^0\rightarrow\pi^-+\mu^++\nu$ and $\bar{K}^0\rightarrow\pi^-+\mu^++\nu$ are

$$M^0=M(K^0\rightarrow\pi^-+\mu^++\nu)=\sum_\lambda g_\lambda\langle\pi^-|\mathcal{J}_B{}^\lambda|K^0\rangle\bar{u}_\nu O_\lambda v_\mu, \tag{4}$$

and

$$\begin{aligned}\bar{M}_0&=M(\bar{K}^0\rightarrow\pi^-+\mu^++\nu)\\ &=\sum_\lambda g_\lambda\langle\pi^-|\mathcal{J}_B{}^\lambda|\bar{K}_0\rangle\bar{u}_\nu O_\lambda v_\mu.\end{aligned} \tag{5}$$

However, by our assumption (iii), (5) must be zero, i.e.,

$$\bar{M}_0=0. \tag{6}$$

Furthermore, by our assumption (ii) we have

$$\langle\pi^0|\mathcal{J}_B{}^\lambda|K^+\rangle=(\tfrac{1}{2})^{\frac{1}{2}}\langle\pi^-|\mathcal{J}_B{}^\lambda|K^0\rangle.$$

Thus, independent of any details of the structure of $\mathcal{J}_B{}^\lambda$ we must have

$$M^+=(1/\sqrt{2})M^0.$$

This means that the transition probability for $K^+\rightarrow\pi^0+\mu^++\nu$ is one half of that of $K^0\rightarrow\pi^-+\mu^++\nu$, i.e.,

$$\omega(K^+\rightarrow\pi^0+\mu^++\nu)=\tfrac{1}{2}\omega(K^0\rightarrow\pi^-+\mu^++\nu). \tag{7}$$

However, K^0 is not a real particle. According to Lee, Oehme, and Yang,[5] K^0 and $\bar{K}^0$ are mixtures of the particles $K_1{}^0$ and $K_2{}^0$.

$$\begin{aligned}|K_1{}^0\rangle&=(|p|^2+|q|^2)^{-\frac{1}{2}}(p|K^0\rangle+q|\bar{K}^0\rangle),\\ |K_2{}^0\rangle&=(|p|^2+|q|^2)^{-\frac{1}{2}}(p|K^0\rangle-q|\bar{K}^0\rangle).\end{aligned} \tag{8}$$

The significance of p and q is the same as in the paper by Lee, Oehme, and Yang. If time reversal invariance holds for H_w, $|p|^2=|q|^2$.

By using (6), we have

$$\begin{aligned}\omega(K_1{}^0\rightarrow\pi^-+\mu^++\nu)&=\omega(K_2{}^0\rightarrow\pi^-+\mu^++\nu)\\ &=(|p|^2+|q|^2)^{-1}|p|^2\omega(K^0\rightarrow\pi^-+\mu^++\nu).\end{aligned} \tag{9}$$

Similarly, for the antiparticle decay

$$K_1{}^0\text{ (or }K_2{}^0)\rightarrow\pi^++\mu^-+\bar{\nu},\quad M(K^0\rightarrow\pi^++\mu^-+\bar{\nu})\equiv 0,$$

due to the condition (iii), and only the $\bar{K}^0$ part contributes. Thus

$$\begin{aligned}M(K_1{}^0\rightarrow\pi^++\mu^-+\nu)&=-M(K_2{}^0\rightarrow\pi^++\mu^-+\nu)\\ &=(|p|^2+|q|^2)^{-1}qM(\bar{K}^0\rightarrow\pi^++\mu^-+\nu).\end{aligned} \tag{10}$$

However, there is no strong final-state interaction in these decays. Therefore, by the TCP theorem the transition probability for $\bar{K}^0\rightarrow\pi^++\mu^-+\bar{\nu}$ summed over final spins and momenta is equal to that of $K^0\rightarrow\pi^-+\mu^++\nu$.[5] Consequently, (10) yields

$$\begin{aligned}\omega(K_1{}^0\rightarrow\pi^++\mu^-+\bar{\nu})&\equiv\omega(K_2{}^0\rightarrow\pi^++\mu^-+\nu)\\ &\equiv(|p|^2+|q|^2)^{-1}|q|^2\omega(\bar{K}^0\rightarrow\pi^++\mu^-+\bar{\nu})\\ &\equiv(|p|^2+|q|^2)^{-1}|q|^2\omega(K^0\rightarrow\pi^-+\mu^++\nu).\end{aligned} \tag{9$'$}$$

Adding (9) and (9′), we have

$$\begin{aligned}\omega(K_1{}^0\rightarrow\pi^\mp+\mu^\pm\pm\nu)&\equiv\omega(K_2{}^0\rightarrow\pi^\mp+\mu^\pm\pm\nu)\\ &\equiv\omega(K^0\rightarrow\pi^-+\mu^++\nu)\equiv 2\omega(K^+\rightarrow\pi^0+\mu^++\nu),\end{aligned} \tag{11}$$

[4] See S. Weinberg *et al.* (to be published).

