ANNALS OF PHYSICS: 13: 126–145 (1961)

Inconsistency of the Local Field Theory of Charged Spin 3/2 Particles*

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The relativistic quantum theory of Fermi Dirac fields of arbitrary spin is investigated and a general theorem is proved which aserts that for fields of half integral spin $>\frac{1}{2}$, the possibility of a consistent quantization requires that the equal-time anticommutators must be functions of the other fields to which the field in question is coupled. The case of spin $\frac{3}{2}$ is studied in detail and the equivalence of various formulations of the theory is shown. The inconsistency of the relativistic local quantum theory of a charged spin $\frac{3}{2}$ field in interaction with an external electromagnetic field is demonstrated by showing that the equal time commutation relations and relativistic covariance of the theory are not compatible. Finally, the mixed spin $\frac{3}{2}$-spin $\frac{1}{2}$ (Bhabha) field is found to be characterized by the same inconsistency.

I. INTRODUCTION

The description of physical states in terms of a relativistic field theory is doubly covariant in the following sense: on the one hand, the complete set of states of the quantized fields in virtue of the Lorentz covariance of the theory yields a representation of the (inhomogeneous) Lorentz group, which is in general reducible. On the other hand, the fundamental field operators themselves form the bases of (perhaps several) irreducible representations of the inhomogeneous Lorentz group. It is the mass and spin parameters of these irreducible representations that are usually called the mass and spin of the fields

* This work is supported in part through AEC Contract AT(30-1)-2098, by funds provided by the U. S. Atomic Energy Commission, the Office of Naval Research and the Air Force Office of Scientific Research.

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that characterize the latter representations. For example the Maxwell field is characterized by zero mass while the states which correspond to an assembly of photons yield a representation of the Lorentz group characterized by a finite mass.

There, at present, is no objective criterion for postulating that a particular particle is to be associated with a fundamental field. However, it is this association that some believe distinguishes elementary particles from composite structures. In this paper it is the fields to which we refer and if we say that a particle of a certain type cannot exist we mean that it cannot be elementary in this sense. We also assume that the primary kinematic characteristics of the particles, its spin, charge, isotopic spin, etc., are also characteristic of the field with which it is associated and that the coupling of the field to others does not alter these features.

Even in the classical theory the introduction of coupling between the fields may materially modify the covariant significance of the set of solutions and one must at each stage examine this aspect. Thus, for example, the requirement that the classical Maxwell field should be the wave function of a null mass particle of unit spin gives rise to the gauge invariance of the equations which requires the coupling of the field to conserved currents. Then the solutions of the classical equations with and without interaction then form a representation which is continuous in passing from one to the other. (In particular the spin parameter of the system is the same for both the free and coupled fields.) There is one other requirement which is basic to the correspondence between the free and coupled fields. That is, that the field without interaction can be formulated in such a way that the introduction of coupling can be restricted to nonderivative types. In this case the structure of the terms involving the field derivatives (the so-called "kinematical" terms) will be the same for both the interacting and noninteracting systems. For quantized fields whose dynamics follows from the action principle, this has the consequence that the commutation relations are independent of the dynamics, *provided* all the field components which are independent kinematically follows from the structure of these terms (*1*). We shall in fact show that in the case of F. D. fields with components which transform with spin $\geqq \frac{3}{2}$, the circumstance just mentioned can never occur for a quantizable theory. In fact, we shall show that the consideration of dynamics is essential in the discussion of the quantization. This is the first indication that the consistency of covariant quantization imposes more stringent requirements than that of a formal Lorentz covariance and a consistent free field quantization.

We use a local Lagrangian together with the action principle to characterize the system since no other complete *dynamical* principle is known for relativistic fields.

In this paper we shall investigate the consequences of the fact that the dynamics is essential for the quantization by studying the spin $\frac{3}{2}$ field and the Bhabha field interacting with an external electromagnetic field. We shall find that in both cases a consistent covariant quantization is impossible.

II. A THEOREM ON THE KINEMATICS OF FERMI-DIRAC FIELDS

Let us first review the basic description which is used for a Fermi–Dirac field which has no internal degrees of spin other than its spin. The Lagrangian for such a system will have the form (1).[1]

$$\mathcal{L} = \frac{i}{4}\left(\psi A^{\mu}\partial_{\mu}\psi - \partial_{\mu}\psi A^{\mu}\psi\right) - \mathcal{H}, \tag{2.1}$$

where $\mathcal{H}$ is an invariant function of the field components ψ and of all the other fields to which they are coupled. (For the latter fields we do not write the "kinematic terms" explicitly.) The field ψ is a set of Hermitian operators which are coupled together in the kinematic term by matrices A^{μ} which we assume to form an irreducible set. For the Lagrangian to be Hermitian, the matrices A^{μ} must be themselves Hermitian. Further, since we consider ψ to be a Fermi–Dirac field, these matrices are also symmetric and real.

From the action principle we obtain the field equations

$$iA^{\mu}\partial_{\mu}\psi = -\frac{\delta_r\mathcal{H}}{\delta\psi} = \frac{\delta_l\mathcal{H}}{\delta\psi}, \tag{2.2}$$

where the subscripts r and l signify, respectively, the right and left derivatives of $\mathcal{H}^{(1)}$. These are formed by factoring to the right or left in the differential of $\mathcal{H}$ the anticommuting variations of the field components. The operator generator of this same class of infinitesimal variations of ψ is

$$G = \frac{i}{2}\int d\sigma\, \psi A^{0}\delta\psi. \tag{2.3}$$

The matrix A^0 may be singular and only the field components which appear in the generator, namely those in the non-null space of A^0 will be candidates for independent variation by G. We say "candidates" since the field equations (2.2) may impose further constraints (i.e., relations between field components which do not involve time derivatives) on these field components so that the variations $\delta\psi$ which appear in (2.3) are not independent. To clarify this situation let us discuss the structure of the field equations (2.2) briefly. Let P_0 be the projection matrix on the null space of A^0 so that $P_0^2 = P_0$ and $P_0A^0 = 0$. Then from (2.2) we have the following equations of constraint:

$$iP_0A^k\partial_k\psi = P_0\frac{\delta_l\mathcal{H}}{\delta\psi}.$$

[1] We use units where $h = c = 1$ and a real space-like metric ($g^{\mu\nu} = (-1, 1, 1, 1)$). Greek indices run from 0 to 3 while Latin indices run from 1 to 3.

