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Structure of the Vertex Function

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An integral representation as a function of invariants is found for the Fourier transform of the matrix element between the vacuum and a one-particle state of the retarded commutator of two currents. A special case is a spectral representation for the vertex as a function of momentum transfer. The threshold in this representation is lower than that found in the usual perturbation theory.

INTRODUCTION

WE shall study the structure of the matrix element of the commutator, the retarded commutator, and the time-ordered product of two field operators taken between the vacuum and a single-particle state. Our technique will be to manipulate these functions in the physical region in such a way as to obtain an integral representation for the function in terms of the invariant momentum parameters that characterize its Fourier transform.

We shall discover that, for a special case of this representation, one can continue analytically certain of the invariant parameters out of the physical region and automatically obtain a spectral relation for the vertex function as a function of the momentum transfer.

The basic representation we shall find for the Fourier transform of the retarded commutator of two currents is

$$\int e^{ikx}\theta(x)dx\langle[j_1(x),j_2(0)]|p\rangle = \int d\mu d\beta \frac{H(\mu,\beta,p^2)}{k^2+2\beta pk-\mu+i\epsilon(pk+\beta p^2)}, \quad (1)$$

in which the variables are limited by

$$0\geqslant\beta\geqslant-1,\quad \mu\geqslant\max\{f(\beta),\,-\beta^2p^2\},$$

where $f(\beta)$ is a function, which we will specify later, determined by the mass spectra of the intermediate states, and $H(\mu,\beta,p^2)$ is uniquely determined by the Fourier transform in the physical region.

THE REPRESENTATION

We shall begin by deriving a representation for the matrix element of the commutator of two currents. We

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shall discuss only boson fields, the extension to the general case being trivial. We consider a matrix element of the form

$$h(x^2,px)=\langle[j_1(x),j_2(0)]|p\rangle, \tag{2}$$

in which $j_1(x)$ and $j_2(x)$ are the currents associated with two fields and the state $|p\rangle$ is a state of energy momentum p_μ; $p^2=\kappa^2$; $p_0>0$. This state may be either a single-particle state or the state produced by operating on the vacuum with a single Fourier component of a field. As a consequence of Lorentz invariance we know that this matrix element depends on the coordinate only through the variables x^2, px. It is also a function of p^2 and of other eigenvalues; the weight functions will depend implicitly on these parameters.

We introduce the Fourier transform of the matrix element with respect to the invariants:

$$h(x^2,px)=\int d\alpha d\beta \exp(i\alpha x^2)\exp(i\beta px)g(\alpha,\beta). \tag{3}$$

The condition that the commutator vanish for space-like intervals, whenever $x^2<0$, is equivalent to the requirement that $g(\alpha,\beta)$ be analytic as a function of α in the lower half α-plane. The part of the Fourier transform, $g(\alpha,\beta)$, that is determined by values of the invariants in the region $p^2x^2\geqslant(px)^2$ is arbitrary but does not enter into the momentum-space Fourier transform that we shall take since this is extended only over real values of the coordinate-space vector. We express the exponential $\exp(i\alpha x^2)$ as the Fourier transform of a Gaussian, absorbing the numerical factors into the weight function:

$$h(x^2,px)=\int dkd\alpha d\beta \exp(-i\bar{k}x) \times\exp(-i\bar{k}^2/4\alpha)\exp(i\beta px)\frac{g(\alpha,\beta)}{\alpha|\alpha|}. \tag{4}$$

Since $g(\alpha,\beta)$ is analytic in the lower half α-plane, we represent it as

$$g(\alpha,\beta)=\int_0^\infty d\lambda^2 \exp(i\lambda^2/4\alpha)\tilde{H}(\lambda^2,\beta). \tag{5}$$

Then the α-integration can be performed, yielding

$$h(x^2,px)=\mathcal{P}\int d\bar{k}\frac{\exp(-ikx)\exp(i\beta px)}{\bar{k}^2-\lambda^2}\tilde{H}(\lambda^2,\beta)d\lambda^2 d\beta. \tag{6}$$

We use the identity

$$\mathcal{P}\int dk\frac{e^{-ikx}}{k^2-\lambda^2}=-i\pi\epsilon(x_0)\int dk\ \epsilon(k_0)e^{-ikx}\delta(k^2-\lambda^2), \tag{7}$$

and introduce the Hilbert transform of $\tilde{H}(\lambda^2,\beta)$ in order to remove the $\epsilon(x_0)$ behavior produced by (7):

$$\tilde{H}(\lambda^2,\bar{\beta})=\mathcal{P}\int H(\lambda^2,\beta)\frac{d\beta}{\bar{\beta}-\beta}. \tag{8}$$

