S. HATSUKADE, *et al.*
16 Aprile 1960
Il Nuovo Cimento
Serie X, Vol. 16, pag. 332-341

Note on Divergenceless Currents and Muon Polarization in $K_{\mu3}$ Decays (*).

S. HATSUKADE, R. E. MARSHAK, S. OKUBO (**) and E. C. G. SUDARSHAN
Department of Physics and Astronomy - University of Rochester

(ricevuto il 2 Gennaio 1960)

Summary. — The hypothesis of conserved current in weak interaction is examined for the strangeness non-conserving decays. This hypothesis enables the unambiguous calculation of the longitudinal polarization of a muon in $K_{\mu3}$ decay as a function of the pion energy. Its dependence on the muon energy is also studied. The result is compared with that of Okun and Martinyan who did not use a conserved current.

1. – Introduction.

The decay of non-strange particles is now more or less quantitatively understood in terms of the chirality invariant $V—A$ interaction (1). The qualitative features of strange particle decays is also consistent with a universal $V—A$ interaction, but precise quantitative tests are lacking chiefly because of the unknown renormalization effects due to the strong interactions. Various authors have, however, proposed (2) direct tests for the current-current structure of this $V—A$ interaction, particularly as regards the isotopic spin transformation properties of the strangeness-violating current. One may, in addition, impose dynamical constraints which do not depend on the renormal-

(*) This work was supported in part by the U. S. Atomic Energy Commission.
(**) Now at University of Naples, Naples.
(1) See, for example, *Present status of weak interactions* by R. E. MARSHAK, *Proceedings of the Kiev Conference on High Energy Physics* (1959).
(2) S. OKUBO, R. E. MARSHAK, E. C. G. SUDARSHAN, W. B. TEUTSCH and S. WEINBERG: *Phys. Rev.*, **112**, 665 (1958).

ization effects and a natural assumption is that the strangeness-violating current is divergenceless (3). We have discussed elsewhere (4) the implications of such a constraint for the pion-neutrino correlation in $K_{\mu 3}$ and K_{e3} decays. In this note we wish to discuss the predictions of this hypothesis for the muon polarization in $K_{\mu 3}$ decay.

The muon polarization in $K_{\mu 3}$ decay is interesting even if one does not consider divergenceless currents in kaon decay. The chirality invariant $(V-A)$ theory (5) suggests that in all weak interactions the fermions tend to come out with negative helicity while antifermions come out with positive helicity (since only the positive chiral projections are involved in the weak coupling). In allowed β-decay this suggestion is borne out since the electrons always possess negative helicity while the positrons possess positive helicity. However, in the $K_{\mu 2}$ and $\pi_{\mu 2}$ decays, the conservation of linear and angular momentum forbids negative helicity for the negative muon and the decay proceeds only by virtue of the chirality-violating mass term; this feature is responsible for the considerable suppression of the two-body electron modes of the kaon and the pion. This suppression no longer obtains in the decay modes $K_{\mu 3}$ and K_{e3} since there are now three particles in the final state. We note that at a given muon energy irrespective of the details of the decay mechanism, the negative

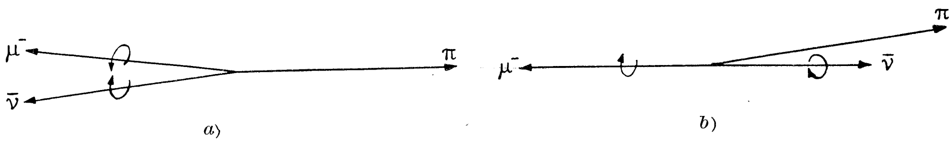

Fig. 1. – *a*) $K_{\mu 3}$ decay configuration of maximum pion energy and « right » muon helicity. *b*) $K_{\mu 3}$ decay configuration of minimum pion energy and « wrong » muon helicity.

muon in $K_{\mu 3}$ decay has negative helicity for the configuration of maximum pion energy while it has positive helicity for the configuration of minimum pion energy (see Fig. 1). If some additional restrictions (like divergenceless currents) are involved, one will be able to specify the detailed manner in which the muon polarization varies for configurations intermediate between these two extremes.

