LOW-ENERGY PARAMETERS OF HYPERON-NUCLEON INTERACTION AND Λ HYPERFRAGMENTS*

S. Iwao and E.C.G. Sudarshan
Department of Physics and Astronomy, University of Rochester, Rochester, New York
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At the present time there are no direct measurements on the structure of the interaction between the nucleon and Λ hyperon and one has to rely on indirect methods to obtain some information. Just as the general features of nuclear forces may be deduced from the properties of complex nuclei, the gross features of the hyperon-nucleon interaction may be deduced from the properties of hyperfragments. Various attempts[1] have been made in this direction but most of them make explicit use of potentials and of wave functions for each hyperfragment.

We have undertaken a systematic study of hyperfragments using the standard method of low-energy nuclear physics. In particular, the so-called "projection theorem" furnishes a powerful tool for the analysis of these complex systems.[2] This theorem enables us to relate the properties of a system of $A-1$ nucleons and a hyperon to those of a system of $A-2$ nucleons in terms of the two-body matrix element of the fundamental two-particle interaction together with fractional parentage coefficients and kinematical factors. The method itself is capable of generalization to many-body forces also, but here we shall confine ourselves to two-body forces. It is clear that in the hyperfragments of low atomic number only certain average properties of the two-body interaction will enter. For the hyperfragments with $A \leq 5$ only the parameters T_Λ, S_1, S_0 enter, which are, respectively, the average hyperon kinetic energy and the ${}^3S$ and ${}^1S$ matrix elements of the Λ-nuclear interaction. The ground-state spins of the $A-1=2,3,4$ systems are 1, 1/2, 0; consequently we can have spin 1/2 or 3/2 for ${}_\Lambda H^3$, spin 0 or 1 for ${}_\Lambda H^4$ and ${}_\Lambda He^4$, and spin 1/2 for ${}_\Lambda He^5$.[3] The experimental binding energies of the Λ hyperon for hyperfragments with $A \leq 5$ are tabulated in Table I.[4] If we use these binding energies to calculate the nucleon–Λ hyperon parameters we get the solution $T_\Lambda = 7.64$ Mev, $S_1 = -2.08$ Mev, and $S_0 = -4.46$ Mev for set (a) and $T_\Lambda = 8.92$ Mev, $S_1 = -2.24$ Mev, and $S_0 = -5.28$ Mev for set (b) in Table I, where both sets favor the antiparallel spins (spins 1/2, 0, 1/2 for $A = 3, 4, 5$). Thus both singlet and triplet interactions are attractive and the average kinetic energy obtained is in fair agreement with the estimate of 6.5 - 8.8 Mev from the momentum distribution of the Λ hyperon obtained from the decay of the hyperfragments.[5]

Table I. The Λ binding energy (in Mev) for the hyperfragments with $A = 3$, 4, and 5.

(a)	${}_\Lambda H^3$	0.12	${}_\Lambda H^4$	2.20	${}_\Lambda He^5$	3.08
(b)	${}_\Lambda H^3$	0.12	${}_\Lambda He^4$	2.36	${}_\Lambda He^5$	3.08

For the configuration favoring parallel spins no solution can be obtained since we obtain the inconsistent set of equations:

$$T_\Lambda + 2S_1 = -0.12 \text{ Mev},$$

$$T_\Lambda + \tfrac{5}{2}S_1 + \tfrac{1}{2}S_0 = -2.20 \text{ or } -2.36 \text{ Mev},$$

$$T_\Lambda + 3S_1 + S_0 = -3.08 \text{ Mev}. \quad (1)$$

One is thus led to conclude that antiparallel spins are favored and that in particular, ${}_\Lambda He^4$ has spin 0.

Having obtained the parameters T_Λ, S_1, S_0 we may investigate the excited states of the hyperfragments with $A = 3, 4$ and, respectively, spins 3/2 and 1. Both of these turn out to be unbound by 3.48 and 0.20 Mev for set (a) and by 4.44 and 0.68 Mev for set (b), respectively. The only bound state of ${}_\Lambda He^4$ has spin 0 and consequently the existence of the mesonic absorption of the negative K meson in helium[6,7]:

$$K^- + He^4 \rightarrow {}_\Lambda He^4 + \pi^-, \quad (2)$$

implies that the K-meson parity (relative to that of the Λ hyperon) is odd. With the same parameters it can also be seen that the hyperdeuteron, $nn\Lambda$, and ${}_\Lambda He^3$ are unbound.

