IL NUOVO CIMENTO Vol. XXI, N. 5 1° Settembre 1961

Local Fields with Terminating Expansions (*).

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(ricevuto il 6 Giugno 1961)

Summary. — The class of field theories in which the current admits of a terminating expansion in a free field is studied. It is shown that, if one imposes the condition of locality, the current in question reduces to a local polynomial and the resulting field has the trivial S-matrix unity.

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1. – One of the challenging problems in the quantum theory of fields is to construct explicitly a local Lorentz invariant model field theory which has an asymptotic particle interpretation and gives rise to an S-matrix different from unity. Such a model, if it existed, would definitely prove that it is possible to have an interacting dynamical system within the framework of the axiomatic theory. Unfortunately, all the known examples of consistent field theories so far have the trivial S-matrix $S \equiv 1$. The examples known to date fall into two groups. The first one is the so called generalized Wightman polynomial (1),

$$\psi(x) = \sum_{i=1}^{n} a_i :\varphi^i(x): , \tag{1}$$

where φ is a free field and the double dot denotes the normal product (2); the

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(*) Supported by the U.S. Atomic Energy Commission.

(1) A. S. Wightman: *Problèmes matématiques de la théorie quantique des champs*, University of Paris lecture notes (1957-58; unpublished).

(2) G. C. Wick: *Phys. Rev.*, **80**, 268 (1950).

second known example is the generalized free field ([3,4]),

$$A(x) = \int_0^\infty \mathrm{d}\mu^2\, \varrho(\mu^2)\, \varphi(x, \mu)\,. \tag{2}$$

Here μ denotes the mass of the free field φ in question, and ϱ is an appropriate non-negative measure.

It can be shown for both models that $S \equiv 1$ identically. A way to see this easily is to notice that both $\psi(x)$ and $A(x)$ are symmetrical with respect to past and future, and the asymptotic limits for both $t \to +\infty$ and $t \to -\infty$ coincide with $\varphi(x)$.

All attempts to generalize these two models so as to include interaction have thus far failed. A suggestion of Wightman's was to take (1) as a current and obtain a local solution of the equation:

$$(\Box^2 + \mu^2)\, B(x) = \sum_1^n a_i :\varphi^i(x): = J(x)\,, \tag{3}$$

$B(x)$ can be expected to have a non-trivial S-matrix in general. WIGHTMAN ([5]) has, however, shown that, in the case the current has the specific form $J(x) = g:\varphi^2(x):$, the resulting $B(x)$ is not local. In the case of the more general $J(x)$ given by (1), ARAKI, HAAG and SCHROER ([6]) have proved that if $J(x)$ is complete, the resulting S-matrix is identically equal to unity. Here a complete field is defined by the property that the only operators that commute with arbitrary polynomials in the field in question are scalar multiples of the unit operator. If $J(x)$ is incomplete, the same authors have shown that the two-particle scattering amplitude must vanish, again emphasizing the difficulties of constructing a field with a non-trivial S-matrix.

It is the purpose of this paper to extend and partially generalize the above mentioned results, also with mainly negative conclusions. In what follows, we will restrict ourselves exclusively to the case where $J(x)$ is complete, as the case of incomplete J is treated in detail in the paper of reference ([6]) and is disposed of as being unphysical. We will take as our model a field whose

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([3]) O. W. GREENBERG: *Bull. Am. Phys. Soc.*, **6**, 306 (1961), Washington Meeting.

([4]) G. F. DELL'ANTONIO: *On the support of a field in p-space*, Northwestern University preprint. We are indebted to Dr. DELL'ANTONIO for a discussion on these points.

([5]) A. S. WIGHTMAN and S. T. EPSTEIN: *Ann. Phys.*, **11**, 201 (1960).