The result of these substitutions is the expression

$$h(x^2,px)=\int dkd\beta\ \epsilon(pk)\exp(-ikx) \times\exp(i\beta px)\delta(\bar{k}^2-\lambda^2)H(\lambda^2,\beta), \tag{9}$$

or, for the Fourier transform of the commutator,

$$f(k^2,pk)=\int e^{ikx}dx\ h(x^2,px) =\int_0^\infty d\lambda^2\int_{-\infty}^\infty d\beta\ \epsilon(pk+\beta p^2) \times\delta((k+\beta p)^2-\lambda^2)H(\lambda^2,\beta). \tag{10}$$

For this representation for the commutator, the retarded Green's function may easily be derived; it is

$$\int e^{ikx}\theta(x)dx\ h(x^2,px) =\int_0^\infty d\lambda^2\int_{-\infty}^\infty d\beta\frac{H(\lambda^2,\beta)}{(k+\beta p+i\epsilon p)^2-\lambda^2}. \tag{11}$$

LIMITS ON THE PARAMETERS

We turn now to the problem of restricting the ranges of the parameters in the representation. We have, as yet, imposed only the condition that the commutator vanish for space-like intervals. Further restrictions arise from the mass-spectra of the intermediate states. If we expand the commutator over a complete set of eigenvectors of the energy-momentum vector, the coordinate-space integration yields a δ-function and we have

$$f(k^2,pk)=(2\pi)^4\sum\delta(k-n_1)\langle j_1|n_1\rangle\langle n_1|j_2|p\rangle -(2\pi)^4\sum\delta(k+n_2-p)\langle j_2|n_2\rangle\langle n_2|j_1|p\rangle. \tag{12}$$

Thus the Fourier transform of the commutator will vanish unless either

$$k=n_1 \quad\text{or}\quad p-k=n_2,$$

where n_1 and n_2 are possible intermediate momenta. In terms of invariants,

$$f(k^2,pk)\neq 0 \quad\text{if}\quad \begin{cases} k^2\geqslant M_1^2 \\ 2pk\geqslant 2\kappa M_1 \end{cases} \quad\text{or}\quad \begin{cases} 2pk\leqslant -M_2^2+p^2+k^2 \\ 2pk\leqslant 2\kappa(\kappa-M_2), \end{cases} \tag{13}$$

where M_1 and M_2 are the lowest masses occurring in each ordering of the commutator. We know the Fourier transform only for real vectors k_μ; thus the function $f(k^2,pk)$ is given only for values of the invariants that lie in the physical region $(pk)^2 \geqslant p^2k^2$. In the nonphysical region, for those real values of the invariants that correspond to a momentum vector k_μ with imaginary components, the function will be determined by the representation (10). In the region $\mathcal{R}$, which consists of all of the (k^2,pk)-plane outside the nonphysical region and outside region (13), $f(k^2p,k)$ vanishes.

For convenience we introduce a new variable $\mu=\lambda^2-\beta^2p^2$ in the representation. Then

$$f(k^2,pk)=\int d\mu d\beta\, H(\mu,\beta)\epsilon(pk+\beta p^2) \times\delta(k^2+2pk\beta-\mu). \quad (14)$$

We allow μ and β to run from $-\infty$ to ∞ and incorporate the causality condition by having $H(\mu,\beta)$ vanish in the region $\mu<-\beta^2p^2$. Each point in the (k^2,pk) plane corresponds to a line integral of the function $\epsilon(pk+\beta p^2)H(\mu,\beta)$ over the straight line $\mu=k^2+2pk\beta$ in the (μ,β) plane.