If the Λ-nuclear potential is of the form

$$V_{N\Lambda}(r) = (1 + a\,\vec{\sigma}_N \cdot \vec{\sigma}_\Lambda)V(r), \quad (3)$$

then the solution obtained corresponds to the choice $a = -0.22$ for set (a) and -0.25 for set (b).

For the p-shell hyperfragments one has to

Table II. The difference of the binding energies of the hyperfragments with p-shell nucleons and ${}_{\Lambda}He^5$ in Mev, where $B_{\Lambda}(4) = 3.08$ Mev.

Hyperfragment	$-B_{\Lambda}(A-1)+B_{\Lambda}(4)$ (experimental)	J	$-B_{\Lambda}(A-1)+B_{\Lambda}(4)$ (theoretical)	Δ_i	J	$-B_{\Lambda}(A-1)+B_{\Lambda}(4)$ (theoretical)	Δ_i
${}_{\Lambda}Li^7$	-2.38	1/2	-2.18	-0.20	3/2	-2.10	-0.28
${}_{\Lambda}Li^8$	-3.03	1	-3.16	0.13	2	-3.14	0.11
${}_{\Lambda}Be^9$	-3.52	1/2	-3.49	-0.03	3/2	-3.53	0.01
${}_{\Lambda}Li^9$	-4.12	3/2	-4.14	0.02	5/2	-4.19	0.07

distinguish between the $p_{1/2}$ shell hyperfragments and $p_{3/2}$ shell hyperfragments (j-j coupling scheme); one has correspondingly four new parameters, two for the $J=1$ and 0 states from the coupling of a $p_{1/2}$ nucleon and $s_{1/2}$ hyperon; and two for the $J=2$ and 1 states from the coupling of a $p_{3/2}$ nucleon and $s_{1/2}$ hyperon. We denote them by P_1, P_0, P_2', P_1', respectively. There are no hyperfragments belonging to the former category at this moment.[4] The numerical values obtained for the latter are $P_2' = -0.551$ Mev, $P_1' = -1.41$ Mev and $P_2' = -1.38$ Mev, $P_1' = -0.051$ Mev for the antiparallel spins and parallel spins, respectively, for nucleons and Λ hyperon. Both sets explain the observed data equally well, to within 5% and 7%, respectively. The data used for this analysis are given in Table II together with the theoretical values. Δ_i in the Table shows the difference between experimental and calculated binding energy. We got a good fit for both sets to within experimental errors.[4] This suggests that our assumptions made here are justified. We do not have to introduce three-body Λ-nuclear forces at this moment to explain the known experimental data. We can treat all the data and unknown hyperfragments from $A=3$ to 13 using the parameters obtained in this paper together with the assumption of the charge symmetry of N-Λ forces. For instance, the binding energies of the Λ hyperon of ${}_{\Lambda}He^7$, ${}_{\Lambda}B^{11}$, and ${}_{\Lambda}B^{12}$ are given as 4.73 Mev, 9.18 Mev, 9.73 Mev, and 4.84 Mev, 9.37 Mev, 9.75 Mev for the antiparallel spins and parallel spins, respectively. These are consistent with the observed values, 3.0 ± 0.7 Mev, 9.9 ± 0.6 Mev, and 9.7 ± 0.6 Mev, respectively, although all three of these values are derived from one event.[4]

Adding the two-body tensor and the spin-orbit N-Λ interaction,

$$bS_{N\Lambda}V(r) + c(\vec{l}_N + \vec{l}_\Lambda)\cdot(\vec{s}_N + \vec{s}_\Lambda)V(r),$$

and using the parameter $a = -0.22$, we obtain the relations $b - 1.7c = 0.37$ and $b - 0.54c = -2.0$ for the antiparallel and parallel spins, respectively. Our parameters give $S_0/S_1 \approx 2$, which is consistent with the estimate 2-3 by Dalitz and Downs.[8]

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