([6]) H. ARAKI, R. HAAG and B. SCHROER: *Nuovo Cimento*, **19**, 90 (1961).

current admits of the following expansion:

$$(4)\qquad J(x) = \sum_{K=2}^{N} \sum_{n+m=K} \int\!\dots\!\int \mathrm{d}^4p_1 \dots \mathrm{d}^4p_n \, \mathrm{d}^4q_1 \dots \mathrm{d}^4q_m \cdot$$
$$\cdot \delta(p_1^2 - \mu^2) \dots \delta(q_m^2 - \mu^2)\theta(p_1^0) \dots \theta(q_m^0) \exp\left[ix(p_1 + \dots + p_n - q_1 \dots - q_m)\right] \cdot$$
$$\cdot f(q_1 \dots q_m | p_1 \dots p_n) a^+(q_1) \dots a^+(q_m) a(p_1) \dots a(p_n) .$$

Here the particle operators are defined in terms of the free field $\varphi(x)$ in the usual manner:

$$(2\pi)^4 a(p)\theta(p_0)\delta(p^2 - \mu^2) = \int \mathrm{d}^4x \exp\left[-ip\cdot x\right]\varphi(x) .$$

and similarly for a^+. The quantities $f(q_1 \dots q_m | p_1 \dots p_n)$ are Lorentz invariant functions of the vectors in question and also are symmetric in $q_1, \dots, q_m$ and $p_1 \dots p_n$ separately. They are further assumed to be sufficiently smooth and continuous and to be bounded by a polynomial at infinity. Apart from these restrictions, they can be arbitrary. It is to be noted that any field for which one of the asymptotic fields is complete admits of a similar expansion (7) in terms of that asymptotic field, with the important difference that the expansion is not in general required to terminate at a finite order. Furthermore, here we do not necessarily identify the free field $\varphi(x)$ with any of the asymptotic fields.

At this point it is convenient to *define* the set of fields, which we denote by $D(x)$, by the following conditions:

$$(5)\quad \begin{cases} a) & D(x) \text{ admits a finite expansion of the form given by (4).} \\ b) & \text{The functions } f(q_1 \dots q_m | p_1 \dots p_n) \text{ appearing in this expansion are finite} \\ & \text{polynomials in the scalar products of the argument vectors.} \\ c) & \text{These functions further satisfy} \\ & n!\, m!\, f(q_1 \dots q_m | p_1 \dots p_n) = (n-1)!\,(m+1)!\, f(q_1 \dots q_m(-p_1) | p_2 \dots p_2) . \end{cases}$$

It is easy to see that when the f's are all constants, $D(x)$ reduces to the Wightman polynomial given by (1). In the general case, $D(x)$ is a local polynomial constructed not only out of the free field $\varphi(x)$ but also the space derivatives thereof; and is therefore a straightforward extension of (1).

All the results proved in reference (6) carry over without any difficulty to the case where the current is given by this slightly more general form.

(7) R. HAAG: *Kgl. Danske Vid. Selskab*, 29, 12 (1955).

2. – The main content of this paper will be the proof of the following result: *If a local covariant field $H(x)$ has a complete current given by* (4), *the S-matrix associated with this field is identically equal to unity.* The essential point of the proof consists in showing that a local $J(x)$ given by (4) must necessarily reduce to the local polynomial $D(x)$ defined above. Since $J(x)$ is then local relative to the free field $\varphi(x)$ and $H(x)$ is local with respect to $J(x)$, if $J(x)$ is assumed to be complete, it will follow that $H(x)$ has the trivial S-matrix unity by the use of Borchers theorem (8), exactly as in reference (6).

We now proceed to prove the main lemma: If $J(x)$ is given by (4) and if $[J(x), J(y)] = 0$ for $(x-y)^2 < 0$, then $J(x)$ must necessarily reduce to the local polynomial defined in (5). To this end, we need the following auxiliary result. Let $\psi(x)$ be a local covariant field and define

$$T(t, \alpha^2, \alpha\cdot\boldsymbol{P}, \alpha\cdot\boldsymbol{Q}, P, Q) = \int \mathrm{d}^3 r \exp\left[i\alpha\cdot\boldsymbol{r}\right] \langle Q\,|\,[J(0), J(x)]\,|\,P\rangle\,. \tag{6}$$

Here Q stands for the many particle state $Q \equiv (q_1, \ldots, q_m)$ and $P \equiv (p_1, \ldots, p_n)$ with $\alpha\cdot\boldsymbol{P}$ denoting the family of variables $\alpha\cdot\boldsymbol{p}_1, \ldots, \alpha\cdot\boldsymbol{p}_n$ and likewise for $\alpha\cdot\boldsymbol{Q}$. $\boldsymbol{r}$ and t represent the space and time components of x in a fixed frame of reference, and α is a three dimensional vector in the same frame. The particular dependence of T on α is required by ordinary rotation invariance. Now if T is given by this expression and exists for real α, it must be an entire function of the vector α in the complex plane, and further the set of functions $\partial^h T/\partial t^h$ must reduce to polynomials in α^2, $\alpha\cdot\boldsymbol{P}$ and $\alpha\cdot\boldsymbol{Q}$ for $t = 0$.