(3) S. S. GERSTEIN and J. B. ZELDOVICH: *Žurn. Ėksp. Teor. Fiz.*, **29**, 698 (1955); *Sov. Phys. Journ. Exp. Theor. Phys.*, **2**, 576 (1957); R. P. FEYNMAN and M. GELL-MANN: *Phys. Rev.*, **109**, 193 (1958).

(4) S. WEINBERG, R. E. MARSHAK, S. OKUBO, E. C. G. SUDARSHAN and W. B. TEUTSCH: *Phys. Rev. Lett.*, **1**, 25 (1958).

(5) E. C. G. SUDARSHAN and R. E. MARSHAK: *Proceedings of the Padua-Venice Conference on Mesons and Newly Discovered Particles* (September, 1957); R. P. FEYNMAN and M. GELL-MANN: *Phys. Rev.*, **109**, 193 (1958); S. OKUBO, R. E. MARSHAK, E. C. G. SUDARSHAN, W. B. TEUTSCH and S. WEINBERG: *Phys. Rev.*, **112**, 665 (1958).

2. – Muon polarization as a function of pion energy.

Within the framework of the current-current structure ([5]), the $K_{\mu3}$ decay matrix element is

$$\langle \pi\mu\nu | H_w | K \rangle = V^\lambda \bar{u}_\mu \gamma_\lambda (1+\gamma_5) u_\nu ,$$

with

$$V^\lambda = \{f(p^\lambda - q^\lambda) + g p^\lambda\} ,$$

where p and q are the kaon and pion momenta respectively and f and g are invariant functions of $(p - q^2)$, so that in the rest system they may be considered as functions of the pion energy. If the kaon is pseudoscalar (scalar), only the vector (axial vector) current contributes to this matrix element. If this current is conserved, it follows that the invariant functions f and g satisfy the relation ([4]):

$$(p - q)^2 f + (p^2 - pq) g = 0 . \tag{1}$$

Consequently, the decay matrix element is now completely determined except for an unknown multiplicative factor depending on the pion energy; we have

$$V^\lambda = \left\{ p^\lambda \left(\frac{q_0}{\Delta} - \frac{m_\pi^2}{m_K q_0} \right) - q^\lambda \right\} f(q_0) , \tag{2}$$

where q_0 is the pion energy and

$$\Delta = m_K - q_0 .$$

The pion and muon energies essentially determine the decay configuration and the muon polarization can now be evaluated as a function of these two quantities. The calculation is straightforward and yields for the longitudinal muon polarization

$$P(k_0, q_0) = -\frac{1}{|\boldsymbol{k}|} \left\{ k_0 + \frac{2(\Delta^2 - a)(k_0 a - m_\mu^2 \Delta) m_\mu^2}{[2k_0 a - (m_\mu^2 + a)\Delta]^2 - a(\Delta^2 - a)(a - m_\mu^2)} \right\} , \tag{3}$$

where

$$a = (m_K - q_0)^2 - q_0^2 + m_\pi^2 = \Delta^2 - |\boldsymbol{q}|^2 .$$

This expression for the polarization is plotted in Fig. 2 as a function of the pion energy q_0 for fixed k_0.

We may now calculate the muon polarization as a function of the pion energy by integrating over the muon energy spectrum for a fixed pion energy.