Since the A^k are linearly related to A^0 by an infinitesimal Lorentz transformation (see Eq. (2.7) below), it follows that $P_0A^kP_0 = 0$ and consequently the constraint equations may be written

$$iP_0A^k\partial_k(1 - P_0)\psi = P_0 \frac{\delta_l\mathcal{H}}{\delta\psi} \tag{2.4}$$

These constraints emerge solely as a consequence of the kinematics of the field ψ (that is, the singular property of A^0). The definition of the kinematically dependent field components $P_0\psi$ is totally independent of the dynamical term $\mathcal{H}$. We shall call (2.4) "primary constraints" to distinguish them from possible "secondary constraints" which define a second set of kinematically dependent components whose structure is dependent on the dynamics.

The left side of (2.4) does not contain the field components $P_0\psi$ and one or both of two possibilities may arise. First, the right side of (2.4) may contain certain (or all) of the field components $P_0\psi$ in such a way that we may express them in terms of the components $(1 - P_0)\psi$. If all of the components are so expressed, the remaining equations (2.2) become equations of motion for the dynamical components $(1 - P_0)\psi$ and they are accordingly capable of arbitrary variations at a single time. The second possibility occurs when some (or all) of the field components $P_0\psi$ are left undetermined by Eq. (2.4) but instead additional constraints are imposed upon some of the components $(1 - P_0)\psi$ at a given time; in this case not all variations $(1 - P_0)\delta\psi$ are independent. There are two sub-alternatives at this point. First, these equations in conjunction with the remaining field equations may degenerate and no longer involve the field ψ. Electrodynamics is such a theory and in general this situation occurs when the mass is zero and the field equations are invariant under some group of transformations involving arbitrary functions, such as gauge transformations. In electrodynamics, the degenerate field equation is the law of charge conservation. The second alternative is that the equations in conjunction with the remaining field equations cause a further separation which results in the existence of new (secondary) equations of constraint. It is possible that these secondary constraints may then determine the remaining components from the set $P_0\psi$ or the whole process may repeat. Thus, if the secondary equations do not fix the undetermined components $(1 - P_0)\psi$, these equations together with the remaining field equations will again separate to produce tertiary equations of constraint. The process will finally terminate since there are a finite number of components for ψ. It is important to emphasize that the secondary (and higher) constraints define kinematically dependent sets of field components whose structure is dependent on the dynamical term.

We shall now show that the last of the alternatives mentioned above *must* occur for the field equations if the field ψ is to be quantized consistently according to the Fermi–Dirac statistics and has spin $\geq 3/2$. For this let us assume that there are no constraints on the components $\tilde{\psi} = (1 - P_0)\psi$ and show that this results in a contradiction; for, in this case, the variations $\delta\tilde{\psi} = (1 - P_0)\delta\psi$ are independent and with the aid of the generator (2.3) we may derive the equal time (anti-) commutation relations (1)

$$\{\tilde{\psi}_\alpha(x), \tilde{\psi}_\beta(y)\} = [(1 - P_0)A^0(1 - P_0)]^{-1}_{\alpha\beta}\delta^{(3)}(x - y), \tag{2.5}$$

where the matrix is inverted in the nonsingular subspace. If these commutation relations are to be consistent, $(1 - P_0)A^0(1 - P_0)$ must be a positive definite matrix since ψ_α form a set of Hermitian operators, hence the left-hand side of (2.5) is positive-definite. We shall now show that $(1 - P_0)A^0(1 - P_0)$ is indefinite if the field ψ has spin $> 1/2$ so that the quantization implied by (2.5) cannot in fact be carried out consistently. The only alternative is to insure that the variations $\delta\tilde{\psi}_\alpha$ are not independent so that (2.5) would not follow; for this it is necessary that the field equations must always impose relations among the field components $A^0\psi$. These relations must be found among the primary constraints and therefore depend on the structure of $\mathcal{H}$. These constraints must restrict the fields ψ in such a manner that the generator (2.3) expressed in terms of a truly independent field components and their variations involves a matrix A (related to A^0) which is positive definite. We can summarize this discussion as follows. The requirement that some of the primary constraints give relations among the components $A^0\psi$, means that secondary constraints must appear. It will turn out that for spins $\geq 3/2$ these secondary constraints are essential whether or not the matrix A^0 is initially assumed to be singular. In any theory involving Fermi–Dirac fields which has the possibility of being consistently quantized the matrix A^0 must always be singular and both primary and secondary constraints must be present.

To demonstrate that A^0 is indefinite we make use of the invariance of the theory under the transformations constituting the homogeneous Lorentz group. If $S_{\alpha\beta}$ are the infinitesimal generators[2] of this group, the Lagrangian matrices A^μ satisfy the relations

$$S^T_{\alpha\beta}A^\mu + A^\mu S_{\alpha\beta} = i(A_\alpha\delta_\beta{}^\mu - A_\beta\delta_\alpha{}^\mu). \tag{2.6}$$

If we specialize to the case of simple Lorentz transformations we get

$$S^T_{4k}A^0 + A^0S_{4k} = A_k\,, \tag{2.7}$$

$$S^T_{4k}A^k + A^kS_{4k} = A^0, \tag{2.8}$$

where $iS_{0k} = S_{4k}$ is a real symmetric matrix. If we substitute (2.7) in (2.8) we

[2] The matrices: $iS_{\alpha\beta}$ have *real* elements since they describe the transformation of Hermitian field components into Hermitian field components. S_{0k} is symmetric and S_{kl} is antisymmetric.

find

$$S_{4k}^2 A^0 + A^0 S_{4k}^2 + 2S_{4k}A^0 S_{4k} - A^0 = 0. \tag{2.9}$$

The proof that A^0 is nondefinite follows directly from (2.9). Without loss of generality we may assume that the set A^μ is irreducible, because if they were not, one set of components of ψ could be covariantly separated from the rest and hence the theory would reduce to that of two fields coupled in some way and the quantization could be treated separately for each field. If A^0 is assumed positive, then it has a lowest eigenvalue $\lambda \geqq 0$.

Let P_λ be the projection matrix which satisfies

$$P_\lambda(A^0 - \lambda) = 0 = (A^0 - \lambda)P_\lambda .$$

Then from (2.9) it follows that

$$2P_\lambda S_{4k}(1 - P_\lambda)(A^0 - \lambda)(1 - P_\lambda)S_{4k}P_\lambda = -\lambda P_\lambda((2S_{4k})^2 - 1)P_\lambda . \tag{2.10}$$

Since $(1 - P_\lambda)(A^0 - \lambda)(1 - P_\lambda)$ is positive definite the left side is positive semidefinite. The right side is negative semidefinite. Hence, both sides must vanish. Since S_{4k} is a real matrix, the vanishing of the left side means that

$$P_\lambda S_{4k}(1 - P_\lambda) = 0.$$

But if $P_\lambda \neq 1$, this leads to the result that $P_\lambda A^\mu(1 - P_\lambda) = 0$ which contradicts the assumption of the irreducibility of the set A^μ. Consequently,

$$(1 - P_\lambda)(A^0 - \lambda)(1 - P_\lambda)$$

is indefinite unless $P_\lambda = 1$. In the latter case we have from the right side of (2.10)

$$(2S_{4k})^2 = 1.$$

That is, the spin of the field is ½. Thus, if A^0 is to be positive it must be a multiple of the unit matrix and the spin of the field must be ½.