To prove the first part of the statement, we observe that since ψ is a local field, the domain of integration over r is restricted by the inequality $|r| \leqslant |t|$, hence α can be continued to any arbitrary complex vector for fixed t. The second statement follows from the fact that for $t = 0$, the integrand has support only at $r = 0$, and therefore reduces to the superposition of a δ-function and finite order derivatives thereof.

Let us now take $J(x)$ given by (6) for the local field in question, and also take $Q \equiv 0$, $P \equiv (p_1, \ldots, p_{2(N-1)})$, where the particle states are defined in terms of the particle operators of $\varphi(x)$. This is easily seen to be the highest non-vanishing matrix element. Evaluating the expression in question explicitly in terms of the expansion functions f, we get

$$\begin{aligned} T(t, \alpha^2, \alpha\cdot\boldsymbol{p}_1, \ldots, \alpha\cdot\boldsymbol{p}_L, P_1, \ldots P_L) = \\ = \sum_{(V)} \frac{C_{(V)}}{\mathscr{K}^0_{(V)}} \exp\left[it\left(\sum P^0_{(V)} + \mathscr{K}^0_{(V)}\right)\right] f(\mathscr{K}_{(V)}\,|\,P_{(V)}) f(\,|\,\mathscr{K}_{(V)} P_{(V)}) + \\ + \frac{C'_{(V)}}{\mathscr{K}^0_{(V)}} \exp\left[it\left(\sum P^0_{(V)} - \mathscr{K}_{(V)}\right)\right] f(\,|\,\mathscr{K}'_{(V)} P_{(V)}) f(\mathscr{K}'_{(V)}\,|\,P_{(V)}). \end{aligned} \tag{7}$$

(8) H. J. BORCHERS: *Nuovo Cimento*, **15**, 784 (1960).

Here $L=2(N-1)$ and the complexion (L) stands for the set of indices $(1, 2, ..., L)$, (U) is a proper subset of (L) and (V) is the complement of (U) with respect to (L). The notation $\sum_{(U)}$ indicates that the summation is to be carried over all such subsets (U), and $\sum P^0_{(U)}$ denotes the sum of all p^0's with indices belonging to (U). The four-vectors denoted by $\mathscr{K}$ are defined by the equation:

$$(8)\qquad \begin{cases} \mathscr{K}^0_{(U)} > 0\,, \quad \mathscr{K}^2_{(U)} = \mu^2\,, \quad \mathscr{K}'^0_{(U)} > 0\,, \quad \mathscr{K}'^2_{(U)} = \mu^2\,, \quad \boldsymbol{\mathscr{K}}_{(U)} + \boldsymbol{\alpha} + \sum \boldsymbol{P}_{(U)} = 0, \\ -\boldsymbol{\mathscr{K}}'_{(U)} + \boldsymbol{\alpha} + \sum \boldsymbol{P}_{(U)} = 0\,, \quad \mathscr{K}^0_{(U)} = \mathscr{K}'^0_{(U)}\,. \end{cases}$$

The C's are simple numerical constants which are irrelevant for our purposes.

The first point to observe about this equation is that from the assumed existence and proper behaviour of the f-function, it follows that T and all the derivatives of T with respect to time exist. The second point is that there is no integration left on the right side of (7); therefore, if we consider this equation together with all the other equations obtained by differentiating both sides with respect to time, they form a linear set of equations for the particular combinations of f-functions appearing in terms of T. Now consider the set of equations:

$$(9)\qquad \left(\frac{\partial^h T}{\partial t^h}\right)_{t=0} = \sum_{(U)} \frac{C_{(U)}}{\mathscr{K}^0_{(U)}}\, i^h \Big(\sum P^0_{(U)} + \mathscr{K}^0_{(U)}\Big)^h \cdot$$
$$\cdot f(\mathscr{K}_{(U)} | P_{(U)}) f(|\mathscr{K}_{(U)} P_{(U)}) + \frac{C'_{(U)}}{\mathscr{K}'^0_{(U)}}\, i^h \Big(\sum P^0_{(U)} - K'^0_{(U)}\Big)^h f(|\mathscr{K}'_{(U)} P_{(V)}) f(\mathscr{K}'_{(U)} | P_{(U)})\,.$$