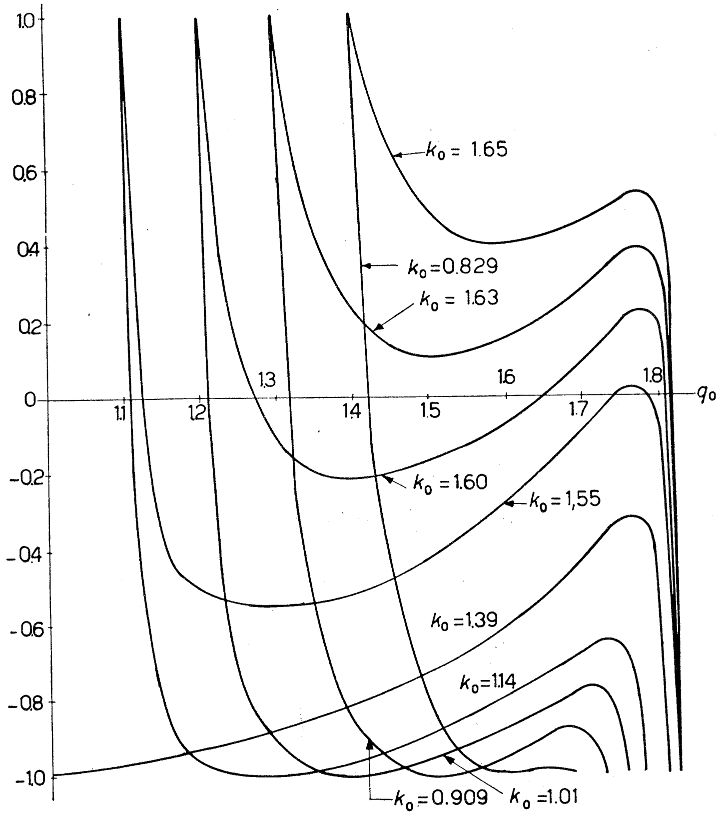

Fig. 2. – Muon longitudinal polarization $P(k_0, q_0)$ as a function of pion and muon energies.

This integration is facilitated by the observation that after the angle integrations are carried out, the kinematic weight factor for the configuration (q_0, k_0) is a constant in the allowed domain and zero outside. The allowed domain is indicated in Fig. 3 and corresponds to the restriction

$$(4)\qquad (\Delta - |\boldsymbol{q}|) + \frac{m_\mu^2}{(\Delta - |\boldsymbol{q}|)} \leqslant 2k_0 \leqslant (\Delta + |\boldsymbol{q}|) + \frac{m_\mu^2}{\Delta + |\boldsymbol{q}|}\,.$$

. Integrating (3) over k_0 gives:

$$(5)\qquad P(q_0) = \frac{F(q_0, k_0') - F(q_0, k_0'')}{G(q_0, k_0') - G(q_0, k_0'')}\,,$$

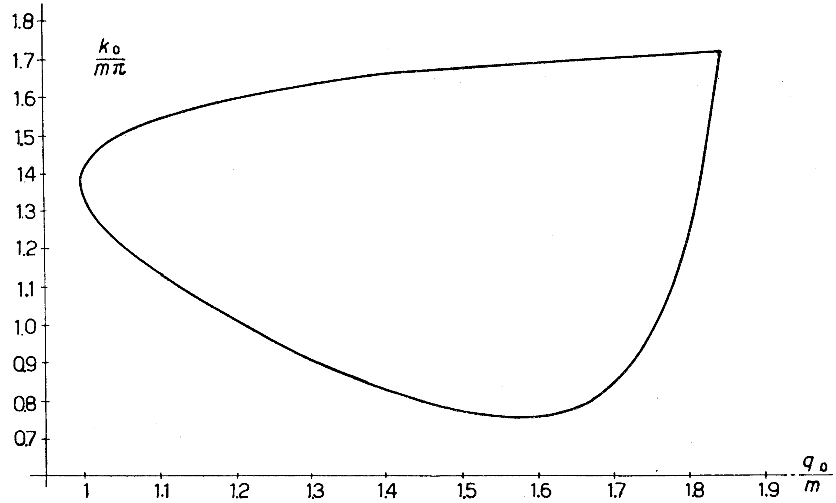

Fig. 3. – The allowed decay configurations.

