CONSEQUENCES OF SU_3 INVARIANCE[+]

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Invariance Properties of Interactions.

Invariance properties of interactions have been studied for many years now; and it is fair to say that the insight gained into dynamics of particles is in a large measure due to our understanding of their invariance properties. The study of compact invariance groups has by now achieved something of a canonical scheme: This involves the definition of the (compact) group, the multiplet assignment to known (and sometimes unknown or even nonexistent) particles, the reduction of Kronecker products of (irreducible) representations and finally the derivation of consequences of the invariance under the group. All this is in complete analogy with the isotopic spin invariance of strong interactions.

As a typical example of the experimental consequences of a symmetry group, we may consider the relations imposed by charge independence on the reaction:

$$\pi + \Sigma \longrightarrow \rho + \Sigma \ .$$

When the distinct amplitudes are written down, there are 19 amplitudes; but from charge independence we know that there are only three independent (complex) amplitudes corresponding to total isotopic spins 0, 1, 2. Hence there must exist 14 algebraic relations between the 19 scattering cross sections. These can be obtained by expanding the initial and final states in terms of states with definite total isotopic spin using Clebsch-Gordan coefficients. The individual cross section then depend on the three absolute magnitudes of the amplitudes for definite isotopic spin and the three (linearly dependent) phase differences. We list the 19 reactions:

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$\Sigma^+\pi^+ \rightarrow \Sigma^+\rho^+$	a	$\Sigma^-\pi^- \rightarrow \Sigma^-\rho^-$	a′
$\Sigma^+\pi^0 \quad \Sigma^+\rho^0$	b	$\Sigma^-\pi^0 \rightarrow \Sigma^-\rho^0$	b′
$\Sigma^+\pi^0 \rightarrow \Sigma^0\rho^+$	c	$\Sigma^-\pi^0 \rightarrow \Sigma^0\rho^-$	c′
$\Sigma^+\pi^- \rightarrow \Sigma^+\rho^-$	d	$\Sigma^-\pi^+ \rightarrow \Sigma^-\rho^+$	d′
$\Sigma^+\pi^- \rightarrow \Sigma^0\rho^0$	e	$\Sigma^-\pi^+ \rightarrow \Sigma^0\rho^0$	e′
$\Sigma^+\pi^- \rightarrow \Sigma^-\rho^+$	f	$\Sigma^-\pi^+ \rightarrow \Sigma^+\rho^-$	f′
$\Sigma^0\pi^+ \rightarrow \Sigma^+\rho^0$	g	$\Sigma^0\pi^- \rightarrow \Sigma^-\rho^0$	g′
$\Sigma^0\pi^+ \rightarrow \Sigma^0\rho^+$	h	$\Sigma^0\pi^- \rightarrow \Sigma^0\rho^-$	h′
$\Sigma^0\pi^0 \rightarrow \Sigma^0\rho^0$	k		

There are 13 relations linear in the cross sections:

$$a = a' \qquad e = e'$$
$$b = b' = h = h' \qquad f = f'$$
$$c = c' = g = g' \qquad j = j'$$
$$d = d' \qquad k = a \quad b \quad b \quad c + d \quad e + f.$$

There is in addition, one nonlinear relation which is best expressed in parametric form

$$b = a + c - 2\sqrt{ac} \quad \cos(\phi_1 - \phi_2)$$
$$f = c + e - 2\sqrt{ce} \quad \cos(\phi_2 - \phi_3)$$
$$k = e + a - 2\sqrt{ea} \quad \cos(\phi_3 - \phi_1)$$

There are thus 19 - 14 = 5 independent quantities determining the 19 cross sections and a total of 14 relations between them. We note that only one of these relations is nonlinear in the cross section. The computations could have been greatly simplified by noting that the 19 reactions arrange themselves into 9 pairs related by charge symmetry and a self-charge symmetric reaction. Hence the qualities of the corresponding pairs of cross sections follow from charge symmetry. The additional 5 relations require more than charge symmetry for its derivation. We shall shortly outline another method of deriving these 4 linear relations. We note that the derivation of these results depend only on the knowledge of the Clebsch-Gordan coefficients.

These methods generalize to more complicated invariance groups. In the sequal we shall confine our attention to invariance under the SU_3 group.[1] In generalization of the charge symmetry operation we have five discrete operations which, together with the identity, constitute the Weyl (reflection) subgroup.[2] All consequences of invariance under the Weyl subgroup are automatically valid for invariance under the SU_3 group.