[5] Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).

where we used Eq. (7). This conclusion is independent of time reversal invariance.†

Exactly the same formulas follow in the case of the electron decay; thus:

$$\omega(K_1^0\to\pi^\mp+e^\pm\pm\nu)\equiv\omega(K_2^0\to\pi^\mp+e^\pm\pm\nu)$$
$$\equiv 2\omega(K^+\to\pi^0+e^++\nu). \quad (12)$$

Experimentally,[6] $\omega(K^+\to\pi^0+\mu^++\nu)\simeq\omega(K^+\to\pi^0+e^++\nu)$ and so we must have

$$\omega(K_2^0\to\pi^\mp+\mu^\pm\pm\nu)\simeq\omega(K_2^0\to\pi^\mp+e^\pm\pm\nu)$$
$$\equiv\omega(K_1^0\to\pi^\mp+e^\pm\pm\nu)\simeq\omega(K_1^0\to\pi^\mp+\mu^\pm\pm\nu). \quad (13)$$

The approximate equality of $K_2^0\to\pi^\mp+\mu^\pm\pm\nu$ and $K_2^0\to\pi^\mp+e^\pm\pm\nu$ is not inconsistent with experiments.[7] Both decay modes together account for practically all K_2^0 decays. From the branching ratio of $K^+\to\pi^0+\mu^++\nu$ mode and the lifetimes of K^+ and K_1^0, we can determine the absolute transition rate of $K^+\to\pi^0+\mu^++\nu$ and hence the branching ratio of the $K_1^0\to\pi^\pm+\mu^\mp\pm\nu$ mode from (11). We predict:

$$\frac{K_1^0\to\pi^\mp+\mu^\pm\pm\nu}{K_1^0\to\text{any mode}}\simeq\frac{K_1^0\to\pi^\mp+e^\pm\pm\nu}{K_1^0\to\text{any mode}}\simeq 6.2\times10^{-4}, \quad (14)$$

which is consistent with the experimental upper limit.[8]

For K_2^0 decay, one can present a similar argument. However, due to the uncertainty of the lifetime of K_2^0, it is more convenient to express the results in terms of the partial lifetime for each mode using (11) and the partial lifetime of $K^+\to\pi^0+\mu^++\nu$. We obtain for the (absolute) transition rate:

$$\omega(K_2^0\to\pi^\mp+\mu^\pm\pm\nu)\simeq\omega(K_2^0\to\pi^\mp+e^\pm\pm\nu)$$
$$\simeq 6.6\times10^6\ \text{sec}^{-1}. \quad (15)$$

Experimentally, the only other observed mode[9] of $K_2^0\to\pi^++\pi^-+\pi^0$ gives only a small branching ratio compared to the μ or e decay modes. Thus, the total transition rate into the various decay modes $K_2^0\to\pi^\mp+\mu^\pm\pm\nu$ and $K_2^0\to\pi^\mp\pm e^\pm\pm\nu$, is expected to be

$$1/\tau(K_2^0)\sim 1.3\times10^7\ \text{sec}^{-1}. \quad (16)$$

The transition rate (16) is actually a lower limit and hence

$$\tau(K_2^0)<7.5\times10^{-8}\ \text{sec} \quad (17)$$

for the mean life of the K_2^0 particle. On the other hand, the experimental mean life of K_2^0 is found to be[7]

$$7.5\times10^{-8}\ \text{sec}<\tau<12.5\times10^{-8}\ \text{sec}.$$

Thus, our theory is consistent with experiment and gives the lifetime very nearly equal to the lower limit. This implies that the K_2^0 decay modes other than $\pi^\pm+\mu^\pm\pm\nu$ and $\pi^\pm+e^\pm\pm\nu$ (including $\pi^++\pi^-+\pi_0$) are quite small,‡ which is also consistent with experiment.[9]