We may thus draw the conclusion that the kinematics of all Fermi–Dirac fields of spin 3/2 will necessarily involve the dynamics of the field if they have the possibility of a quantization. This is consequent on the indefinite nature of A^0 in all such theories which necessitates the imposition of secondary constraints. It is for this reason that the free field quantization is completely without interest since even the equal-time commutation relations for the free and for the coupled fields are distinct (no interaction representation exists). We shall further find that in the case of spin 3/2 though it is possible to quantize the free field consistently, the commutation relations for a charged spin 3/2 field coupled to an external electromagnetic field only by its charge (i.e., "minimally") are inconsistent for *any* nonvanishing external field.

III. CLASSIFICATION OF SPIN 3/2 KINEMATICS

An explicit example of the theorem presented in the previous section is provided by the kinematics of a spin 3/2 field. In accordance with the general action principle the system is completely specified by the local Lagrangian density (2.1). If we restrict ourselves to a free field, the dynamical term becomes,

$$\mathcal{H} = \tfrac{1}{2} m \psi B \psi,$$

where because of the Fermi statistics B is a numerical matrix satisfying the relation

$$-B^T = B = B^+,$$

and whose structure will depend upon the precise nature of the system. In the usual theory (2), ψ is a 16-component entity and is the direct sum of the 12-component $\mathfrak{D}(1, \tfrac{1}{2}) + \mathfrak{D}(\tfrac{1}{2}, 1)$ representation and the 4-component

$$\mathfrak{D}(\tfrac{1}{2}, 0) + \mathfrak{D}(0, \tfrac{1}{2})$$

representation of the homogeneous Lorentz group. Relativistically, the field $\mathfrak{D}(1, \tfrac{1}{2}) + \mathfrak{D}(\tfrac{1}{2}, 1)$ is not the only irreducible representation which exhibits a maximum spin of 3/2 since there exists the 8-component $\mathfrak{D}(\tfrac{3}{2}, 0) + \mathfrak{D}(0, \tfrac{3}{2})$ representation. We shall consider the most general equation which involves the 8-component entity in addition to the 16-component vector spinor which thus contains 8 + 12 + 4 components.

A very convenient method of treating such a field is provided by the theory of (totally) symmetric multispinors. The number of independent components of a third-rank symmetric multispinor ϕ_{abc} is 4.5.6/1.2.3 = 20. Here ϕ_{abc} transforms as the direct product of 3 spinors $\psi_a(1)\psi_b(2)\psi_c(3)$. It can easily be shown that they correspond to the direct sum of the 8 and 12 component representations mentioned above. The projection matrix to the $\mathfrak{D}(1, \tfrac{1}{2}) + \mathfrak{D}(\tfrac{1}{2}, 1)$ part of the multispinor is,

$$\tfrac{1}{4}[3\delta_{aa'}\delta_{bb'}\delta_{cc'} - \delta_{aa'}\overset{5}{\gamma}_{bb'}\overset{5}{\gamma}_{cc'} - \overset{5}{\gamma}_{aa'}\delta_{bb'}\overset{5}{\gamma}_{cc'} - \overset{5}{\gamma}_{aa'}\overset{5}{\gamma}_{bb'}\delta_{cc'}].$$

We shall need one more result from the theory of multispinors,[3] namely that all invariants that can be constructed can be expressed in terms of the two fundamental invariants $\delta_{aa'}\delta_{bb'}\delta_{cc'}$ and $\delta_{aa'}\overset{5}{\gamma}_{bb'}\overset{5}{\gamma}_{cc'} + \overset{5}{\gamma}_{aa'}\delta_{bb'}\overset{5}{\gamma}_{cc'} + \overset{5}{\gamma}_{aa'}\overset{5}{\gamma}_{bb'}\delta_{cc'}$. As a consequence, the most general form for the Lagrangian matrices A^μ, B are,

[3] This and many other results have been derived by the authors and J. Schwinger (unpublished).

$$A^\mu = \sum (\beta\gamma^\mu)_{aa'}\beta_{bb'}\beta_{cc'} + \lambda \sum (\beta\gamma^\mu)_{aa'}(\beta\gamma^5)_{bb'}(\beta\gamma^5)_{cc'} \\ - i\omega \sum [(\beta\gamma^\mu)_{ab}\beta_{cd'} + (\beta\gamma^\mu)_{a'b'}\beta_{c'd}] + \kappa(\beta\gamma^\mu)_{dd'}, \tag{3.1}$$

$$B = \beta_{aa'}\beta_{bb'}\beta_{cc'} + \lambda' \sum \beta_{aa'}(\beta\gamma^5)_{bb'}(\beta\gamma^5)_{cc'} + \theta\beta_{dd'}, \tag{3.2}$$

where the sum is overall possible permutation of the Dirac matrices and where $\gamma_\mu{}^* = -\gamma_\mu$, $\gamma_\mu{}^T = -\beta\gamma_\mu\beta$, $\beta = \gamma^0 = -\gamma_0$ which is a Majorana representation. We have absorbed an overall normalization into the definition of the unit of length. The index d refers to the 4 component single-index spinor $\mathfrak{D}(\frac{1}{2}, 0) + \mathfrak{D}(0, \frac{1}{2})$ which together with ϕ_{abc} comprises the 24 components of ψ.

Let us first consider the simple case where the 4-component spinor is not present. In this case $\omega = \kappa = \theta = 0$ and the vector matrix A^μ assumes the simpler form,

$$A^\mu = \sum (\beta\gamma^\mu)_{aa'}(\beta_{bb'}\beta_{cc'} + \lambda(\beta\gamma^5)_{bb'}(\beta\gamma^5)_{cc'}).$$

Again, it can be shown that the two factors correspond to a decomposition of ψ according to spin $\frac{3}{2}$ or spin $\frac{1}{2}$ under space rotations. One immediately notices that the indefiniteness of the spin $\frac{3}{2}$ is unaltered; both the spin $\frac{3}{2}$ and spin $\frac{1}{2}$ parts have indefinite submatrices in general.