The complete set of relations would follow if we use the Clebsch-Gordan decomposition of the initial and final states and use the (generalized) Clebsch-Gordan coefficients. There is but one serious drawback to the use of such a method; one must know the Clebsch-Gordan coefficients already. If we know these and have a direct line to the experiments (and a ready set of fudge factors) one is in business; but some of you may have sentimental objections to such a tedius and unsporting method of calculation. From a purely practical point of view, even if the Clebsch-Gordan coefficients are known for a reaction involving several particles, the coupling schemes become sufficently cumbersome to prompt the search for other schemes.

The Weyl Subgroup

A significant step in this direction is the observation that if one considers charge independence together with invariance under the Weyl reflections, these are equivalent to the invariance under the SU_3 group. Most of the consequences of charge independence are sufficiently familiar to be read off; and hence if we know the Weyl reflection properties of the transition amplitudes we can deduce all the consequences. We can further show that it is sufficient to include a suitable single Weyl reflection. Now the usual multiplet assignments of the octet and decuplet are represented graphically as hexagonal and triangular arrays:

$$\begin{array}{ccc} & N^{\circ} \quad N^{+} & \\ \Sigma & (\ \Sigma^{\circ}, \Lambda\ ) & \Sigma^{+} \\ & \Xi^{-} \quad \Xi^{\circ} & \end{array}$$

Octet Hexagon

$$\begin{array}{c} N_{*} \quad N_{*}^{\circ} \quad N_{*}^{+} \quad N_{*}^{++} \\ Y_{1*} \quad Y_{1*}^{\circ} \quad Y_{1*}^{+} \\ \Xi_{*}^{-} \quad \Xi_{*}^{\circ} \\ \Omega^{-} \end{array}$$

Decuplet Triangle

We shall choose the Weyl reflection which involves the interchange of Σ^{+} and N^{+}. This interchanges the roles of Ξ° and N°, and of Σ^{-} and Ξ^{-}, but takes Σ°, Λ into linear combinations of themselves. The complete transformation[5] under this Weyl reflection, W is given by;

$$\begin{aligned} \Sigma^{+} &\to -N^{+} \\ \Sigma &\to +\Xi^{-} \\ \Xi^{\circ} &\to -N^{\circ} \\ \Sigma^{\circ} &\to 1/2\ \Sigma^{\circ} - \sqrt{3}/2\ \Lambda \\ \Lambda &\to \sqrt{3}/2\ \Sigma^{\circ} - 1/2\ \Lambda \end{aligned}$$

For the decuplet the operation of W is simple since there are no particles with multiple weights; the intercharge of particles can be read off from the triangular array. We obtain in this manner for "elastic" baryon-meson scattering equalities of the type

$$w(N^\circ + \pi^+ \to \Sigma^+ + K^\circ) = w(\Xi^\circ + K^+ \to N^+ + \bar{K}^\circ)$$
$$w(N^\circ + \pi^+ \to N_*^{++} + \pi^-) = w(\Xi^\circ + K^+ \to N_*^{++} + K^-).$$

On the other hand the Weyl reflection of the reaction

$$N^+ + \pi^\circ \longrightarrow N_*^{++} + \pi^-$$

relates this amplitude to a linear combination of the amplitudes for the reactions

$$\Sigma^+ + \pi^\circ \to N_*^{++} + \bar{K}; \qquad \Sigma^+ + \eta \to N_*^{++} + \bar{K}$$

Thus the only relations linear in the cross section following four Weyl reflections are of the form:

$$w(N^+ + \pi^\circ \longrightarrow N_*^{++} + \pi^-) + w(N^+ + \eta \longrightarrow N_*^{++} + \pi^-)$$
$$= w(\Sigma^+ + \pi^\circ \longrightarrow N_*^{++} + \bar{K}^-) + w(\Sigma^+ + \eta \longrightarrow N_*^{++} + K^-)$$

Finally, if we consider the reaction

$$\Sigma^\circ + \pi^\circ \to \Sigma^\circ + \pi^\circ \qquad (A)$$

and the effect of W on it, we see that this amplitude mixes with the amplitudes.