If f's are considered as the unknown quantities, the determinant of the above set of linear equations clearly does not vanish identically, and therefore one can solve them in terms of $(\partial^h T/\partial t^h)_{t=0}$. Using the fact that $(\partial^h T/\partial t^h)_{t=0}$ is a polynomial in α^2, $\boldsymbol{\alpha}\cdot\boldsymbol{p}_1$, ..., $\boldsymbol{\alpha}\cdot\boldsymbol{p}_L$, we easily get

$$(10)\qquad f(\mathscr{K}_{(U)} | P_{(V)}) f(|\mathscr{K}_{(U)} P_{(U)}) = R(\alpha^2, \boldsymbol{\alpha}\cdot\boldsymbol{p}_1, ..., \boldsymbol{\alpha}\cdot\boldsymbol{p}_L; p_1, ..., p_L)\,.$$

Here R is a rational function of the first set of variables separated by a semicolon from the rest, and (U) is supposed to run over all possible configuration of indices. We now want to prove that R is actually a polynomial in the indicated variables. To this end, we rewrite eq. (7) in a slightly different form:

$$(11)\qquad T = \sum_{(U)} \exp\Big[it \sum P^0_{(U)}\Big] \left[\cos\{t\mathscr{K}^0_{(U)}\} F_{(U)} + \frac{\sin\{t\mathscr{K}^0_{(U)}\}}{\mathscr{K}^0_{(U)}} F'_{(U)}\right],$$

where

$$F_{(U)} = \frac{1}{\mathscr{K}^0_{(U)}} [C_{(U)} f(\mathscr{K}$$

$$F'_{(U)} = i[C_{(U)} f(\mathscr{K}_{(U)} |$$

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where

$$F_{(U)} = \frac{1}{\mathscr{K}^0_{(U)}} [C_{(U)} f(\mathscr{K}_{(U)} | P_{(V)}) f(|\mathscr{K}_{(U)} P_{(U)}) + C'_{(U)} f(|\mathscr{K}'_{(U)} P_{(V)}) f(\mathscr{K}'_{(U)} | P_{(U)})],$$

$$F'_{(U)} = i[C_{(U)} f(\mathscr{K}_{(U)} | P_{(V)}) f(|\mathscr{K}_{(U)} P_{(U)}) - C'_{(U)} f(|\mathscr{K}'_{(U)} P_{(V)}) f(\mathscr{K}'_{(U)} | P_{(U)})].$$

Because of their definition, F's also are rational functions in the variables α^2, $\boldsymbol{\alpha}\cdot\boldsymbol{p}_1, \ldots, \boldsymbol{\alpha}\cdot\boldsymbol{p}_L$, $\mathscr{K}^0_{(U)}$. It is also easy to verify that F and F' are even functions of $\mathscr{K}^0_{(U)}$ and therefore the square root that arises from the definition of $\mathscr{K}^0_{(U)}$ must disappear. Hence, we can conclude that F and F' are rational functions of the quantities α^2, $\boldsymbol{\alpha}\cdot\boldsymbol{p}_1, \ldots, \boldsymbol{\alpha}\cdot\boldsymbol{p}_L$: Now, take one of the components of $\boldsymbol{\alpha}$, say α_x and decompose F and F' in partial fractions with respect to α_x.

$$\text{(12)} \qquad \begin{cases} F_{(U)} = \sum_i \dfrac{E^i_{(U)}}{(\alpha_x - \gamma_i)^{n_i}} + P_{(U)}\,, \\ F'_{(U)} = \sum_i \dfrac{E'^i_{(U)}}{(\alpha_x - \gamma_i)^{n_i}} + P'_{(U)}\,. \end{cases}$$

Here E's and γ's do not depend on α_x, and P's are polynomials in further, all γ's are required to be distinct. We will now prove that all E's must vanish, leaving only the polynomial. To see this, we use the fact that T is an entire function of $\boldsymbol{\alpha}$, and therefore in the final sum the numerators must combine so as to cancel the denominators. Using the fact that γ's are distinct, this leads to a condition of the following form:

$$\text{(13)} \qquad \sum_{(U)} \exp\left[it \sum P^0_{(U)}\right] \left[\cos\{t\mathscr{K}^0_{(U)}\} E^i_{(U)} + \frac{\sin\{t\mathscr{K}^0_{(U)}\}}{K^0_{(U)}} E^{i'}_{(U)}\right] = 0$$

for $\alpha_x = \gamma_i$ for all i.