For a consistent quantization one is thus forced to do the following: put $\lambda = 1$ so that the spin $\frac{3}{2}$ submatrix becomes non-negative. This makes A^0 singular, but the singular part which corresponds to the 8-component

$$\mathfrak{D}(\tfrac{3}{2}, 0) + \mathfrak{D}(0, \tfrac{3}{2})$$

part of the field, decouples, that is, the entire set A^μ becomes reducible and we may therefore omit these components. Since no choice of ω and κ with $\lambda = +1$ will make the roots of

$$(A^0)^2 + (2 - \kappa)A^0 - 2\kappa = 18\omega^2$$

positive, these quantities must be chosen so as to produce primary constraints. For this purpose one has to choose ω so that A^0 becomes singular: one obtains,

$$\omega^2 = -\tfrac{1}{9}\kappa. \tag{3.6}$$

At this stage A^0 is singular but still is indefinite. The singular nature of A^0 leads to the primary constraints but in view of the indefiniteness further restrictions are necessary to permit quantization. This implies restrictions on the structure of the mass term (i.e., secondary constraints). We obtain these only if

$$\theta = \tfrac{1}{2}\kappa_0. \tag{3.7}$$

In particular, the matrix A^0 (which enters into the generator of changes in the field variables) is given by

$$A^0 = \sum \delta_{aa'}[\beta_{bb'}\beta_{cc'} + \lambda(\beta\gamma^5)_{bb'}(\beta\gamma^5)_{cc'}]. \tag{3.3}$$

For a consistent quantization of the half-integral spin field ψ, it is necessary that A^0 be positive definite or that secondary constraints occur. By elementary means one can show that A^0 satisfies the characteristic equation

$$[(A^0)^2 - 2(1+\lambda)A^0 - 3(1-\lambda)^2](A^0 + 1 + \lambda) = 0, \tag{3.4}$$

corresponding to the eigenvalues

$$-(1+\lambda), \qquad 1 + \lambda \pm 2(\lambda^2 - \lambda + 1)^{1/2}.$$

Thus, at least one of the roots is negative for arbitrary values of λ. By a straight forward, but tedious analysis of the matrix algebra one can show that the root $-(1+\lambda)$ corresponds to the part of the field which transforms with spin $\frac{1}{2}$ under spatial rotations while the two other eigenvalues correspond to spin $\frac{3}{2}$. From the structure of the matrix it is thus clear that except for $\lambda = +1$ the submatrix of A^0 for the part transforming according to the spin $\frac{3}{2}$ representation under spatial rotations is indefinite and a consistent quantization is impossible. Let us hence consider the more complicated case with $\omega \neq 0$, $\kappa \neq 0$. The characteristic equation now becomes

$$[(A^0)^2 - 2(1+\lambda)A^0 - 3(1-\lambda)^2] \\ [(A^0)^2 + (1 + \lambda - \kappa)A^0 - 18\omega^2 - \kappa(1+\lambda)] = 0. \tag{3.5}$$

For the free field (3.7) leads to the vanishing of one of the parts of ψ which transforms according to spin $\frac{1}{2}$ under spatial rotations, and to determination of the other spin $\frac{1}{2}$ but as a nonlocal function of the spin $\frac{3}{2}$ field.

In this fashion we see that of all possible spin $\frac{3}{2}$ equations for "free" fields, those which can be consistently quantized belong to a restricted class for which the Lagrangian matrix A^0 is singular and the dynamics is so restricted that of the 12 components which one would normally expect to be dynamical actually only 8 survive as true dynamical variables. The theory involves apart from the mass m, a single constant ω which can take any *nonzero* value. A theory of precisely this type has been known for several years (2). However, the basic quantity in it is a vector-spinor. In the Appendix we show the complete equivalence of the two formulations.

IV. THE CHARGED SPIN $\frac{3}{2}$ FIELD

We have found that the only form of a spin $\frac{3}{2}$ theory which has the possibility of consistent quantization in the absence of interactions is the vector-spinor equation or is equivalent to it. For the purpose of the investigation of the quantization of this equation when the field is charged and coupled to the electromagnetic field it is most convenient to work with the vector-spinor theory in essentially the form which was given by Rarita and Schwinger (2). For clarity, we shall give an independent discussion of some of the material given in Section III in terms of the vector-spinor formulation. In this case the Lagrangian matrices are

$$(A^{\mu})_{\lambda\sigma} = \beta(\gamma^{\mu}g_{\lambda\sigma} + W(\delta_{\lambda}{}^{\mu}\gamma_{\sigma} + \delta_{\sigma}{}^{\mu}\gamma_{\lambda}) + K\gamma_{\lambda}\gamma^{\mu}\gamma_{\sigma}), \tag{4.1}$$

where we have suppressed the spinor indices. W and K are real parameters which we shall specify shortly. The form (4.1) is the most general with the necessary properties of symmetry and Hermeticity required respectively by the F.D. statistics of the field and the Hermitian character of the field Lagrangian. A^0 satisfies the equation

$$(A^0 - 1)((A^0)^2 - 2(2K - W)A^0 - (2K + 3W^2 + 2W + 1)) = 0$$

and hence is indefinite as required by the general theorem. We may make A^0 singular by letting

$$K = -\tfrac{1}{2}(3W^2 + 2W + 1)$$

(this corresponds to the condition $\omega^2 = -\frac{1}{9}\kappa$ in Section III) when we find

$$A^0(A^0 - 1)(A^0 + 2(3W^2 + 3W + 1)) = 0,$$

so A^0 is still indefinite. The field components which transform as a spin $\frac{3}{2}$ set we shall find are those of the eigenvalue 1 which is accordingly eightfold $(2(2\frac{3}{2} + 1))$ degenerate. The eigenvectors of the eigenvalues 0 and $-2(3W^2 + 3W + 1)$ turn out to correspond to spinors ($S = \frac{1}{2}$) and the degeneracies are consequently four and four. However, the canonical variables which correspond to the spin $\frac{3}{2}$ field components are not simply the set of field components described by the eigenvalue $+1$ of A^0 as we shall see below.