$\Sigma^\circ + \pi^\circ \to \Lambda + \eta$	(B)	$\Lambda + \pi^\circ \to \Sigma^\circ + \eta$	(E)
$\Sigma^\circ + \eta \to \Sigma^\circ + \eta$	(C)	$\Lambda + \eta \to \Lambda + \pi^\circ$	(F)
$\Sigma^\circ + \eta \to \Lambda + \pi^\circ$	(D)	$\Lambda + \eta \to \Sigma^\circ + \pi^\circ$	(G)
		$\Lambda + N \to \Lambda + \eta$	(H)

Applying W in turn to these seven amplitudes, we get a homogeneous system of equations, the solution of which gives the five relations.

$$A = H; \quad B = G; \quad C = F; \quad D = E; \quad A = B + C + D$$

four of which yield relations linear in the cross sections. Similiarly in the reaction octet + octet $\longrightarrow$ decuplet + octet, we have the four coupled amplitudes

$\Sigma^\circ + \pi^\circ \to Y_{1*}^\circ + \pi^\circ$	(A)	$\Sigma^\circ + \eta \to Y_{1*}^\circ + \eta$	(C)
$\Lambda + \pi^\circ \to Y_{1*}^\circ + \eta$	(B)	$\Lambda + \eta \to Y_{1*}^\circ + \pi^\circ$	(D)

Applying W we get the homogeneous system

$$\begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = 0$$

with the solution

$$-A = B = C = D$$

yielding three relations linear in the cross sections.

Using these methods we have worked out the implications of invariance under SU_3 for the reactions octet + octet ⟶ octet + octet and octet + octet ⟶ decuplet + octet[6] as well as the decays octet ⟶ octet + octet and decuplet ⟶ octet + octet. The catalogue of the reactions and the relations following from SU_3 invariance which are linear in the cross sections are presented in the Appendix. These relations hold either for the differential or for the total cross sections at any energy provided the mass differences and other violations of the SU_3 invariance may be neglected.

Approximate Invariance Properties

But before we start neglecting these invariance breaking effects, we ought to estimate them. One way which has been tried is to use the actual phase space and appropriate form factors which depend on the actual masses rather than the ideal masses. But we wish to go beyond this. We shall outline a method which is generally applicable and which can accommodate breakdown of the symmetry in first or higher orders. It has the advantage of being applicable in essentially the same form to any arbitrary compact group and is purely automatic in the sense that no prior information about the Clebsch-Gordan coefficient for the group is essential. It is based on a general theory suggested by Smushkevich's method of deriving consequences of charge independence.[7] The proofs of this class of theorems[8] have been worked out by us but is omitted from this presentation. To illustrate the method we consider an example[9] from charge independence: The decay of the isotopic spin 3/2 nucleon resonance into a pion and a nucleon.

$$N_*^{++} \longrightarrow p + \pi^+ \qquad (\Gamma_1)$$
$$N_*^{+} \longrightarrow p + \pi^\circ \qquad (\Gamma_2)$$
$$N_*^{+} \longrightarrow \eta + \pi^+ \qquad (\Gamma_3)$$
$$N_*^{\circ} \longrightarrow p + \pi^- \qquad (\Gamma_4)$$
$$N_*^{\circ} \longrightarrow \eta + \pi^\circ \qquad (\Gamma_5)$$
$$N_* \longrightarrow \eta + \pi^- \qquad (\Gamma_6)$$

If charge independence were strictly valid we would have equal total widths.

$$\Gamma_1 = \Gamma_2 + \Gamma_3 = \Gamma_4 + \Gamma_5 = \Gamma_6$$

In addition we can further show that the weighted "count" of any component of an irreducible multiplet is independent of the component. From the nucleon doublet we get the weighted proton and neutron counts to be equal.

$$\Gamma_1 + \Gamma_2 + \Gamma_4 = \Gamma_3 + \Gamma_5 + \Gamma_6$$

From the pion triplet, equating weighted counts we get

$$\Gamma_1 + \Gamma_3 = \Gamma_2 + \Gamma_5 = \Gamma_4 + \Gamma_6$$

These six equations are compatible and give the unique solution

$$2\Gamma_1 = 3\,\Gamma_2 = 6\,\Gamma_3 = 6\,\Gamma_4 = 3\,\Gamma_5 = 2\,\Gamma_6$$