The parabolic boundary of the nonphysical region in the (k^2,pk) plane, $(pk)^2=p^2k^2$, generates a set of lines that have as their envelope the curve A: $\mu=-\beta^2p^2$, and touch the envelope at the point $\beta=-pk/p^2$ at which the ϵ-function changes sign. All other points in the physical region correspond to lines on which the ϵ-function changes sign in the interior of the region in which $H(\mu,\beta)$ is zero. The boundary $k^2=M_1^2$, $2pk>\kappa M_1$ of $\mathcal{R}$ corresponds to the set of lines of positive slope passing through the point $\beta=0$, $\mu=M_1^2$ and lying between the line $\beta=0$ and a line tangent to the curve A. The other boundary $-2pk=M_2^2-p^2-k^2$ corresponds to a set of lines produced by the clockwise rotation of a line through the point $\beta=-1$, $\mu=M_2^2-p^2$ from the line $\beta=-1$ to a line tangent to the curve A. The other points in the (k^2,pk) plane at which $f(k^2,pk)$ vanishes correspond to all other lines lying entirely in the region swept out by these two sets of lines. If $H(\mu,\beta)$ vanishes sufficiently rapidly for large β, then a necessary condition for all these line integrals to vanish is that the function $H(\mu,\beta)$ vanishes in the region covered by these lines. The support of $H(\mu,\beta)$ lies in the region $\mathcal{S}$, the complement of the region covered by lines generated by points in $\mathcal{R}$. Only in this region can $H(\mu,\beta)$ be arbitrary. An alternative derivation of a necessary condition involving a different characterization of $H(\mu,\beta)$ is given in the Appendix.

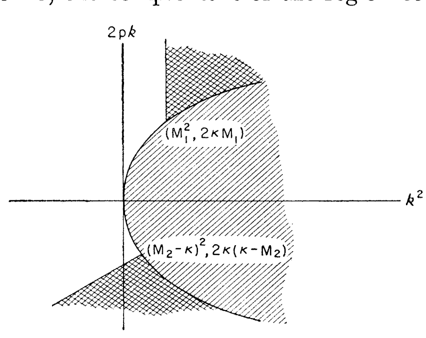

FIG. 1. The Fourier transform of the commutator as a function of k^2 and $2pk$. The part of the physical region in which the function is nonzero is cross-hatched; the nonphysical region is shaded.

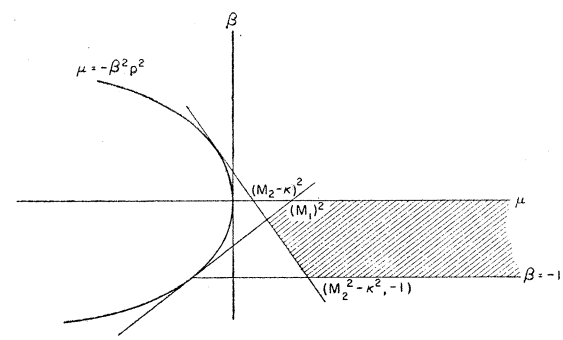

FIG. 2. The (μ,β)-plane. The weight function $H(\mu,\beta)$ is nonzero in the shaded region $\mathcal{S}$.

A typical situation is illustrated in Fig. 1, and the corresponding domain for $H(\mu,\beta)$ is shown in Fig. 2.

The two physical regions produced by the two orderings of the commutator may or may not be disjoint.

Case A: If $M_1+M_2\geqslant\kappa$, the two regions are disjoint. The region $\mathcal{R}$ will touch the boundary of the nonphysical region, and the region $\mathcal{S}$ will include only values of μ bounded below by the lines arising from the vertices of the region $\mathcal{R}$. Then $H(\mu,\beta)$ is nonvanishing only if

$$\mu\geqslant\max\{M_1^2+2\kappa M_1\beta,\ (M_2-\kappa)^2+2\kappa(\kappa-M_2)\beta\}, \quad (15)$$
$$0\geqslant\beta\geqslant-1.$$

Case B: If $M_1+M_2<\kappa$, the two physical regions overlap. Then the region $\mathcal{R}$ is bounded by two straight lines. Their intersection generates a line which is a lower bound for μ, but which always cuts the parabola $\mu=-\beta^2p^2$. The region $\mathcal{S}$ is then

$$\mu\geqslant\max\{M_1^2+\beta(M_1^2-M_2^2+\kappa^2),\ -\beta^2p^2\}, \quad (16)$$
$$0\geqslant\beta\geqslant-1.$$

