

ISOTOPIC SPIN SELECTION RULES  
AND  $K_2^0$  DECAY\*

S. Okubo and R. E. Marshak  
University of Rochester,  
Rochester, New York

and

E. C. G. Sudarshan  
Harvard University,  
Cambridge, Massachusetts  
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One of the interesting theoretical points connected with the decays of the strange particles is to determine whether there exists some kind of isotopic spin selection rule for these decays or not. Currently, there are two main view points on this issue. In the first point of view, one chooses the interaction Hamiltonian responsible for the weak decays to obey the familiar  $\Delta I = \frac{1}{2}$  selection rule.<sup>1</sup> Though that rule was originally intended to apply for the nonleptonic modes, it may be extended to the leptonic modes also (if we assign zero isotopic spin for leptons). In the second point of view,<sup>2</sup> the weak interaction Hamiltonian is a current-current interaction and the interacting nonleptonic currents are taken to have definite transformation properties: the strangeness-conserving current transforms as an isovector and the strangeness-violating current transforms as an isospinor. For simplicity we call this the " $I = \frac{1}{2}$  current" rule. Both the extended  $\Delta I = \frac{1}{2}$  selection rule and the  $I = \frac{1}{2}$  current rule predict the same results for leptonic decays. However, for the nonleptonic decays of the strange particles, the  $I = \frac{1}{2}$  current rule gives a  $\Delta I = \frac{3}{2}$  component in the Hamiltonian as well as a  $\Delta I = \frac{1}{2}$  component. Thus, generally, the two rules will make different predictions for these decays and there seems to be no way to make an experimental<sup>3</sup> choice between them at the present time. However, we note that if we believe in the present ansatz of charged current-current couplings generating weak interaction, the second viewpoint will be theoretically favored, since the over-all  $\Delta I = \frac{1}{2}$  rule requires neutral current-current couplings also.

In the present note, we show that if the extended  $\Delta I = \frac{1}{2}$  selection rule is true for both leptonic and nonleptonic modes, then we can calculate almost uniquely the lifetime of  $K_2^0$  and the various branching ratios of  $K_2^0$  decay from the known decay rates of  $K^+$  decay. Unfortunately, the present experimental results are not sufficiently accurate but we may hope to make this test more

incisive in the near future. In what follows, we assume spin zero for the  $K$  meson and  $CP$ -invariance of the theory as well as the  $\Delta I = \frac{1}{2}$  rule. We can derive the following identity between the transition probabilities without much difficulty:<sup>4,5</sup>

$$w(K_2^0 \rightarrow 3\pi) = w(K^+ \rightarrow 3\pi). \quad (1)$$

Furthermore, in a previous note,<sup>2</sup> we proved the relations

$$\begin{aligned} w(K_2^0 \rightarrow \pi^\pm + e^\mp + \nu) &= 2w(K^+ \rightarrow \pi^0 + e^+ + \nu), \\ w(K_2^0 \rightarrow \pi^\pm + \mu^\mp + \nu) &= 2w(K^+ \rightarrow \pi^0 + \mu^+ + \nu). \end{aligned} \quad (2)$$

Since these are the major decay modes of  $K_2^0$ , we can immediately calculate the lifetime of  $K_2^0$  and various branching ratios from the known data for  $K^+$  decay.<sup>6</sup> The result is

$$\tau_{\text{theoretical}}(K_2^0) = 5.16 \times 10^{-8} \text{ sec}, \quad (3)$$

which must be compared with the experimental value<sup>7</sup>

$$\tau_{\text{experimental}}(K_2^0) = (9.0_{-2.5}^{+3.5}) \times 10^{-8} \text{ sec}. \quad (4)$$

The calculated branching ratios are

$$\begin{aligned} K_2^0 \rightarrow \pi^- + \mu^+ + \nu, & \quad 0.17 \\ K_2^0 \rightarrow \pi^+ + \mu^- + \bar{\nu}, & \quad 0.17 \\ K_2^0 \rightarrow \pi^- + e^+ + \nu, & \quad 0.18 \\ K_2^0 \rightarrow \pi^+ + e^- + \bar{\nu}, & \quad 0.18 \\ K_2^0 \rightarrow 3\pi, & \quad 0.30. \end{aligned} \quad (5)$$

Furthermore, if we make the assumption that the final state in 3-pion decay is completely symmetric with respect to the space wave functions of the pions, which is approximately justified by the experimental results, then<sup>8</sup>

$$\frac{w(K_2^0 \rightarrow \pi^0 + \pi^0 + \pi^0)}{w(K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0)} = \frac{3}{2}. \quad (6)$$

Then, the branching ratios for  $K_2^0 \rightarrow 3\pi$  is further subdivided to give

$$\begin{aligned} K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0, & \quad 0.12 \\ K_2^0 \rightarrow \pi^0 + \pi^0 + \pi^0, & \quad 0.18. \end{aligned} \quad (7)$$

Thus, there is some discrepancy between the calculated and experimental lifetimes [Eqs. (3) and (4)]. If we take this seriously, then we must discredit Eq. (1) and/or Eq. (2), namely the extended  $\Delta I = \frac{1}{2}$  selection rule for  $K_2^0$  decay.