Of course this result could have been read off from the well-known Clebsch- Gordan in this case. However, let us now state that the invariance under the isotopic spin rotations is not exact but that it is broken by a transition matrix which transforms the sum of an invariant operator and the third component of a vector (regular representation) operator. The weighted counts are not independent of the component then but depend on the magnetic quantum number linearly. We have the following identities: From N_* components we get

$$\Gamma_1 - (\Gamma_2 + \Gamma_3) = (\Gamma_2 + \Gamma_3) - (\Gamma_4 + \Gamma_5) = (\Gamma_4 + \Gamma_5) - \Gamma_6.$$

The nucleon components give no relation. The pion components give

$$(\Gamma_1 + \Gamma_3) - (\Gamma_2 + \Gamma_5) = (\Gamma_2 + \Gamma_5) - (\Gamma_4 + \Gamma_6),$$

which lead to the solution:

$$\Gamma_4 = \Gamma_2 - 1/3\,\Gamma_1;$$

$$\Gamma_5 = \Gamma_2 + 2\,\Gamma_3 - 2/3\,\Gamma_1; \quad \Gamma_6 = 3\,\Gamma_2 + 3\,\Gamma_3 - 2\,\Gamma_1.$$

In the second order of perturbation the weighted counts depend quadratically on the magnetic quantum number: only the nucleon isobar gives a constraint and we have the unique relation:

$$\Gamma_1 - \Gamma_6 = 3\,(\Gamma_2 + \Gamma_3 - \Gamma_4 - \Gamma_5).$$

We note that the relations given by the weighted counts is the same as the mass (or width) formula for the corresponding multiplet since Γ_1, $(\Gamma_2 + \Gamma_3)$, $(\Gamma_4 + \Gamma_5)$, Γ_6 are respectively the widths of the nucleon isobars N_*^{++}, N_*^{+}, N_*^{0}, N_*^{-} respectively. On the other hand the second order relation is valid both for the real and imaginary parts of the complex masses of these isobar components to the expected order in electromagnetic effects.

Decay of Resonances and SU_3 Invariance

We are now prepared to consider the consequences of approximate SU_3 invariance. The only added technical tool is the information about the dependence of the (diagonal) matrix elements of the perturbation of the unitary symmetry on the two magnetic quantum numbers Y and I^2 (hypercharge and total isotopic spin squared). Fortunately these are now well known to first and second order in the guise of the two Okubo mass formulae[10]. As an illustration of the derivation of the consequence we consider the decay of the decuplet baryon resonance into a baryon octet and a meson octet.[9] There are twelve groups of reactions ($N_* N\pi$), ($N_* \Sigma K$), ($Y_{1*} N\bar{K}$), ($Y_{1*} \Lambda\pi$), ($Y_{1*} \Sigma\pi$), ($Y_{1*} \Sigma\eta$), ($Y_{1*} \Xi K$), ($\Xi_* \Lambda\bar{K}$), ($\Xi_* \Sigma\bar{K}$), ($\Xi_* \Xi\eta$), ($\Xi_* \Xi\pi$) and ($\Omega \Xi\bar{K}$). We shall deal with the sums of the relative weights for all decays of resonance into baryon plus meson within each of these groups, denoting these sums in the order given by W_1 up to W_{12}. By the rules outlined we get in zeroth order (i.e., exact SU_3 invariance) the complete solution

$$
\begin{aligned}
& 3w_1 = 12w_3 = 8w_4 = 12w_5 = 12w_8 = 12w_9 = 6w_{12} \\
= \; & 3w_2 = 12w_7 = 8w_6 = 12w_{10} = 12w_{11}
\end{aligned}
$$

In the first order using the first order Okubo (mass) formula for the decuplet and the octets we get

$$
\begin{aligned}
& 1/4\,(w_1 + w_2) - 1/3(w_3 + w_4 + w_5 + w_6 + w_7) \\
= \; & 1/3\,(w_3 + w_4 + w_5 + w_6 + w_7) - 1/2\,(w_8 + w_9 + w_{10} + w_{11}) \\
= \; & 1/2\,(w_8 + w_9 + w_{10} + w_{11}); \\
& (w_1 + w_3) + (w_7 + w_{10} + w_{11} + w_{12}) = 3\,(w_4 + w_8) + \\
& 1/3\,(w_2 + w_5 + w_6 + w_9) \\
& (w_2 + w_7) + (w_3 + w_8 + w_9 + w_{12}) = 3\,(w_6 + w_{10}) + 1/3\,(w_1 + w_4 + w_5 + w_{11}).
\end{aligned}
$$