Case A is the normal case. For a vertex arising from the interaction of stable particles, the stability condition will make the sum of the intermediate masses greater than any of the external masses. In this case the sign of the ϵ-function associated with a point (k^2,pk) depends only on which physical region contains the point. The support of $f(k^2,pk)$ generated by the representation is bounded by straight lines, all of which lie wholly in the complement of region $\mathcal{R}$. Thus in case A we have shown that $f(k^2,pk)$ vanishes in the nose of the nonphysical region, up to the line defined by

$$\mu=M_1(M_2-\kappa),\quad \beta=-(M_1+\kappa-M_2)/2\kappa, \quad (17)$$

which corresponds to the tip of the region $\mathcal{S}$. Furthermore, for extreme values of the relative masses, the representation demands that the contributions from certain matrix elements vanish. If $M_2-\kappa>M_1$, then the term $\sum\langle j_1|n\rangle\langle n|j_2|p\rangle$ in which the sum is over all eigenvalues consonant with a fixed n_μ, vanishes for $M_1^2\leqslant n^2\leqslant(M_2-\kappa)^2$. Similarly, if $M_1>M_2+\kappa$, then $\sum\langle j_2|n\rangle\langle n|j_1|p\rangle$ vanishes for $M_2^2\leqslant n^2\leqslant(M_1-\kappa)^2$.

An analogous representation can easily be derived for the matrix element of a commutator between two single-particle states with the same momentum.

GREEN'S FUNCTIONS

We have derived the representation (1) with limits (15) or (16). Now we shall write more symmetric representations, in terms of the momenta k_1 and k_2 associated with each current. In case A, we shall write the time-ordered Green's function. Since in this case the time-ordered Green's function differs from the retarded function only by the sign of the $i\epsilon$ in the denominator of (1) associated with the physical region arising from the reversed order of operators in the commutator, the function we want is given by dropping the sign factor from the $i\epsilon$ term. If we define

$$\int e^{ik_1x}e^{ik_2y}dxdy\langle(j_1(x),j_2(y))_+|p\rangle =\delta(k_1+k_2-p)G_+(k_1^2,k_2^2,p^2), \quad (18)$$

then, changing the limits on μ to incorporate the term $-\beta p^2$ arising from the change in the momentum variables,

$$G_+(k_1^2,k_2^2,p^2)=\int\frac{d\mu d\xi_1 d\xi_2\,\delta(\xi_1+\xi_2-1)H(\xi_1,\xi_2,\mu,p^2)}{\xi_1k_1^2+\xi_2k_2^2-\mu+i\epsilon},$$
$$\mu\geqslant\max\{\xi_1M_1^2+\xi_2(M_1-\kappa)^2,\ \xi_2M_2^2+\xi_1(M_2-\kappa)^2\},$$
$$\xi_1,\ \xi_2\geqslant 0;\quad p^2\leqslant(M_1+M_2)^2. \quad (19)$$

For case B we use the similar retarded function: the only change in (19) is that $i\epsilon$ becomes $i\epsilon\{k_1^2-k_2^2+p^2(\xi_1-\xi_2)\}$ and the limits are

$$\mu\geqslant\max\{\xi_1M_1^2+\xi_2M_2^2,\ p^2\xi_1\xi_2\},$$
$$\xi_1,\ \xi_2\geqslant 0;\quad p^2\geqslant(M_1+M_2)^2. \quad (20)$$

SPECTRAL REPRESENTATIONS

If we specialize the representation, putting a second particle on the energy shell, we obtain a spectral representation for the vertex as a function of momentum transfer. Consider two identical particles: let $k=p-p'$ where $(p')^2=p^2=\kappa^2$ (then $2pk=k^2$), and introduce a new variable $\sigma=\mu/(1+\beta)$ in (1); then, in the physical region where $k^2<0$, the vertex is

$$\Gamma(k^2)=\int_{\sigma_0}^{\infty}\frac{d\sigma\tilde{H}(\sigma)}{k^2-\sigma-i\epsilon}, \quad (21)$$

where

$$\tilde{H}(\sigma)=\int_{\mathcal{S}}d\beta(1+\beta)H(\sigma(1+\beta),\beta), \quad (22)$$

and the value of σ_0 is the minimum value of σ in the region $\mathcal{S}$ and is determined by (17):

$$\sigma_0=2\kappa M_1(M_2-\kappa)/(\kappa+M_2-M_1). \quad (23)$$

Here M_1 is the lowest mass that the odd-particle current can couple to the vacuum, and M_2 is the lowest mass that can be created by the current of the particle of mass κ. For stability $M_2>\kappa$. In the pion-nucleon vertex, $M_1=3m_\pi$, $M_2=\kappa+m_\pi$, and $\sigma_0=3m_\pi^2\kappa/(\kappa-m_\pi)$. In the first-order electromagnetic vertex, $M_1=2m_\pi$, $\sigma_0=2m_\pi2\kappa/(2\kappa-m_\pi)$. The relation (21), which has been derived for negative momentum transfer, defines an analytic function for all values of the momentum transfer with a cut along the positive real axis. We can observe that the weight function is real and that, above $\sigma=(2\kappa)^2$, it is related to the amplitude for pair creation in an external field.