The $\mathscr{K}$'s are in general distinct since $\boldsymbol{p}$'s are arbitrary. It is easy to see that such an equation cannot be satisfied for arbitrary p's and arbitrary t unless all E's vanish, proving our original assertion. The same argument can be applied to the other components of $\boldsymbol{\alpha}$, with the conclusion that F and F' are polynomials in $\boldsymbol{\alpha}$; or equivalently, in the variables α^2, $\boldsymbol{\alpha}\cdot\boldsymbol{p}_1, \ldots, \boldsymbol{\alpha}\cdot\boldsymbol{p}_L$.

Solving for the f's in terms of F's, we get

$$\text{(14)} \qquad f(\mathscr{K}_{(U)} | P_{(V)}) f(|\mathscr{K}_{(U)} P_{(U)}) =$$
$$= Q^{(1)}_{(U)}(\alpha^2, \boldsymbol{\alpha}\cdot\boldsymbol{p}_1, \ldots, \boldsymbol{\alpha}\cdot\boldsymbol{p}_L; p_1, \ldots, p_L) + \mathscr{K}^0_{(U)} Q^{(2)}_{(U)}(\alpha^2, \boldsymbol{\alpha}\cdot\boldsymbol{p}_1, \ldots, \boldsymbol{\alpha}\cdot\boldsymbol{p}_L; p_1, \ldots, p_L),$$

where Q's are polynomials in the first set of variables.

We now take all the vectors P_i with i belonging to set (V) to lie completely

along the time axis. For this particular configuration of $P_{(V)}$, $f(\mathscr{K}_{(U)}|P_{(V)})$ is only function of $\mathscr{K}^0_{(U)}$ from Lorentz invariance, and we denote it by $1/\chi(\mathscr{K}^0_{(U)})$. We also switch our independent variable from α to $\mathscr{K}_{(U)}$, clearly without changing the polynomial character of Q's, and also drop the index (U), with the result

$$f(|\mathscr{K}P_{(U)}) = \chi(\mathscr{K}^0)[S_1(\mathscr{K}^2, \mathscr{K}\cdot\boldsymbol{P}_{(U)}; P_{(U)}) + \mathscr{K}^0 S_2(\mathscr{K}^2, \mathscr{K}\cdot\boldsymbol{P}_{(U)}; P_{(U)})]\,, \tag{15}$$

where S_1 and S_2 are polynomials in $\mathscr{K}^2$ and $\mathscr{K}\cdot\boldsymbol{P}_{(U)}$. Since $f(|\mathscr{K}P_{(U)})$ is a symmetric function of $\mathscr{K}$ and of each of the P's that constitute the set $P_{(U)}$, it follows that the right hand side of (15) is not only a polynomial in the variables $\mathscr{K}\cdot\boldsymbol{P}_{(U)}$, but also in the variables of the form $\boldsymbol{p}_i\cdot\boldsymbol{p}_j$. At this point, we invoke the Lorentz invariance of f, which requires it to be only a function of the covariant scalar products $\mathscr{K}\cdot P_{(U)}$, $p_i\cdot p_j$, etc. This, combined with the previous statement about the polynomial dependence of f on the three-dimensional scalar products, establishes the result that f is a polynomial in the four-dimensional scalar product $\mathscr{K}\cdot p_i$, $p_i\cdot p_j$, etc.

The same method can be applied to mixed elements by first taking $Q\equiv(p_1)$, $P\equiv(p_2, \ldots, p_{2(N-1)})$ and continuing in the same fashion. One can then easily establish the result that $f(p_1, \ldots, p_s|p_{s+1}, \ldots, p_N)$ is a polynomial in the variables $p_i\cdot p_j$ and also verify the crossing relations given in (5*c*). We have, therefore, proved that the term of the highest order in the expansion (5) must be a local polynomial in $\varphi(x)$, if $J(x)$ is a local current.