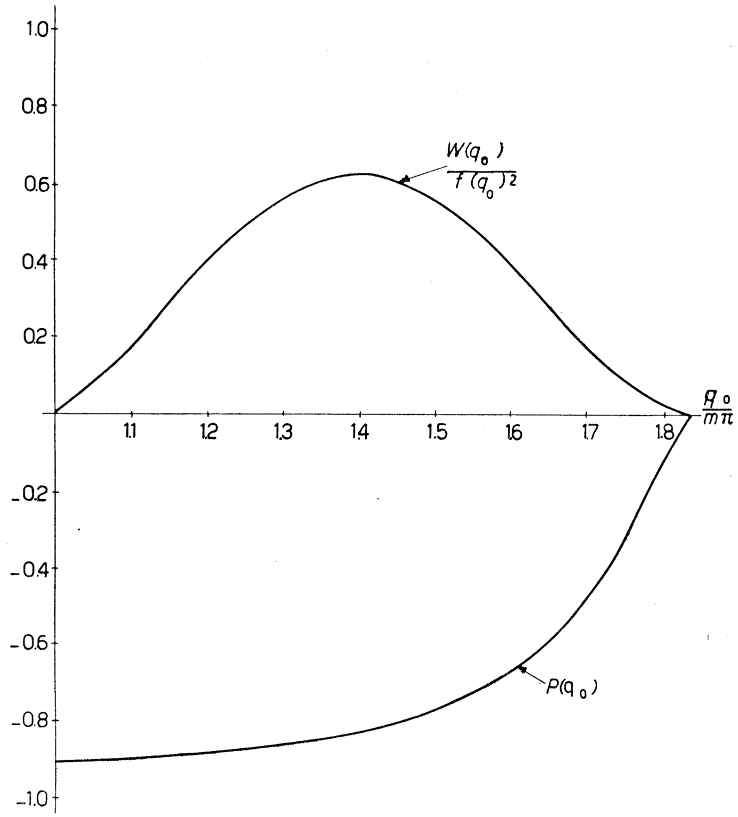

Fig. 4. – Integrated muon longitudinal polarization and differential energy spectrum as a function of pion energy.

1360

where

$$F(q_0, k_0) = \tfrac{2}{3}a^2 k_0^2 |\boldsymbol{k}| - (a + m_\mu^2) a \Delta k_0 |\boldsymbol{k}| +$$
$$+ \{\tfrac{1}{2}(a + m_\mu^2)^2 \Delta^2 - \tfrac{1}{2}a(a - m_\mu^2)(\Delta^2 - a) + (\Delta^2 - a) a m_\mu^2 + \tfrac{4}{3}a^2\}|\boldsymbol{k}| -$$
$$- \{(a + m_\mu^2)a + (\Delta^2 - a)m_\mu^2\} m_\mu^2 \log(k_0 + |\boldsymbol{k}|),$$

$$G(q_0, k_0) = \frac{1}{2} a(\Delta^2 - a)(a - m_\mu^2)k_0 - \frac{2}{3a}\left\{k_0 a - \frac{1}{2}(a + m_\mu^2)\Delta\right\}^3$$

and k_0' (or k_0'') is the maximum (or minimum) value of k_0 given by eq. (4).

This function is plotted in Fig. 4 along with the differential pion energy spectrum divided by the square of the (unknown) function $|f(q_0)|^2$. One notices that the polarization is large and negative over most of the spectrum.

3. – Muon polarization as a function of muon energy.

It is perhaps of greater interest experimentally to determine the muon polarization as a function of the muon energy. Since the matrix element for decay is known only within a multiplicative factor depending on the pion energy, this polarization computation can be carried through only if some form is assumed for the function $f(q_0)$. The two obvious choices are to take either $f(q_0)$ or $g(q_0)$ to be a constant. The integration over the pion energies now gives, respectively:

$$P_f(k_0) = -\frac{k_0}{|\boldsymbol{k}|} + \frac{m_\mu^2\{F_1(\Delta'', k_0) - F_1(\Delta', k_0)\}}{|\boldsymbol{k}|\{G_1(\Delta'', k_0) - G_1(\Delta', k_0)\}}, \tag{6}$$

where

$$F_1(\Delta, k_0) = \tfrac{1}{2}(2m_K k_0 - m_\mu^2)\Delta^2 - (5m_K^2 k_0 - m_\pi^2 k_0 - 2m_K m_\mu^2)\Delta +$$
$$+ (4m_K k_0 - m_\mu^2)(m_K^2 - m_\pi^2)\log \Delta + (m_K^2 - m_\pi^2)^2 \frac{k_0}{\Delta},$$