Two additional points may be noted: firstly, making use of the *TCP* theorem one could assert that the energy spectra as well as the angular correlations in the various muon modes $K_{1,2}^0\to\pi^++\mu^-+\bar\nu$, $K_{1,2}^0\to\pi^-+\mu^++\nu$, $K^+\to\pi^0+\mu^++\nu$, $K^-\to\pi^0+\mu^-+\bar\nu$ are identical, irrespective of the validity of time reversal invariance. Similarly the various electron modes have identical energy spectra and angular correlations.[10] Secondly, if one further makes use of the observed absence of $\theta_2^0\to2\pi$ decay, one deduces that[11]

$$\alpha\equiv\left|\frac{|p|^2-|q|^2}{|p|^2+|q|^2}\right|\lesssim 0.1,$$

and consequently that the modes $K_{1,2}^0\to\pi^++\mu^-+\bar\nu$, $K_{1,2}^0\to\pi^-+\mu^++\nu$, and $K_{1,2}^0\to\pi^++e^-+\bar\nu$, $K_{1,2}^0\to\pi^-+e^++\nu$ are equally frequent, a conclusion not inconsistent with the scanty data on the relative frequencies of these modes. This argument is independent of time-reversal invariance. In this connection it is to be noticed that the greatest source of uncertainty in the theoretical predictions stems from the branching ratios of the K^+ decay modes.

From an experimental point of view, the branching ratio of $K_1^0\to\pi^\mp+\mu^\pm\pm\nu$ will be quite difficult to measure, because of the particle mixture aspect of the K^0 particle. A more accurate measurement of the lifetime of K_2^0 will constitute a direct test of the present theory. We must reiterate that it is not necessary to assume the universality of the Fermi interaction in our discussion and the $\mathcal{J}_B{}^\lambda$ coupled to the $\mu\nu$ and $e\nu$ pairs can be different.

Similar relations connecting the transition amplitudes of lepton decays of hyperons also follow from the assumed isospinor character of the interaction current. In the case of Σ decay, one can compare the partial lifetimes for the decays of Σ^0 and Σ^- (rather than give a branching ratio); one has

$$\omega(\Sigma^0\to p+e^-+\bar\nu)/\omega(\Sigma^-\to n+e^-+\bar\nu)=\tfrac{1}{2}.$$

† *Note added in proof.*—Equations (11) and (12) have been derived by I. Yu. Kobzarev and L. B. Okun under the more restricted assumption of time reversal invariance (private communication from L. B. Okun); see also, L. B. Okun, Proceedings of the Conference on Mesons and Recently Discovered Particles, Padua-Venice, 1958.

[6] See, e.g., M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7.

[7] Bardon, Fuchs, Lande, Lederman, Chinowsky, and Tinlot, Phys. Rev. **110**, 780 (1958).

[8] Eisler, Plano, Samios, Schwartz, and Steinberger, Nuovo cimento **5**, 1700 (1957).

[9] Kenneth Lande, thesis, Columbia University, 1958 (unpublished).

‡ *Note added in proof.*—This is already in conflict with the assumption of the over-all $\Delta I=\frac{1}{2}$ selection rule *and* the assumption of a totally symmetric final state of 3 pions, since in this case we have[6] $\omega(K_2^0\to3\pi)\cong\omega(K^+\to3\pi)$. Also, in this case the calculated lifetime for K_2^0 would be much shorter than found experimentally.

[10] In a previous paper,[4] we state that the spectrum shape and angular correlations would, in general, be different for the $K_{\mu3}{}^0$ and $K_{\mu3}{}^\pm$. This statement gets modified if one makes a specific hypothesis about the isotopic spin transformation properties, as we have done here.

[11] Steven Weinberg, Phys. Rev. **110**, 782 (1958).

In the decay of the cascade particles again, one has

$$\omega(\Xi^-\rightarrow\Sigma^0+e^-+\bar{\nu})/\omega(\Xi^0\rightarrow\Sigma^++e^-+\bar{\nu})=\tfrac{1}{2}.$$

These relations are independent of the (unknown) lepton momenta (and even of the assumption of a local interaction): but in view of the apparent absence of the lepton modes, they do not seem to be of any immediate interest.

These considerations are irrelevant for the $K_{\mu 2}$ and $\Lambda\rightarrow p+e^-+\bar{\nu}$ modes.