In the second order, using the second order Okubo formula we get the single relation

$$
\begin{aligned}
& 1/4\,(w_1 + w_2) - w_{12} = (w_3 + w_4 + w_5 + w_6 + w_7) - \\
& 3/2\,(w_8 + w_9 + w_{10} + w_{11}).
\end{aligned}
$$

A more complicated example is given by the decay of the Y octet baryon resonance into the baryon octet and the meson octet. There are seventeen isotopic groups of reactions ($NN\eta$), ($NN\pi$), ($N\Lambda K$), ($N\Sigma K$), ($Y_0 N\bar{K}$), ($Y_0 \Lambda\eta$), ($Y_0 \Sigma\pi$), ($Y_0 \Xi K$), ($Y_1 N\bar{K}$), ($Y_1 \Lambda\pi$), ($Y_1 \Sigma\pi$), ($Y_1 \Xi K$), ($Y_1 \Sigma\eta$), ($\Xi\Lambda\bar{K}$), ($\Xi\Sigma\bar{K}$), ($\Xi\Xi\eta$), ($\Xi\Xi\pi$) with total weights defined as before denoted by w_1 to w_{17}. In the zeroth order of perturbation the relations are:

$$
\begin{aligned}
&3w_6 = w_7 = w_{13} = w_{10}\\
&w_{12} = w_{15} = w_2\\
&w_4 = w_9 = w_{17} = 1/2\, w_{11} - w_2 + 3w_{10}\\
&w_{16} = w_3 = w_5 = 1/6w_{11} + 1/3w_2 - 1/3w_{10}\\
&w_1 = w_8 = w_{14} = 1/3w_{11} + 2/3w_{10} - 1/3w_2
\end{aligned}
$$

In the first order of perturbation of the unitary symmetry, we have the following relation:

$$
\begin{aligned}
&(w_1 + w_2 + w_3 + w_4) + (w_{14} + w_{15} + w_{16} + w_{17})\\
= \;&3(w_5 + w_6 + w_7 + w_8) + 1/3(w_9 + w_{10} + w_{11} + w_{12} + w_{13});\\
&(w_1 + w_2 + w_5 + w_9) + (w_8 + w_{12} + w_{16} + w_{17})\\
= \;&3\,(w_3 + w_6 + w_{10} + w_{14}) + 1/3\,(w_4 + w_7 + w_{11} + w_{13} + w_{15});\\
&(w_3 + w_4 + w_8 + w_{12}) + (w_5 + w_9 + w_{14} + w_{15})\\
= \;&3\,(w_1 + w_6 + w_{13} + w_{16}) + 1/3\,(w_2 + w_7 + w_{10} + w_{11} + w_{17}).
\end{aligned}
$$

In the second order of perturbation there are no relations (beyond those obtained from charge independence).

Having obtained these relations it is up to us to check them against experiment. We note immediately that several of these total weights are inaccessible experimentally since the corresponding decays are not allowed energetically. Four groups of decays $(\Xi_*\Xi\pi)$, $(Y_{1*}\Lambda\pi)$, $(Y_{1*}\Sigma\pi)$ and $(N_*N\pi)$ are energetically allowed for the δ decuplet decay and seven groups $(\Xi\Xi\pi)$, $(\Sigma\bar{K}N)$, $(\Sigma\Lambda\pi)$, $(\Sigma\Sigma\pi)$, $(\Lambda\bar{K}N)$, $(\Lambda\Sigma\pi)$ and $(NN\pi)$ are allowed for the γ octet decays. It follows that only the zeroth order predictions can be tested from this data. A direct application of the results do not agree with experiment; to fit one may introduce a form factor (fudge factor?) of the form $[p^2/(p^2 + X^2)]^{\ell}$ in the squaredmatrix element (following Glashow and Rosenfeld[11]) where p is the momentum in the center of mass of the decay products. This comparison is tabulated in the accompanying table and is essentially identical in degree of fit with the perturbation calculation of Glashow and Rosenfeld.[11] There is fair agreement with the "predictions" from SU_3 invariance. We note in passing that the other weights, while not accessible directly from decays, are in principle measurable from pole extra-polation of suitable transition amplitudes. We note also that the same relations hold between the production cross sections for the unitary singlet and decuplet or an octet in the collision of two octets.