$$G_1(\Delta, k_0) = -\frac{m_K}{2}\{3m_\mu^2 + 4m_K^2 - 8m_K k_0\}\Delta^2 +$$
$$+\left\{\left(6m_K^2 + \frac{3}{2}m_\mu^2\right)(m_K^2 - m_\pi^2) + 2m_K^2 m_\mu^2 - \frac{1}{2}m_\mu^4 - 4m_K(2m_K^2 - 2m_\pi^2 - m_\mu^2)k_0 - 8m_K^2 k_0\right\}\Delta +$$
$$+ (m_K^2 - m_\pi^2)\{8m_K k_0^2 + 2(m_K^2 - m_\pi^2 - m_\mu^2)k_0 - m_K(3m_K^2 - 3m_\pi^2 + 2m_\mu^2)\}\log \Delta -$$
$$- \frac{(m_K^2 - m_\mu^2)^2}{2}\{m_K^2 - m_\pi^2 + m_\mu^2 - 4k_0^2\}\frac{1}{\Delta},$$

with

$$\Delta'' = m_K - q_0'',$$
$$\Delta' = m_K - q_0',$$

and q_0', q_0'' are the maximum and minimum values of q_0 for a given k_0, and

$$(7) \qquad P_g(k_0) = -\frac{k_0}{|\boldsymbol{k}|} + \frac{m_\mu^2\{F_2(a'', k_0) - F_2(a', k_0)\}}{m_K|\boldsymbol{k}|\{G_2(a'', k_0) - G_2(a', k_0)\}},$$

$$F_2(a, k_0) = \frac{1}{2}(2m_K k_0 - m_\mu^2)a^2 + \{-4m_K(m_K^2 + m_\pi^2)k_0 - 3m_\mu^2(m_K^2 - m_\pi^2) + 4m_\mu^2 m_K^2\}a + $$
$$+ (m_K^2 - m_\pi^2)\{(m_K^2 - m_\pi^2)(2m_K k_0 - 3m_\mu^2) + 4m_\mu^2 m_K\}\log a + m_\mu^2(m_K^2 - m_\pi^2)^3\frac{1}{a},$$

$$G_2(a, k_0) = \frac{1}{2}(8m_K k_0 - 4m_K^2 - 3m_\mu^2)a^2 +$$
$$+ \{8m_K(m_K^2 - m_\pi^2 + m_\mu^2)k_0 - 16m_K^2 k_0^2 - (2m_K^2 - 6m_\pi^2)m_\mu^2\}a -$$
$$- (m_K^2 - m_\pi^2)m_\mu^2(3m_K^2 - 3m_\pi^2 + 2m_\mu^2 - 8m_K k_0)\log a + (m_K^2 - m_\pi^2)^2 m_\mu^4\frac{1}{a},$$

where

$$a = m_K^2 + m_\pi^2 - 2m_K q_0$$

and a' and a'' denote the values of a at q_0' and q_0''.

These expressions are plotted in Fig. 5 along with the differential muon energy spectrum.

These assumptions of a divergenceless current and constancy of $f(q_0)$ or $g(q_0)$ may be compared to the approximation employed by other authors that $f(q_0)$ and $g(q_0)$ are both constants independent of q_0. Then the ratio f/g can be obtained by matching the branching ratio of the $K_{\mu 3}$ and K_{e3} modes ([6]). In this manner, FUJII and KAWAGUCHI ([6]) obtained:

$$(8) \qquad f/g = -4.9 \quad \text{or} \quad 0.56 .$$

This gives two alternative predictions for the decay matrix element and muon polarizations ([7]). We give, for comparison, the muon polarization

$$(9) \qquad P_{fg}(k_0) = -\frac{k_0}{|\boldsymbol{k}|} + \frac{m_\mu^2\{F_3(\Delta'', k_0) - F_3(\Delta', k_0)\}}{|\boldsymbol{k}|\{G_3(\Delta'', k_0) - G_3(\Delta', k_0)\}},$$

$$F_3(\Delta, k_0) = \left\{\frac{1}{2}(2m_K k_0 - m_\mu^2)\Delta^2 - (m_K^2 - m_\pi^2)k_0\Delta\right\}f^2 +$$
$$+ m_K^2(\Delta^2 - 2K_0\Delta)fg + m_K^2\left(\frac{\Delta^2}{2} - k_0\Delta\right)g^2,$$

([6]) R. GATTO: *Phys. Rev.*, **111**, 1426 (1958); S. ONEDA: *Nucl. Phys.*, **9**, 476 (1959); A. FUJII and A. KAWAGUCHI: *Phys. Rev.*, **113**, 1156 (1959).