3. EVIDENCE FROM THE NONLEPTON MODES

The success of the chiral $V-A$ interaction[1] in the lepton processes lends credence to the possibility of constructing the nonleptonic weak interactions as the product of a (chiral) strangeness-conserving current with a (chiral) strangeness-nonconserving current, these being identified with the current operators appearing in the leptonic weak interactions. However, we shall again only examine the consequences of the weaker assumption that this weak interaction Hamiltonian transforms as the direct product of the isovector J_B (strangeness conserving[12]) and the isospinor $\mathcal{J}_B$ (strangeness nonconserving) currents. The weak-interaction Hamiltonian, and consequently the transition amplitude, transforms as a linear combination of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ contributions. In this sense our postulate of an isospinor $\mathcal{J}_B$ is essentially different from the $\Delta I=\frac{1}{2}$ selection rule discussed by various authors.[13] The (absolute) amplitudes corresponding to the $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ contributions depend on the details of the strong interaction and can, under suitable conditions, simulate a $\Delta I=\frac{1}{2}$ selection rule. (Note that, so far, we have not made any assumptions of universality of coupling, since the consequences of this assumption are intimately connected with a proper treatment of the strong interactions.) The decay of the Λ hyperon and the θ are examples. In the first case,[3] a suitable weak interaction can be written down which leads to predictions identical with the $\Delta I=\frac{1}{2}$ selection rule in Born approximation, and these Born approximation results are relatively unaffected in view of the weakness of the pion-nucleon interaction in the $J=\frac{1}{2}$ angular momenta states. For θ decay, a strong final-state pion-pion interaction[14] may be invoked to suppress the $\Delta I=\frac{3}{2}$ transition amplitude relative to the $\Delta I=\frac{1}{2}$.

From these considerations it might be supposed that all the available experimental data on nonleptonic decays are consistent with the isospinor current $\mathcal{J}_B$, and this is very nearly true. The analysis in most cases can only be carried out to the extent of showing that "comparable" $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ amplitudes can explain the data except in those cases (like θ^0) where some special circumstance may be responsible for the effective suppression of one channel. The only known exception is the decay of the θ^0 particle; namely, the numerical values of the branching ratio $\omega(\theta^0\rightarrow\pi^0+\pi^0)/\omega(\theta^0\rightarrow\pi^+ +\pi^-)$ and the ratio of lifetimes of θ^+ and θ^0 lead to difficulty. One cannot simultaneously reconcile both of these numbers with arbitrary $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ amplitudes; however, they can be explained by including a $\Delta I=\frac{5}{2}$ transition amplitude which can be as small as 10% of the $\Delta I=\frac{1}{2}$ amplitude. While such a large correction is difficult to understand as an electromagnetic effect, the difficulty is aggravated if one believes in the over-all $\Delta I=\frac{1}{2}$ selection rule for the process.

We thus find that there is no need to invoke the over-all $\Delta I=\frac{1}{2}$ selection rule and that the hypothesis of an isospinor current $\mathcal{J}_B$ coupled to an isovector (strangeness-conserving) current is equally satisfactory. In the present context the θ^0 decay is a crucial process and the branching ratio should be measured as accurately as possible.

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[12] There is no conclusive evidence at present that components with $I=2$, say, are absent from this current. Such a component would have implications for the β decay of complex nuclei as well as for the ratio of the transition rates for the modes $\Sigma^\pm\rightarrow\Lambda^0+e^\pm\pm\nu$ [see Steven Weinberg, Phys. Rev. (to be published)].

[13] M. Gell-Mann and A. Pais, *Proceedings of the International Conference on High-Energy Physics* (Pergamon Press, Inc., London, 1955); see also reference 6.

[14] It is interesting to note that some information regarding the pion-pion interaction can be obtained from the θ decay mode. If we assume time-reversal invariance, then

$$r=\frac{\omega(\theta_1^0\rightarrow 2\pi^0)}{\omega(\theta_1^0\rightarrow 2\pi^0)+\omega(\theta_1^0\rightarrow\pi^++\pi^-)}=\frac{1+2x^2\pm 2\sqrt{2}x\cos(\delta_2-\delta_0)}{3(1+x^2)},$$

where x is the ratio of the absolute magnitudes of the amplitude for the $I=2$ state to the amplitude for the $I=0$ state. Since x is real, we have the inequality:

$$\cos^2(\delta_2-\delta_0)\geqslant(1-3r)(1-\tfrac{3}{2}r).$$

With the experimental value $r\simeq 0.14$, we have $|\cos(\delta_2-\delta_0)|\geqslant 0.6=\cos 61°$ and this limit is insensitive to r for small values of r. This result depends only on the validity of time reversal invariance (and does not even depend on the spin of θ); it can serve as a test for (phenomenological) theories of the pion-pion interaction.