The rest of the proof proceeds by induction. We can now decompose the current as follows:

$$J(x) = J_{N-1}(x) + P_N(x)\,, \tag{16}$$

where $P_N(x)$ is the local polynomial containing up to N particle operators, and $J_{N-1}(x)$ has an expansion that ends up with $N-1$ particle operators. We can now consider a matrix element of the commutator between vacuum and a state containing $L-2=2N-4$ particles, with the result,

$$\begin{aligned} T(t,\alpha,p_1,\ldots,p_{L-2}) &= \int \mathrm{d}^3r\, \exp[i\alpha\cdot r]\langle 0|[J(0),J(x)]|p_1,\ldots,p_{L-2}\rangle = \\ &= \int \mathrm{d}^3r\, \exp[i\alpha\cdot r]\langle 0|[J_{N-1}(0),J_{N-1}(x)]|p_1,\ldots,p_{L-2}\rangle + \\ &+ \int \mathrm{d}^3r\, \exp[i\alpha\cdot r]\langle 0|[P_N(0),P_N(x)]|p_1,\ldots,p_{L-2}\rangle\,. \end{aligned} \tag{17}$$

The last term on the right hand side satisfies the same analyticity properties that T satisfies, namely, it is an entire function in α and its derivatives with respect to t reduce to polynomials in α at $t=0$. These statements can easily be verified using the well-known properties of the local polynomial. Hence

considering the expression

$$T(t,\alpha,p_1,\ldots,p_{L-2}) - \int$$

instead of T, our previous a identical conclusions. It is leads to the result that $J($ our main lemma is proved a is established.

3. – The result we have of reference (6). It shows th is not allowed to have a te result is also a natural exte ferent from unity, the outg in terms of the incoming nection between a free field that perhaps in local quan requirement of the existenc that it is not possible to id particle states. This also ap tum field theory using an in positive or in the negative

We wish to thank Prof. manuscript and for discus TONIO, Dr. O. W. GREEN various topics related to t

(9) O. W. GREENBERG: pri for a discussion of this topic

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(*) *Traduzione a cura della R*

considering the expression

$$T(t, \alpha, p_1, \ldots, p_{L-2}) - \int \mathrm{d}^3 r \exp [i\alpha \cdot r] \langle 0 | [P_N(0), P_N(x)] | p_1, \ldots, p_{L-2} \rangle ,$$

instead of T, our previous argument can be applied to $J_{N-1}(x)$ unchanged, with identical conclusions. It is now clear that this induction process continued leads to the result that $J(x)$ must be a local polynomial in $\varphi(x)$. Therefore our main lemma is proved and the result stated at the beginning of this paper is established.

3. – The result we have just proved strengthens the negative conclusions of reference ([6]). It shows that in order to get an interaction, the current $J(x)$ is not allowed to have a terminating expansion in terms of a free field. This result is also a natural extension of the theorem that if the S-matrix is different from unity, the outgoing field cannot admit of a terminating expansion in terms of the incoming asymptotic field ([9]). The absence of a finite connection between a free field and an interacting field tends to confirm the belief that perhaps in local quantum field theories with particle interpretation the requirement of the existence of scattering forces the physical states to be such that it is not possible to identify them with a linear space spanned by « free » particle states. This also appears plausible from recent developments in quantum field theory using an indefinite metric, and a definite answer either in the positive or in the negative would be very desirable.

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We wish to thank Prof. I. BIALYNICKI-BIRULA for a critical reading of the manuscript and for discussions; we also wish to thank Dr. G. F. DELL'ANTONIO, Dr. O. W. GREENBERG and Dr. FRANCIS TROYON for discussions of various topics related to the subject of this paper.

([9]) O. W. GREENBERG: private communication. We are indebted to Dr. GREENBERG for a discussion of this topic.

RIASSUNTO (*)

Studiamo la classe di teorie di campo in cui la corrente ammette in un campo libero uno sviluppo dotato di limite finito. Mostriamo che, se si impone la condizione di localizzazione, la corrente in questione si riduce ad un polinomio locale ed il campo risultante ha la banale unità della matrice S.

(*) *Traduzione a cura della Redazione.*