Decays of the $P_{3/2}$ decuplet and $d_{3/2}$ octet and SU_3 invariance

Resonance	Decay Mode	$F = \frac{P}{M}\left(\frac{P^2}{P^2 + x^2}\right)^{\ell}$	Γ th	Γ exp	Weight
Ξ_*(1530)	$\bar{K}N$	0.015	13	< 7	$1/2w_{11}$
Y_* (1385)	$\Lambda\pi$	0.048	41	50	$1/3w_4$
Y_* (1385)	$\Sigma\pi$	0.009	5	< 4	$1/3w_5$
N_* (1238)	$N\pi$	0.058	100*	100*	$1/3w_1$
	$w_4 = 3/8\, w_1$; $w_5 = 1/4\, w_1$; $w_{11} = 1/4\, w_1$				
Ξ (1600)	$\Xi\pi$	0.011	0.6	?	$1/2w_{17}$
Σ (1660)	$\bar{K}N$	0.081	3.0*	3	$1/3w_9$
Σ (1660)	$\Lambda\pi$	0.100	11.0*	11	$1/3w_{10}$
Σ (1660)	$\Sigma\pi$	0.070	13.0*	13	$1/3w_{11}$
Λ (1520)	$\bar{K}N$	0.017	6.2	5	w_5
Λ (1520)	$\Sigma\pi$	0.023	7.6	9	w_7
N (1512)	$N\pi$	0.115	66.4	80	$1/2w_2$

$$w_2 = 3\, w_{10} + 1/2\, w_{11} - w_9;$$
$$w_5 = 2/3\, w_{10} + 1/3\, w_{11} - 1/3\, w_9;$$
$$w_7 = w_{10}; \quad w_{17} = w_9.$$

X = 350 MeV. Starred widths were used in the fitting.
Experimental data reproduced from Glashow and Rosenfeld, P. R. L. 10, 192 (1963)

We have already remarked about the consequences of exact SU_3 invariance in so far as relations linear in the cross sections are concerned. When exact SU_3 invariance is broken by perturbations, most of these relations cease to be valid. For the scattering reaction using the modified Smushkevich method and Okubo mass formulae we get for the octet + octet → octet + octet reaction four relations linear in the cross section in the first order of perturbation and none in the second order. For the octet + octet → decuplet + octet reaction there are five relations in the first order and one relation in the second order. These are, of course, in addition to the relations entailed by charge independence. These relations are also presented in the appendix.

Electromagnetic Properties of Unitary Multiplets

We now turn to the electromagnetic properties of unitary multiplets. Consistent with current thinking on the subject we assume that the electromagnetic current transforms as a generator of the unitary group. This constraint follows if we write down the current in terms of the Heisenberg field operators in accordance with the minimal gauge invariant form. In any case, the current operator transforms in a manner simply related to the first order perturbation of the unitary Hamiltonian and the two tensors are related by an element of the Weyl subgroup that we referred to before. It then follows that if we rewrite the octet and decuplet (or any other multiplet of interest after performing this reflection, the Okubo mass formulae in first and second order can be used to predict electromagnetic properties in first and second order. The transcribed multiplets are as follows

$$\begin{array}{ccc} & \Sigma^+ & N^+ \\ \Xi^0 & (\Sigma^{0\prime\prime},\ \Lambda^{\prime\prime}) & N^0 \\ & \Xi^- & \Sigma^- \end{array}$$

Octet Hexagon

$$\begin{array}{cccc} & & & N_*^{++} \\ & & Y_{1*}^+ & N_*^+ \\ & \Xi_*^0 & Y_{1*}^0 & N_*^0 \\ \Omega^- & \Xi_*^- & Y_{1*}^- & N_*^- \end{array}$$

Decouplet Triangle

where as before, $\Sigma^{0\prime\prime}$ and $\Lambda^{\prime\prime}$ are suitable linear combinations

$$\Sigma^{0\prime\prime} = 1/2\ \Sigma^0 + \sqrt{3}/2\ \Lambda$$

$$\Lambda^{\prime\prime} = \sqrt{3}/2\ \Sigma^0 - 1/2\ \Lambda .$$

We then have the following relations for the magnetic moments (or any electromagnetic form factor)

$$\left.\begin{array}{l}\mu(\Sigma^+) = \mu(N^+)\\ \mu(\Xi^\circ) = \mu(\Sigma^\circ) = \mu(N^\circ)\\ \mu(\Xi^-) = \mu(\Sigma^-)\\ \mu_T(\Sigma^{\circ\prime\prime}, \Lambda^{\prime\prime}) = 0\end{array}\right\}\quad \text{"Charge independence"}$$