([7]) L. OKUN and S. G. MATINIAN: *Žurn. Ėksp. Teor. Fiz.*, **36**, 1317 (1959); *Sov. Phys. Journ. Exp. Theor. Phys.*, **9**, 933 (1959).

$$G_3(\Delta, k_0) = \frac{m_\mu^2 m_K}{2}(\Delta^2 - 2K_0\Delta)f^2 + 2m_\mu^2 m_K\left(\frac{\Delta^2}{2} - k_0\Delta\right)fg + m_K^2\left\{(\Delta^2 - 2k_0\Delta)k_0 - \frac{1}{2}m_K(\Delta^2 - 2K_0\Delta)\right\}g^2\,,$$

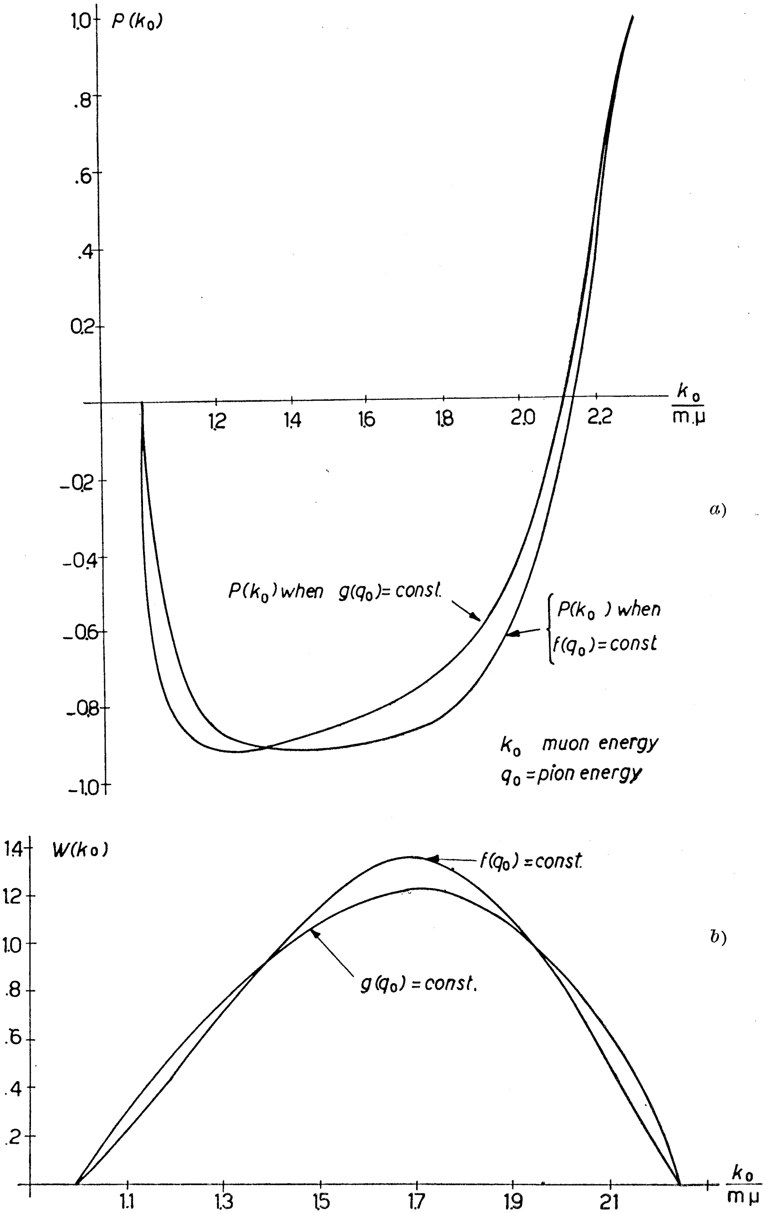

Fig. 5. – Integrated muon longitudinal polarization and energy spectrum as a function of muon energy.

