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Quantum Numbers for System of Nucleons and Antinucleons.

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The most general hermitian operator on the charge state of a nucleon can be expressed as a linear combination of the unit matrix and the three two-by-two isotopic spin matrices τ_1, τ_2, τ_3: It is also well-known that the charge state of a many-nucleon system is completely defined (1) by two quantum numbers I and I_3 which are conserved by virtue of the charge-independence of nuclear forces. Electromagnetic interactions which destroy the conservation of I (but not I_3) represent only minor corrections to the conservation of the «total» isotopic spin I and the classification of a many-nucleon system using I is one which displays the constants of motion.

In this case the isotopic spin I may be defined in one of several equivalent ways, as describing the invariance of the dynamical laws under the following groups of transformations:

a) Unitary transformations in the (2-dimensional) «space» spanned by the charge states of the nucleon.

b) Real rotations in a 3-dimensional «isotopic spin space».

c) Permutations of the charge coordinates of the individual particles of the many-nucleon system. (Because of the Pauli principle, this is equivalent to permutations of the space and spin coordinates of the nucleons).

When both nucleons and antinucleons are present, since the charge states are now increased to four (p, n, $\bar{\text{n}}$, $\bar{\text{p}}$) the most general hermitian matrix is a linear combination of 16 Dirac matrices. It is then clear that I and I_3 no longer suffice to classify the charge states of the system. Whereas I still labels the irreducible representations of group *b*), we need a total of three quantum numbers to label the irreducible representations of the groups in *a*) (2) and/or *c*). The problem is mathematically the same as considered by Wigner in his supermultiplet theory (3). Unfortunately his

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(1) Apart from an irrelevant permutation degeneracy, which is removed as soon as the exclusion principle is taken into account.

(2) Where group (*a*) is now the group of unitary transformations in the (4-dimensional) space spanned by the charge states (proton, neutron, antineutron, antiproton).

(3) E. Wigner: *Phys. Rev.*, **51**, 106 (1937): E. Wigner and E. Feenberg: *Rep. Prog. Phys.*, **8**, 274 (1941).

T, S, Y (sometimes denoted by P, P', P'') bear no simple relation to the total isotopic spin I. A more satisfactory choice could be a set containing the total isotopic spin I, the « total » baryon number K (4) and a suitably chosen third number H which could be constructed explicitly. To specify a definite component of this irreducible representation (*i.e.*, the complete wave function) in charge space one needs, in general, three « third components » I_3, K_3 and H_3 (cfr. reference (3)).

For a system of one nucleon and one antinucleon, this full set of labels (I, K, H) is redundant; and the classification (as well as K and H themselves) is completely defined in terms of I and the multiplicative quantum numbers G (5) and these are constants of motion. This nucleon-antinucleon system is however an exception like the deuteron in supermultiplet theory (6). In the general case it can be shown that *there are no additive quantum numbers for the nucleon-antinucleon-pion system in addition to I, I_3 and K_3 which are strictly conserved (if one accepts the usual charge-independent* PS(PS) *or* PS(PV) *interaction*). This circumstance is due to the particular dynamics of the system. It is of interest to examine the possibility of approximate conservation of some of the additional quantum numbers related to the charge degrees of freedom at least in certain restricted domains of energy and/or for specific reactions.

Regarding an experimental test of such a possibility, the simplest nontrivial system must contain at least 3 particles with both nucleons and antinucleons. Such a process is the nonannihilation scattering of an antinucleon by deuterons. Consider the reactions

(A) $\quad \bar{\text{n}}+\text{d}\rightarrow\bar{\text{n}}$

(B) $\quad \bar{\text{n}}+\text{d} \quad \bar{\text{p}}+\text{p}+\text{p}$

A $\quad \bar{\text{p}}+\text{d} \quad \bar{\text{p}}+\text{p}+\text{n}$

B $\quad \bar{\text{p}}+\text{d}$

The final states are classified in Table

TABLE I

Quantum numbers for final states		Statistical Weight	R
K	Overall symmetry of charge space		
1			
	intermediate symmetry; Youn pattern [21]		
	intermediate symmetry; Y pattern [21]		

Charge-independence in the usual sense only requires that the ratio R of the differential cross-sections

$$R \quad \frac{\sigma(\text{A})}{\sigma(\text{B})} \quad \frac{\sigma(\text{A}')}{\sigma(\text{B}')}$$

be restricted by

(4) K is related to the baryon number K, (number of nucleons minus antinucleons) as T is related to T_3. See B. TOUSCHEK: *Nuovo Cimento*, **8**, 181 (1958); O. HARA and Y. FUJII: *Progr. Theor. Phys.*, **17**, 313, 819, 820, 822 (1957).

(5) T. D. LEE and C. N. YANG: *Nuovo Cimento*, **3**, 749 (1956): the same number has been introduced with somewhat different notation by D. AMATI and B. VITALE: *Nuovo Cimento*, **2**, 719 (1955) and C. J. GOEBEL: *Phys. Rev.*, **103**, 258 (1956).

(6) The Bartlett and Heisenberg forces, which in general destroy the validity of the quantum numbers which specify a supermultiplet, do not destroy the validity of the supermultiplet picture in the case of the deuteron.

3370

If the final state belongs to a definite irreducible representation one obtains the values listed in the last column of the table. An experimental value $R > 2$ would not be very significant; however, a value $2 > R > \frac{1}{2}$ should further enable us to infer a resonance in the ($\frac{1}{2}$, $\frac{1}{2}$, symmetric) state.

The point we wish to emphasize is that even within the framework of charge-independent interactions not all final states are necessarily equivalent and the additional quantum numbers labelling these states may be sufficiently well conserved, at least for a certain range of processes. That the existence of new selection rules related to conservation of quantum numbers like strangeness *necessarily* requires the introduction of additional internal degrees of freedom seems to us unwarranted. There is wealth of internal symmetry labels for many-particle systems which could be exploited in a suitable composite model of strange particles. We believe that this problem has not been sufficiently investigated to provide a definite verdict on the success of such a program.

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3371