The thresholds for these spectral or dispersion relations are lower than those of the usual perturbation theory. However it has been shown by Karplus, Sommerfield, and Wichmann,[1] Nambu,[2] and Oehme[3] that the use of a wider set of intermediate masses in perturbation theory will lower the threshold. Our thresholds are lower bounds to the true threshold. It is quite possible that use of the fact that the intermediate states are made up of several particles will raise the thresholds.

ANALYTICITY IN THE MASSES

We can use this representation to study another aspect of the analytic behavior of the vertex. Let us treat p^2 as a time-like momentum transfer and consider the function (19) or (20) as a function of the mass $k_1^2=k_2^2=M^2$. Then the function is analytic for negative M^2 and can be continued as a function of M^2 up to the minimum value of μ. In the case of equal intermediate masses, this minimum value occurs in (19) when $p^2=M_1^2=M_2^2$ and is $\frac{1}{2}M_1^2$. Thus in the meson-nucleon vertex one can continue only up to $M^2\leqslant(\kappa+m_\pi)^2/2$. This would be on the mass shell only if $m_\pi\geqslant(\sqrt{2}-1)\kappa$, which is the condition obtained by Bremermann, Oehme, and Taylor.[4] We do not need a continuation of this type to obtain (21), since we have worked in the physical region and have obtained the spectral representation directly.

ACKNOWLEDGMENTS

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[1] Karplus, Sommerfield, and Wichmann, Phys. Rev. **111**, 1187 (1958).
[2] Y. Nambu, Nuovo cimento **9**, 610 (1958).
[3] R. Oehme, Phys. Rev. **111**, 1430 (1958).
[4] Bremermann, Oehme, and Taylor, Phys. Rev. **109**, 2178 (1958).

APPENDIX

We shall give an alternative derivation of the representation (1) and prove that the function $H(\mu,\beta)$ vanishes outside the region $\mathcal{S}$ if $f(k^2,pk)$ vanishes in $\mathcal{R}$. It is convenient to give the proof in two stages: First we show that if the function $f(k^2,pk)$ vanishes in the region $\mathcal{R}_0$: $(k-p)^2<0$, $k^2<0$, then $H(\mu,\beta)$ vanishes outside the region $\mathcal{S}_0$: $\mu\geqslant-\beta^2p^2$, $0\geqslant\beta\geqslant-1$. No assumption concerning the mass spectrum, beyond the axiom of the completeness of positive-energy states, is involved in requiring the vanishing of $f(k^2,pk)$ in $\mathcal{R}_0$. It is now a simple matter to incorporate the detailed mass-spectrum restrictions and to show that if $f(k^2,pk)$ vanishes in $\mathcal{R}$, then $H(\mu,\beta)$ vanishes outside $\mathcal{S}$.

Since the Fourier transform $\tilde{f}(x)$ of the function

$$f(k-\tfrac{1}{2}p)=f(k^2,pk)$$
$$=\int dx e^{i(k-\frac{1}{2}p)x}\langle[j_1(x/2),\, j_2(-x/2)]|p\rangle$$

vanishes for space-like x and the function itself vanishes in $\mathcal{R}_0$, according to Theorem 2 of Jost and Lehmann[5] it has a representation

$$f(k^2,pk)=\int_{u^2<p/^24}d^3u\int_0^\infty dt^2\left\{\varphi_1(\mathbf{u},t^2)+\varphi_2(\mathbf{u},t^2)\frac{\partial}{\partial k_0}\right\}$$
$$\times\Delta_t(k-\tfrac{1}{2}p-\mathbf{u}),$$

with suitable weight functions φ_1 and φ_2, where Δ_t is the invariant commutator function for a "mass" t. In this case, the weight functions φ_1 and φ_2 depend only on $\mathbf{u}^2$. We may now make use of the following identity given by Dyson[6]:

$$\int\Delta_t(q-\mathbf{u})\delta(\mathbf{u}^2-b^2)d^3u=\pi\int_{-b}^{b}I_0[t(b^2-v^2)^{\frac{1}{2}}]\Delta_t(q-\tilde{v})dv,$$

where $\tilde{v}$ is a four-vector with vanishing space components, and time component equal to v. Since in the Lorentz frame chosen p has only a time component, κ, we may put

$$\tilde{v}=\alpha p,$$

and obtain

$$f(k^2,pk)=\pi\int_0^\infty dt^2\int_0^{p^2/4}db^2\int_{-b/\kappa}^{b/\kappa}d\alpha\, I_0[t(b^2-\alpha^2p^2)^{\frac{1}{2}}]$$
$$\times\left\{\kappa\varphi_1(b^2,t^2)+\varphi_2(b^2,t^2)\frac{\partial}{\partial\alpha}\right\}\Delta_t(k-(\alpha+\tfrac{1}{2})p).$$

[5] R. Jost and H. Lehmann, Nuovo cimento **5**, 1598 (1957).
[6] F. J. Dyson, Phys. Rev. **111**, 1717 (1958), Eq. (14).

If we now use the identity

$$\Delta_t(q)=\epsilon(q_0)\int^\infty d\lambda^2\,\Delta_t(\sqrt{\lambda^2})\delta(q^2-\lambda^2),$$

and introduce the parameters

$$\beta=-(\alpha+\tfrac{1}{2}),\quad \mu=\lambda^2-\beta^2p^2,$$

it follows that

$$f(k^2,pk)=\int_1^0 d\beta\int_{-\beta^2p^2}^\infty d\mu\, H(\mu,\beta)\epsilon(pk+\beta p^2)$$
$$\times\delta(k^2+2pk\beta-\mu),$$

where

$$H(\mu,\beta)=\pi\int_0^\infty dt^2\int_0^{p^2/4}db^2\,\Delta_t[(\mu+\beta^2p^2)^{\frac{1}{2}}]$$
$$\times I_0\{t[b^2-(\beta+\tfrac{1}{2})^2p^2]^{\frac{1}{2}}\}\theta(b^2-(\beta+\tfrac{1}{2})^2p^2)$$
$$\times\{\kappa\varphi_1(b^2,t^2)-\varphi_2(b^2,t^2)\partial/\partial\beta\}.$$

This concludes the first stage of the proof and yields a completely general representation including causality and the positive-definite nature of the energy spectrum.

For arbitrary functions φ_1 and φ_2, $H(\mu,\beta)$ will, in general, not be a distribution in the Schwartz sense. It is here that we must confine ourselves to the narrower class of Schwartz distributions. (This corresponds to an implicit restriction on φ_1 and φ_2, which in the Jost-Lehmann representation could be taken as arbitrary functions.) For these distributions we can apply the arguments of Jost and Lehmann to incorporate explicit mass restrictions and show that the function $H(\mu,\beta)$ which vanishes outside $\mathcal{S}_0$ vanishes in the region outside $\mathcal{S}$, provided $f(k^2,pk)$ vanishes in $\mathcal{R}$. For any point in $\mathcal{R}$, the line integrals of $\epsilon(pk+\beta p^2)H(\mu,\beta)$ over all lines that do not intersect the *convex* region $\mathcal{S}$ vanishes if $f(k^2,pk)$ vanishes in $\mathcal{R}$; but for all the lines generated by points belonging to $\mathcal{R}$, $\epsilon(pk+\beta p^2)$ changes sign inside the parabola where $H(\mu,\beta)$ vanishes. Consequently we can replace $\epsilon(pk+\beta p^2)H(\mu,\beta)$ by an appropriately modified $\bar{H}(\mu,\beta)$ for suitable sets of these lines, which cover the entire region between $\mathcal{S}$ and $\mathcal{S}_0$. A lemma of Jost and Lehmann[5] which states that for a function which is nonvanishing only in a bounded domain, a necessary and sufficient condition that all line integrals over straight lines that do not intersect a convex region vanish is that the function vanishes outside the convex region, then permits us to conclude that $\bar{H}(\mu,\beta)$ itself vanishes in the region between $\mathcal{S}$ and $\mathcal{S}_0$. Consequently $H(\mu,\beta)$ vanishes everywhere outside $\mathcal{S}$.