1363

where $K_0 = (m_K^2 - m_\pi^2 + m_\mu^2)/2m_K$ is the maximum muon energy, in Fig. 6. It may be possible to discriminate among the theoretically predicted expressions P_f, P_g and P_{fg} by direct measurement of the muon polarization. It

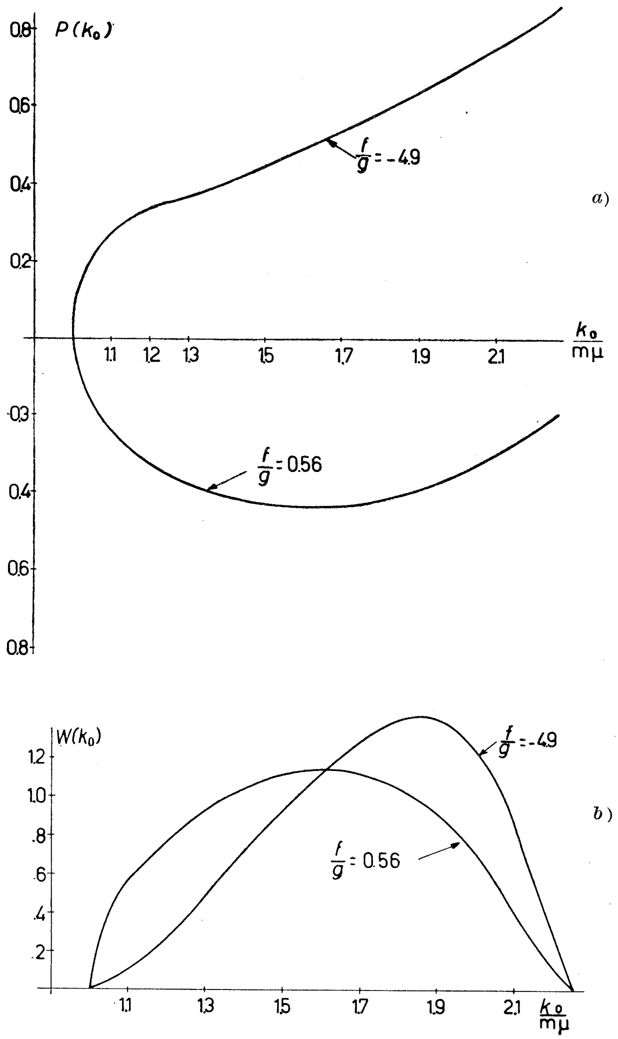

Fig. 6*a*). – Integrated muon longitudinal polarization as a function of muon energy under the assumption of constant $f(q_0)$ and $g(q_0)$. *b*) Muon energy spectrum under the assumption of constant $f(q_0)$ and $g(q_0)$.

appears that the polarization measurement may be easier than the pion-neutrino correlation measurement for testing the divergenceless current hypothesis. It may be noted that independent of the isotopic spin transformation properties of the interaction current, the polarization in the decays $K \to \pi^0 + \mu^- + \bar{\nu}$ and $K^0 \to \pi^+ + \mu^- + \bar{\nu}$ are the same under the « divergenceless »-current hypothesis.

Finally, it should be remarked that we have derived the relation between the functions $f(q_0)$ and $g(q_0)$ under the assumption of a divergenceless current [2]. However, a strictly conserved divergenceless current is not really required; it is sufficient that the divergence of the current has a vanishing matrix element between the one-kaon and one-pion states [8].

([8]) Compare the discussion by S. OKUBO: *Nuovo Cimento*: **13**, 292 (1959).

RIASSUNTO (*)

Si esamina l'ipotesi della corrente conservata nelle interazioni deboli nei decadimenti che non conservano la stranezza. Questa ipotesi rende possibile un calcolo non ambiguo della polarizzazione longitudinale del muone nel decadimento $K_{\mu 3}$ in funzione dell'energia del pione. I risultati vengono confrontati con quelli di Okun e Martinyan che non fecero uso della corrente conservata.

(*) *Traduzione a cura della Redazione.*