$$\mu(\Sigma^{\circ\prime\prime}) + 3\mu(\Lambda^{\prime\prime}) = 2\mu(\Sigma^+) + 2\mu(\Sigma^-) \qquad \text{"Mass formula"}$$

$$\mu(\Sigma^+) + \mu(N^+) + \mu(N^\circ) + \mu(\Sigma^-) + \mu(\Sigma^{\circ\prime\prime}) + \mu(\Lambda^{\prime\prime}) \qquad \text{"trace condition"}$$
$$+ \mu(\Xi^\circ) + \mu(\Xi^-) = 0$$

Substituting for $\Sigma^{\circ\prime\prime}$, $\Lambda^{\prime\prime}$ we get

$$\mu(\Sigma^{\circ\prime\prime}) = 1/4\mu(\Sigma^\circ) + 3/4\mu(\Lambda) + 3/2\mu_T(\Sigma^\circ, \Lambda)$$
$$\mu(\Lambda^{\prime\prime}) = 3/4\mu(\Sigma^\circ) + 1/4\mu(\Lambda) - 3/2\mu_T(\Sigma^\circ, \Lambda)$$
$$-\mu_T(\Sigma^{\circ\prime\prime}, \Lambda^{\prime\prime}) = -\sqrt{3}/4\mu(\Sigma^\circ) + \sqrt{3}/4\,\mu(\Lambda) - 1/2\mu_T(\Sigma^\circ, \Lambda).$$

These relations then lead to the constraints:

$$\mu(\Sigma^+) = \mu(N^+)$$
$$\mu(\Sigma^-) = (\Xi^-)$$
$$\mu(\Xi^\circ) = \mu(N^\circ) = 2\mu(\Lambda) = -2\mu(\Sigma^\circ) = +2/\sqrt{3}\,\mu_T(\Sigma^\circ, \Lambda)$$
$$\mu(\Sigma^+) + \mu(\Sigma^-) = 2\mu(\Sigma^\circ)$$

which have been derived by other methods before.[10, 12, 13] For the meson octet essentially the same relations are obtained (for the electric form factor) except that we have the additional constraints

$$F(K^+) = -F(K^-)$$
$$F(K^\circ) = -F(\bar{K}^\circ) = 0$$

We note that no significant experimental data, exist for comparison through the Λ magnetic moment may be determined soon; the transition moment $\mu_T(\Sigma^\circ, \Lambda)$ gives for the Σ° lifetime a value of the order 10^{-19} sec though the form factor at this momentum transfer is rather poorly known. The fact that K° is heavier than K^+ has sometimes been quoted as evidence against a vanishing charge form factor for the K°, but this obtains only if the kaon–photon intermediate state dominates the mass difference and this is by no means necessary. For the baryon decuplet the results are even simpler, yielding identical charge and magnetic form factors for all the particles except for a factor proportional to the electric charge. In particular the neutral particles have identically zero electromagnetic form factors.

Let us now turn to the electromagnetic properties to the second order. We observe that these properties can be correlated using the second order (mass) formula for the Weyl reflected multiplets. The electromagnetic self energies are given by

$$\epsilon(\Sigma^+) = \epsilon(N^+)$$
$$\epsilon(\Xi^\circ) = \epsilon(N^\circ)$$
$$\epsilon(\Xi^-) = \epsilon(\Sigma^{\circ\prime\prime}) = \epsilon(\Sigma^-)$$
$$\epsilon_T(\Sigma^{\circ\prime\prime}, \Lambda^{\prime\prime}) = 0$$

for the baryon octet; these are all of the "charge independent" type since in second order the Okubo(mass) formula yields no constraint. As before

$$\epsilon(\Sigma^{\circ\prime\prime}) = 1/4\,\epsilon(\Sigma^\circ) + 3/4\,\epsilon(\Lambda) + \sqrt{3}/2\,\epsilon_T$$
$$-\epsilon_T(\Sigma^{\circ\prime\prime}, \Lambda^{\prime\prime}) = -\sqrt{3}/4\,\epsilon(\Sigma^\circ) + \sqrt{3}/4\,\epsilon(\Lambda) - 1/2\,\epsilon_T.$$