The parameter W in the Lagrangian matrices is still not fixed. If we make the point transformation of the field components

$$\psi_{\alpha}' = \psi_{\alpha} + \frac{\mu}{4}\gamma_{\alpha}\gamma^{\beta}\psi_{\beta} \tag{4.2}$$

then the fields ψ' will be characterized by Lagrangian matrices A' which have the same structure as the set A except that W is replaced by

$$W' = W(1 \quad \mu) \quad \frac{\mu}{2}$$

Such a transformation merely mixes the two classes of spin $\frac{1}{2}$ components but leaves the set of spin $\frac{3}{2}$ components invariant. Consequently, the particular value of W is without physical significance (except for $\mu = 1$ when the transformation is singular; this choice corresponds to $\omega = 0$ which was excluded in Section III). All the Lagrangian matrices are hence equivalent. In most of the following choose for convenience $W = -1$. In this case the projection matrices for the two sets of spin ½ components are

$$(P_{-2})_{\mu\nu} = \tfrac{1}{2}(\delta_{\mu\alpha} + g_{\mu\alpha})(-\tfrac{1}{3}\gamma_\alpha\gamma_\beta)\tfrac{1}{2}(g_{\beta\nu} + \delta_{\beta\nu})$$

and

$$(P_0)_{\mu\nu} = \tfrac{1}{2}(\delta_{\mu\nu} - g_{\mu\nu}),$$

that is, the corresponding sets of components are $\gamma^k\psi^k$ and ψ^0 which we see are indeed characterized by spin ½ for 3 dimensional rotations. The spin $\tfrac{3}{2}$ components, which are independent of the choice of W, are

$$\psi^k_{3/2} = (\delta_{kl} + \tfrac{1}{3}\gamma_k\gamma_l)\psi^l. \tag{4.3}$$

There is a one-parameter family of invariant matrices,

$$B_{\alpha\beta} = \beta(g_{\alpha\beta} + T\gamma_\alpha\gamma_\beta),$$

which also have the necessary antisymmetry. If we write for the invariant Hamiltonian

$$\mathcal{H} = \mathcal{H}' + \tfrac{1}{2}m\psi B\psi,$$

then the field equations will be

$$A^0(-i\partial_0)\psi + [A^k(-i\partial_k \;\; + mB]\psi = -\frac{\delta\mathcal{H}'}{\delta\psi},$$

so we find the equations of constraint

$$P_0[A^k(-i\partial_k \;\; + mB](1 - P_0)\psi + mP_0BP_0\psi = -P_0\frac{\delta\mathcal{H}'}{\delta\psi} \tag{4.4}$$

A consistent quantization requires that further restrictions be imposed upon $(1 - P_0)\psi$. Accordingly, P_0BP_0 must be taken as singular if such restrictions are to emerge from (4.4). We find that $P_0BP_0 = 0$ if

$$T = \tfrac{1}{4}[(1 + 3W)^2 + 3(1 + W)^2]$$

(which corresponds to the choice $\theta = \tfrac{1}{2}\kappa$ of Section III). Thus, we choose this value for T and thereby also find a unique mass term. Further, under the transformation (4.2), $T \to T' = \tfrac{1}{4}[(1 + 3W')^2 + 3(1 + W')$ so the choice of W is still completely free. With this choice of T, the equations of constraint assume the form

$$P_0(A^k(-i\partial_k) \quad mB)(\;\; - P_0)\psi = -P_0\frac{\delta\mathcal{H}'}{\delta\psi}, \tag{4.5}$$

so if $\mathcal{H}'$ has the proper structure we will obtain four constraints on the twelve components $(1 - P_0)\psi$ which should leave only eight kinematically independent components which are characterized by spin $\frac{3}{2}$.

We could, at this point, construct the commutation relations for the set of kinematically independent free field components by letting $\mathcal{H}' = 0$ which characterizes the free field. However, we find it more convenient to take the free field as a special case of the charged field in interaction with an external electromagnetic field.

We may represent a charged field by considering two neutral fields ψ_1 and ψ_2. We take these to form a two-component set and let

$$A^\mu \rightarrow A^\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

We may also introduce the antisymmetrical Hermitian matrix

$$q = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

so that we may construct a nontrivial vector density

$$j^\mu = \tfrac{1}{2}\psi q A^\mu \psi.$$

It would not be possible to construct a nontrivial conserved vector density with a single Hermitian field because of the necessary symmetry of A^μ and the basic anticommutativity of the Fermi fields. Because q acts as the generator of infinitesimal rotations in the two-dimensional charge space under which the Lagrangian is invariant, the current j^μ will be conserved. We may mention here that this j^μ is the most general conserved vector, i.e., if $\mathcal{L}$ is invariant under $\delta\psi = i\delta\lambda q\psi$, then j^μ is conserved, other "possible" j^μ's are conserved only with Lagrangians which are more special. We shall couple the charged field to the electromagnetic field only through the charge, thus, in the usual way, we let

$$\mathcal{H}' = -ej^\mu \mathcal{A}_\mu,$$

where $\mathcal{A}_\mu$ is the potential which characterizes the external electromagnetic field. This coupling is of course equivalent to the replacement

$$-i\partial_\mu \rightarrow -i\partial_\mu - eq\mathcal{A}_\mu = \pi_\mu$$

in the kinematic term in $\mathcal{L}$.

In the absence of other interactions, the field equations are

$$A^0(-i\partial_0 - eq\mathcal{A}_0)\psi = -[A^k(-i\partial_k - eq\mathcal{A}_k) + mB]\psi,$$

and hence the equation of constraint (4.5) is

$$P_0(A^k\pi_k + mB)(1 - P_0)\psi = 0, \tag{4.6}$$

which reads

$$(-\tfrac{2}{3}\gamma\cdot\pi + m)\gamma_l\psi^l - \pi_k(\delta_{kl} + \tfrac{1}{3}\gamma_k\gamma_l)\psi^l = 0$$

so that the spin $\frac{1}{2}$ field $\gamma_l\psi^l$ is defined *nonlocally* in terms of the spin $\frac{3}{2}$ field components. In spite of the nonlocal character of this constraint we shall find that all of the components of the field satisfy *local* commutation relations.

If we insert (4.6) into the field equations for ψ^0 and $(\delta_{kl} + \frac{1}{3}\gamma_k\gamma_l)\psi_l$ we obtain the secondary set of constraints which will express the fields ψ^0 in terms of the kinematically independent components. These can be combined with the primary constraints (4.6) so that we may write at a given space-time point the covariant constraint

$$\left\{\frac{W}{2}\,eq\sigma F - \frac{3m^2}{2}\,(2W+1)\right\}\gamma_\sigma\psi^\sigma = ieq\gamma^\lambda F_{\lambda\sigma}\psi^\sigma \tag{4.7}$$

(which we have given for an arbitrary choice of W).