Suppose that for a moment we take the first point of view, so that we discard the  $I = \frac{1}{2}$  current rule for the leptonic modes and believe in the restricted  $\Delta I = \frac{1}{2}$  rule, (i.e., for nonleptonic modes only). Then, Eq. (1) will still hold and presumably Eq. (6), also. Then, we can still calculate the absolute transition rate

$$w(K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0) = 2.4 \times 10^6 \text{ sec}^{-1}.$$

On the other hand, the experimental upper limit<sup>9</sup> to the charged three-pion mode is  $\sim 14\%$  and using the experimental lifetime [see Eq. (4)], we get

$$2.1 \times 10^6 \text{ sec}^{-1} \gg w(K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0).$$

This is at variance with the calculated value given above. This could be an argument against the  $\Delta I = \frac{1}{2}$  rule for nonleptonic modes.<sup>10</sup> However, the experimental values quoted above do not appear very reliable and it is desirable to have more accurate measurements. Furthermore, the calculated value, Eq. (3), may increase by as much as 20%, if we use different data<sup>11</sup> for the relative abundance of the  $K_{\mu 3}$  and  $K_{e 3}$  modes of the positive  $K$  meson. For these reasons, it seems too early to discredit the  $\Delta I = \frac{1}{2}$  rule.

Next, we would like to comment on the  $I = \frac{1}{2}$  current rule. In this case, we have both  $\Delta I = \frac{1}{2}$  and  $\Delta I = \frac{3}{2}$  for the nonleptonic modes, and as a result, we no longer have relations like Eq. (1); but we still have Eq. (2). Consequently, we cannot calculate the absolute lifetime of  $K_2^0$  as Eq. (3), but only an upper limit, which was the result of the previous paper.<sup>2</sup> In this case, however, instead of Eq. (6), we have<sup>12</sup>

$$0 \leq \frac{w(K_2^0 \rightarrow \pi^0 + \pi^0 + \pi^0)}{w(K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0)} \leq \frac{3}{2}. \quad (8)$$

If the final 3-pion space wave function is completely symmetric, we will have Eq. (6) again.

Finally, it is worthwhile to observe that  $K_1^0 \rightarrow 3\pi^0$  (not  $K_2^0 \rightarrow 3\pi^0$ ) is absolutely forbidden, independent of isotopic spin selection rules, if  $CP$ -invariance holds.<sup>13, 14</sup> This could be checked experimentally. Furthermore,  $K_1^0 \rightarrow \pi^+ + \pi^- + \pi^0$  (again not  $K_2^0$ ) will be forbidden, provided the final 3-pion space wave function is completely symmetric. This conclusion is again independent of the  $\Delta I = \frac{1}{2}$  rule.<sup>13, 14</sup> Thus, its presence will give a proof of the existence of a nonsymmetrical space wave function for the 3-pion final state in the decay.

We wish to thank Professor R. H. Dalitz for an interesting discussion of some of these points.

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<sup>1</sup>For a review of this subject, see: M. Gell-Mann and A. H. Rosenfeld, Annual Review of Nuclear Science (Annual Reviews, Inc., Stanford, 1957), Vol. 7.

<sup>2</sup>Okubo, Marshak, Sudarshan, Teutsch, and Weinberg, Phys. Rev. **112**, 665 (1958). L. B. Okun, Proceedings of the Padua-Venice Conference on Mesons and Newly Discovered Particles, 1957 (to be published).

<sup>3</sup>For the case of  $\Lambda$ -decay, see Okubo, Marshak, and Sudarshan, Phys. Rev. (to be published).

<sup>4</sup>This can be derived by slightly modifying the discussion by Dalitz: R. H. Dalitz, Reports on Progress in Physics (The Physical Society, London, 1957), Vol. XX, p. 163.

<sup>5</sup>M. Gell-Mann and A. H. Rosenfeld, Annual Review of Nuclear Science (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 464. However, we are not necessarily to assume the complete symmetry of the final 3-pion states, as it seems to be stated there.

<sup>6</sup>We used the data for  $K^+$  given by Gell-Mann and Rosenfeld (see reference 1.).

<sup>7</sup>M. Bardon et al., Phys. Rev. **110**, 780 (1958).

<sup>8</sup>This was also derived by Gell-Mann and Rosenfeld (see reference 5).

<sup>9</sup>L. M. Lederman: quoted by R. H. Dalitz, Reference 10.

<sup>10</sup>A similar argument was given by R. H. Dalitz at the Conference on Weak Interactions, Gatlinburg, Tennessee, October, 1958 (unpublished).

<sup>11</sup>R. W. Birge et al., Nuovo cimento **4**, 834 (1956).

<sup>12</sup>We used the method of Dalitz: R. H. Dalitz, Proc. Phys. Soc. (London) **A69**, 527 (1956).

<sup>13</sup>The state of zero charge and total isotopic spin  $I$  of  $N$  pions is identically an eigenstate of the charge conjugation operator with eigenvalue  $(-1)^{I+N}$ . So, if the  $K$  meson has zero spin and  $CP$ -invariance holds, the final three-pion state in the decay of  $K_1^0$  ( $K_2^0$ ) must have even (odd) isotopic spin. Since symmetric isotopic spin wave functions of 3 pions are possible only for odd isotopic spin ( $I=1$  or  $I=3$ ), the final space wave function for  $K_1^0 \rightarrow 3\pi$  must automatically be unsymmetric.

<sup>14</sup>Note added in proof: It has been called to our attention that these conclusions have been stated also by A. Pais and S. B. Treiman, Phys. Rev. **106**, 1106 (1957).