The commutation relations may now be derived using the generator

$$G = \frac{i}{2}\int d\sigma\,\bar\psi A^0\delta\psi = \frac{i}{2}\int d\sigma\left[\bar\psi_k\left(\delta_{kl} + \frac{1}{3}\gamma_k\gamma_l\right)\delta\psi_l + \frac{2}{3}\,\bar\psi_k\gamma^k\delta(\gamma_l\psi^l)\right]$$

together with the constraint (4.6) on the variations $A^0\delta\psi$ on a given surface

$$(-\tfrac{2}{3}\gamma\cdot\pi + m)\delta(\gamma_k\psi^k) - \pi_k(\delta_{kl} + \tfrac{1}{3}\gamma_k\gamma_l)\delta\psi_l = 0$$

Thus, we find (with the notation $\psi_k^{3/2} = (\delta_{kl} + \frac{1}{3}\gamma_k\gamma_l)\psi_l$)

$$\{\psi_k^{3/2}(x), \bar\psi_k^{3/2}(y)\}$$
$$= (\delta_{kn} + \tfrac{1}{3}\gamma_k\gamma_n)(\delta_{nm} + \tfrac{2}{3}\pi_n\Delta\pi_m)(\delta_{ml} + \tfrac{1}{3}\gamma_m\gamma_l)\cdot\delta(x-y) \tag{4.8}$$

where $\Delta = (m^2 - \frac{2}{3}eq\sigma\cdot H)^{-1}$, which we notice is *local* in spite of the non-local character of the constraint which expresses $\gamma_k\psi^k$ in terms of $\psi_k^{3/2}$. In (4.8) σ is the vector formed from the antisymmetrical tensor

$$\sigma_{kl} = \frac{i}{2}\,[\gamma_k, \delta_l]$$

and H is the magnetic field strength.

With the use of the constraint equation (4.6) we may derive the commutation relations for the kinematically dependent components of the field. We obtain, for example

$$\{\gamma_k\psi^k(x), \bar\psi_l^{3/2}(y)\} = (m + \tfrac{2}{3}\gamma\cdot\pi)\Delta\pi_m(\delta_{ml} + \tfrac{1}{3}\gamma_m\gamma_l)\cdot\delta(x-y) \tag{4.9}$$

$$-\{\gamma_k\psi^k(x), \gamma_l\bar\psi^l(y)\} = \tfrac{3}{2}[(m + \tfrac{2}{3}\gamma\cdot\pi)\Delta(m - \tfrac{2}{3}\gamma\cdot\pi) - 1]\delta(x-y). \tag{4.10}$$

The remaining commutation relations are also local since ψ^0 is locally given in terms of $\gamma_k\psi^k$ and $\psi_k^{3/2}$ by (4.7).

The consistency of the commutation relations depends upon the positive definiteness of the matrices on the right-hand side of (4.8) and (4.10). For simplicity consider (4.10). If $\varphi(x)$ is an arbitrary complex spinor function, then

$$\int \varphi^*(x)\{\gamma_k\psi^k(x), \gamma_l\psi^l(y)\}\varphi(y)\, dxdy = M^\dagger M + MM^\dagger$$
$$= \frac{3}{2}\int \varphi^*(x)\left[\left(m + \frac{2}{3}\gamma\cdot\pi\right)\Delta\left(m - \frac{2}{3}\gamma\cdot\pi\right) - 1\right]\varphi(x)\, dx \tag{4.11}$$

where $M = \int\varphi^*(x)\gamma_k\psi^k(x)\, dx$ so as a consequence of the positive definiteness of the operator on the left, the right side of (4.11) must be positive for arbitrary spinors $\varphi(x)$.

Let us define

$$(m - \tfrac{2}{3}\gamma\cdot\pi)\varphi(x) = \psi(x)$$

then

$$M^\dagger M + MM^\dagger = \frac{3}{2}\int\psi^*(x)[\Delta(x)\delta(x - y) - \bar{\Delta}(x, y)]\psi(y)\ (dx)(dx), \tag{4.12}$$

where

$$\bar{\Delta}(x, y) = (x \mid \{m^2 + (\tfrac{2}{3})^2(\gamma\cdot\pi)(\gamma\cdot\pi)^\dagger\}^{-1} \mid y) \tag{4.13}$$

and consequently is a positive definite operator. Accordingly, for the positivity of (4.11), Δ must be a positive definite function. However since

$$\Delta(x) = (m^2 - \tfrac{2}{3}eq\sigma\cdot H)^{-1},$$

it is only positive if $\frac{2}{3} \mid eH \mid < m^2$ everywhere. Now we may quantize the field in any Lorentz system and in each the commutation relations will have the form specified, with H the magnetic field strength as measured in that frame. But for a given external field we may always find a frame where $\frac{2}{3} \mid eH \mid < m^2$ is violated. Accordingly, the commutation relations can be made to exhibit an inconsistency in any nonvanishing external field.

Since the commutation relations can be demonstrated to be inconsistent in some frame (where the magnetic field is sufficiently strong) they must of course be inconsistent in any frame if the field equations and quantization are formally Lorentz covariant. But, it can be seen that a necessary and sufficient condition for the positivity of any anticommutator is that $\Delta - \bar{\Delta}$ be non-negative, and it is easily demonstrated that for a constant magnetic field $\Delta - \bar{\Delta}$ is positive if $\mid eH \mid < m^2$. It is clear that these two circumstances are incompatable with the

Lorentz covariance of the theory since the strength of an electromagnetic field depends upon the Lorentz frame in which it is observed.

V. THE BHABHA EQUATION

An equation which describes a particle with two mass states and spins $3/2$ and $1/2$ respectively has been given by Bhabha (3) who formulated his theory in terms of spinors with a field of 20 components which is the direct sum of the representation $\mathfrak{D}(1, 1/2) + \mathfrak{D}(1/2, 1)$ and the representation $\mathfrak{D}(1/2, 0) + \mathfrak{D}(0, 1/2)$ twice. We shall use a reformulation of the theory due to Gupta (4) which makes use of a vector spinor coupled to a Dirac spinor. It is advantageous to consider the group of transformations admitted by such a coupled system to see the equivalence of various formulations and to choose the simplest set of parameters for direct algebraic manipulations. By virtue of the existence of this transformation group, without any loss of generality, a single coupling constant can be used to characterize the system in the following fashion. Denote the irreducible parts of the field by ψ, ϕ_1, and ϕ_2. Any arbitrary coupling of ϕ_2 with both ϕ_1 and ψ can be reduced to a coupling of ϕ_2 with only ψ. The terms involving ϕ_2 only, as well as those involving the coupling of ϕ_1 and ψ can be brought to standard form as in the previously discussed case of spin $3/2$. The only essentially new coupling constant is now the coupling of ϕ_1 and ϕ_2 and, of course, the mass ratio of the spin $3/2$ and spin $1/2$ states. The standard vector-spinor scheme that we use has a coupling scheme somewhat different from the above outline but we wish to stress that the choice of this specific form involves no loss of generality; and since the reduction from the general form to the standard form is purely geometrical, the presence of gauge-invariant electromagnetic interactions does not destroy the validity of the above arguements.

The standard vector-spinor Lagrangian density for the Bhabha field in an external electromagnetic field is

$$\begin{aligned}\mathfrak{L} = -\tfrac{1}{2}\{&\psi^\mu\beta(\pi^\alpha\gamma_\alpha + m)\psi_\mu - \tfrac{1}{3}\psi^\mu\beta(\gamma_\alpha\pi^\alpha + \pi_\mu\gamma^\nu)\psi_\nu \\ &+ \tfrac{1}{3}\psi^\mu\beta\gamma_\mu(\pi^\alpha\gamma_\alpha - m)\gamma^2\psi_2 - \phi\beta(\gamma^\alpha\pi_\alpha + \kappa)\phi \\ &- \lambda[\phi\beta\pi^\alpha\psi_\alpha + \psi^\alpha\beta\pi_\alpha\phi]\}\end{aligned} \tag{5.1}$$

(all derivatives should be antisymmetrized). The equations of motion follow in the standard fashion. The matrix A^0 is again a singular matrix but now of rank 16 with the eigenvalues 1, $-2/3$, 0, 1 and the dynamics is restricted so as to impose additional constraints and to eliminate the degree of freedom corresponding to the negative eigenvalue (for "free" fields). The constraint equations are obtained in a similar fashion to the vector-spinor case and serve to define the parts $\gamma^\mu\psi_\mu$ and ψ^0 in terms of the "transverse" vector-spinor

$$\psi^{3/2} = \psi_k + \tfrac{1}{3}\gamma_k\gamma_l\psi_l$$

and the spinor ϕ in the form

$$\gamma^\mu\psi_\mu = \quad 3\lambda\phi - \left[\frac{m^2}{2-3\lambda^2} + \frac{1}{3}\,eq\sigma\cdot H\right]^{-1} \cdot\left\{eqH\times\gamma\cdot\psi^{3/2} + \frac{3m\lambda(m+\kappa)}{2-3\lambda^2}\,\phi\right\}$$

$$\psi^0 = \beta\left\{\frac{-3\lambda}{2}\,\phi + \left(m - \frac{2}{3}\,\gamma\cdot\pi\right)^{-1}\left(\pi\cdot\psi^{3/2} + \frac{3m\lambda}{2}\,\phi\right)\right\}. \qquad (5.3)$$

Here, for simplicity, we have restricted ourselves to the case of a external magnetic field. The 8 independent components of $\psi_k^{3/2}$ and the 4 of ϕ form the 12 independent dynamical variables necessary to describe the 4 + 8 degrees of freedom.

The generator of variations in the fields can be written down using the above constraint equation in the Lagrangian density. It is

$$G = \frac{i}{2}\int d\sigma\left\{\psi_k^{3/2}\delta\psi_k^{3/2} + \left(1 + \frac{3\lambda^2}{2}\right)\phi\delta\phi - \frac{2}{3}\,\xi\bar{\Delta}\delta\xi\right\}$$

where $\bar{\Delta}$ is defined by (4.13) and

$$\xi = \pi\cdot\ ^{3/2} + \frac{3m\lambda}{2}\,\phi.$$

We can insure that $\psi_k^{3/2}$ and $\delta\psi_k^{3/2}$ are purely transverse by inserting the projection operator

$$\mathcal{P}_{kl} = \delta_{kl} + \tfrac{1}{3}\gamma_k\gamma_l\,,$$

wherever $\psi_k^{3/2}$ and $\delta\psi_k^{3/2}$ appear. A partial diagonalization can be accomplished by the substitution

$$\phi' = \phi + m\lambda E^{1/2}\bar{\Delta}\pi\cdot\psi^{3/2},$$

where

$$E = 1 + \frac{3\lambda^2}{2} \quad \frac{3m^2\lambda^2}{2}\,\bar{\Delta}.$$

The generator now becomes

$$G = \frac{i}{2}\int d\sigma\;\{\phi' E\delta\phi' + \psi_k^{3/2}S_{kl}\,\delta\psi_l^{3/2}\},$$

$$S_{kl} = \left[\mathcal{P}\left(1 - \frac{2}{3}\,\pi C\pi\right)\mathcal{P}\right]_{kl}, \qquad C = \left(1 + \frac{3\lambda^2}{2}\right)\bar{\Delta}E^{-1}.$$

The commutation relations can now be written down by inspection:

$$\{\psi_k^{3/2}(x), \psi_l^{3/2}(y)\} = \{\mathcal{P}(1 + \tfrac{2}{3}\pi D\pi)\mathcal{P}\}_{kl}\cdot\delta, \qquad D = (C^{-1} - \tfrac{2}{3}\pi\mathcal{P}\pi)^{-1},$$
$$\{\phi'(x), \phi'(y)\} = E^{-1},$$
$$\{\phi'(x), \psi_k^{3/2}(y)\} = 0.$$

We now proceed to show that these commutation relations are inconsistent in the presence of a strong external electromagnetic field in the sense that the matrix appearing on the right-hand side of the anticommutator is not positive-definite. For this purpose, split S_{kl}^{-1} in the following fashion:

$$S^{-1} = U + V, \qquad \begin{aligned} U &= \mathcal{P}\{1 - \tfrac{2}{3}\pi(C^{-1} - D^{-1})^{-1}\pi\}\mathcal{P}, \\ V &= \mathcal{P}\tfrac{2}{3}\pi\{D + (C^{-1} - D^{-1})^{-1}\}\pi\mathcal{P}. \end{aligned}$$

Since $U^2 = U$ and $UV = 0$ it is sufficient to show that V possesses at least one negative diagonal element. For this purpose, choose

$$\varphi_k = \mathcal{P}_{kl}\pi_l C\cdot w,$$

where w is an arbitrary spinor function. The diagonal matrix element with respect to φ_k of the coefficient of $\delta(x - y)$ in the anticommutator for $\psi_k^{3/2}$ is simply $\tfrac{3}{2}\int w^*(D - C)w$. Now, C is positive definite:

$$C = \left(1 + \frac{3\lambda^2}{2}\right)\left\{m^2 + \left(1 + \frac{3\lambda^2}{2}\right)\left(m - \frac{2}{3}\gamma\cdot\pi\right)^+\left(m - \frac{2}{3}\gamma\cdot\pi\right)\right\}^{-1},$$

but D is indefinite

$$D = \left[\frac{m^2}{1 + 3\lambda^2/2} - \frac{2}{3}eq\sigma\cdot H\right]^{-1}$$

and can be negative for sufficiently large values of H. Hence for this particular case V and, consequently, S^{-1} are not positive definite. The quantized theory thus exhibits the same type of inconsistency as in the simpler case of spin $\tfrac{3}{2}$.

VI. CONCLUSIONS

It is well to recapitulate the results obtained. The general theorem proved in Section II shows that the matrix A^0 which is fundamental in the (anti-) commutator is always in indefinite matrix with the single exception of the Dirac theory. This implies that in a consistent quantum theory of half-integral spin fields, where the commutator matrix must be positive definite, there must be secondary constraints. These occur because the primary constraints in general lead to equations which produce certain relations among the "dynamical" field components. The delicate balance which, for the free fields, produces a theory which is compatable with the requirements of Lorentz covariance is upset in the presence of an interaction with an external electromagnetic field. Our demonstration has been in a sense indirect since we have shown that the equal time commutation relations cannot be consistent with positive definiteness requirements in all Lorentz frames. A direct demonstration of the lack of covariance would consist in showing that the generators of the infinitesimal Lorentz transformations on the field components do not satisfy the structure relations associated with the group. Because of the involved nature of this calculation we shall reserve this more direct demonstration for a later communication.

One should stress that the anticommutator continues to be local in the presence of the interaction, at least in the cases studied by us in detail. It may also be pointed out that the lack of consistency of the quantization of half integral spin fields of higher spin manifests itself already at the kinematical level (but because "kinematics" here involves dynamics). This implies that no interaction representation exists in these cases.

That the quantization of higher spin fields is not satisfactory has been generally felt, but we have been unable to find any proof of an inconsistency in the literature. The only systematic attempt in this direction seems to have been that of Weinberg and Kusaka (5) who claim to show that in the presence of interactions with an external electromagnetic field all higher spin fields (>1), both integral and half integral, acquire *nonlocal* (anti-) commutators which do not vanish for space like separations, thus violating causality. We have, however, explicitly carried through the quantization of the spin $\frac{3}{2}$ field in the presence of an external electromagnetic field and have shown that the anticommutators are *local* in all cases.

Of course, charged Fermi systems of spin $\geq \frac{3}{2}$ and higher do exist, but they do not admit a formulation in terms of a local action principle nor is their electromagnetic structure described solely in terms of the gauge-invariant replacement $\partial_\mu \rightarrow \partial_\mu - eqA_\mu$ in their wave equations, i.e., by a local interaction. Those charged systems that we know are complex nuclei which are "composite structures" in current theory. An elementary particle (such as the proton, for example, is believed to be) while exhibiting a complex "low-frequency" structure presumably preserves its kinematic properties for arbitrarily high-frequency perturbations which extract the "bare" particles or "field"; needless to say this is the criterion for distinguishing fundamental fields furnished by local relativistic quantum field theory. The results of the present investigation suggest that *any* charged particle of spin $\frac{3}{2}$ should be a composite structure in the sense that for arbitrarily high frequency measurements no charged field with the kinematic structure of spin $\frac{3}{2}$ will survive. The final answer to the question of whether local fields describe "elementary" particles must, of course, be found in nature.

Acknowledgments

The authors are indebted to Professor Julian Schwinger for numerous discussions and invaluable aid on these questions. One of us (K.J.) would like to acknowledge the hospitality extended to him at the Institute for Theoretical Physics of the University of Copenhagen where a portion of this work was done.

APPENDIX. THE EQUIVALENCE OF THE MULTISPINOR AND VECTOR SPINOR FORMULATIONS

The basic field quantity ψ^μ used in the vector spinor formulation carries an explicit vector index and can be covariantly decomposed into two parts

$$\psi^\mu = (\psi^\mu + \tfrac{1}{4}\gamma^\mu\gamma_\nu\psi^\nu) - \tfrac{1}{4}\gamma^\mu\gamma_\nu\psi^\nu = \varphi^\mu - \tfrac{1}{4}\gamma^\mu\varphi,$$

which correspond to the representations $\mathfrak{D}(1, \tfrac{1}{2}) + \mathfrak{D}(\tfrac{1}{2}, 1)$ and $\mathfrak{D}(\tfrac{1}{2}, 0) + \mathfrak{D}(0, \tfrac{1}{2})$, respectively. The most general Lagrangian matrices are, respectively,

$$(A^\mu)_{\alpha\beta} = \beta^{\gamma\mu}g_{\alpha\beta} + W\beta(\gamma_\alpha\delta_\beta{}^\mu + \gamma_\beta\delta_\alpha{}^\mu) + K\beta\gamma_\alpha\gamma^\mu\gamma_\beta\,,$$

$$B_{\alpha\beta} = \beta g_{\alpha\beta} + T\beta\gamma_\alpha\gamma_\beta$$

where the W, K, T terms correspond to the ω, κ, θ terms of the multispinor Lagrangian matrices. To see the correspondence more explicitly consider the representation of ϕ^μ by a multispinor:

$$\varphi_\gamma{}^\mu = \tfrac{1}{6}\sum P_{abc}(\beta\gamma^\mu)_{ab}\delta_{c\gamma}\chi_{abc}\,,$$

where $\tfrac{1}{6}\sum P$ is the symmetrising operator. The inverse transformation is given by

$$\chi_{abc} = \tfrac{3}{4}\cdot\tfrac{1}{6}\cdot\sum P_{abc}(\beta\gamma_\mu)_{ab}\delta_{c\gamma}\varphi_\gamma{}^\mu.$$

To work out the complete set of correspondence relations, we rewrite the vector spinor Lagrangian matrices in terms of φ^μ and ϕ to obtain:

$$\psi^\mu\{\beta\gamma^\alpha g_{\mu\nu} + W\beta(\gamma_\mu\delta_\nu{}^\alpha + \gamma_\nu\delta_\mu{}^\alpha) + K\beta\gamma_\mu\gamma^\alpha\gamma_\nu\}\psi^\nu = \varphi^\mu\beta\gamma^\alpha g_{\mu\nu}\varphi^\nu$$
$$+ \varphi^\mu[W\delta_\mu{}^\alpha + \beta\gamma^\alpha\beta\gamma_\mu]\varphi + \varphi[W\delta_\nu{}^\alpha + \tfrac{1}{4}\gamma_\nu\gamma^\alpha]\varphi^\nu + (K + \tfrac{1}{2}W - \tfrac{1}{8})\varphi\beta\gamma^\alpha\varphi.$$

If one makes use of the correspondence between tranceless vector spinors and totally symmetric multispinors, the identification with the multispinor Lagrangian is immediate.

We notice that there is a one parameter family of transformations on both the fields and Lagrangian matrices simultaneously which leaves the physical content of the theory completely unaltered. In the χ, φ(or φ^μ, φ) forms, this is simply a scale transformation of ϕ relative to $\chi(\varphi^\mu)$ and a compensating change in the A^μ, B. The parts involving ϕ once (or twice) get multiplied by the reciprocal factor once (or twice). In the usual vector spinor form the significance is more obscure. (see (4.2)) Since the transformation is purely geometrical, the presence of interactions does not interfer with the transformation group.

In the construction of interactions, the coupling matrices, I, must be chosen so as to preserve the existence of secondary constraints, that is, they must satisfy the equality

$$P_0 I P_0 = 0.$$

This is true, in particular, of gauge invariant electromagnetic interactions.

RECEIVED: October 13, 1960

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