As they stand they yield relations between the Compton scattering amplitudes,[13] but the actual masses of the particles involve the mass differences due to other perturbations of the unitary symmetry. These intrinsic masses satisfy:

$$M(N^\circ) = M(N^+)$$
$$M(\Xi^-) = M(\Xi^\circ)$$
$$M(\Sigma^-) = M(\Sigma^\circ) = M(\Sigma^+)$$
$$M_T(\Sigma^\circ, \Lambda) = 0$$

independent of the details of the violation of unitary symmetry. Since the observed masses are of the form $m = M + \epsilon$ we get the two familiar relations,[10,12,13]

$$m(N^+) - m(N^\circ) + m(\Xi^\circ) - m(\Xi^-) + m(\Sigma^-) - m(\Sigma^+) = 0$$
$$\sqrt{3}m_T(\Sigma^\circ\Lambda) = m(N^\circ) - m(N^+) - m(\Sigma^\circ) + m(\Sigma^+)$$

Experimentally the left hand side of the first relation is about 0.6 MeV. If the mass formula

$$3M(\Lambda) + M(\Sigma) = 2M(N) + 2M(\Xi)$$

is valid we have another relation:

$$2\sqrt{3}\, m_T(\Sigma^0 \Lambda) = 2m(N^0) + 2m(\Xi^0) - 3m(\Lambda) - m(\Sigma^0)$$

For the meson octets the first of these relations is empty and the second relation cannot be tested. The analogue of the third relation yields:

$$m^2(\eta) = 1/3 \left\{ 4m^2(K^+) - 2m^2(\pi^+) + m^2(\pi^0) \right\}.$$

The mass formulae for the vector meson octet is in poor agreement in contrast to the baryon octet. Certain special mechanisms responsible for such a breakdown involving the unitary singlet vector meson φ are discussed by Sakurai[14] and by Okubo[15] for the vector octet. For the baryon decuplet we have the following relations between masses in the first order of perturbation of the unitary symmetry:

$$m(\Omega^-) = 3m(Y_*^-) - 2m(N_*^-);$$
$$m(\Xi_*^-) = 2m(Y_*^-) - m(N_*^-);$$
$$m(N_*^+) - m(N_*^0) = m(Y_*^+) - m(Y_*^0)$$
$$m(N_*^0) - m(N_*^-) = m(Y_*^0) - m(Y_*^-) = m(\Xi_*^0) - m(\Xi_*^-).$$
$$m(N_*^{++}) = 3m(N_*^+) - 3m(N_*^0) + m(N_*^-).$$

At the present time these relations are untested experimentally, including the last relation which follows from charge independence alone. We remark here that for the baryon multiplets the correction to the first order mass formula is of the order of the electromagnetic mass differences.

Remarks.

In conclusion we make the following comments.

(i) While we have worked here exclusively with invariance under SU_3 and perturbations thereon, the methods developed here are perfectly general and can be used in connection with any arbitrary compact group.

(ii) Even within the framework of SU_3 invariance we can consider meson production and multimeson annihilation processes of baryon–antibaryon systems; and if ever the higher-dimensional irreducible representations are realized by resonance multiplets the computations in these cases can be done in essentially the same manner.

(iii) The theorems on weighted counts which have been briefly alluded to provide a class of exact results in the theory of compact groups interesting in their own right. This subject is systematically developed in a paper[8] to be published shortly.

(iv) It is, nevertheless, remarkable that the mass formulae agree so well. Perhaps the high mass intermediate states dominate the mass differences;

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DISCUSSION

LIPKIN: In response to Dr. Sudarshan's remarks about various permutations of Lipkin, Levison, Meshkov, I would just like to say that being firm believers in that we should practice what we preach and since SU_3 involves the permutations of three objects, we tried them first on ourselves. I should also like to go along strongly with Dr. Sudarshan's point that many of these prediction that one sees published as a result of complicated calculations using group theory can be obtained in a more simple way and I will go further and say that the ones that are really interesting to experimental physicists are the ones that are obtained simply, becuase if you just go through and grind out the calculation you find that you can relate cross sections for everything under the sun and the general theory related 10 cross sections for in terms of 10 parameters of which 9 are related to experiments that no one can possibly do in the laboratory. You then find that there are few things when you have 2 reactions that are equal and then when you look at them carefully, you see that these are just the ones that could have been obtained very easily by other means. And I think now there's enough written or going to be written so that experimentalists should not be afraid of all this group theory and anyone who wants to do an experiment and would like to know what SU_3 says about its possible consequences should be able to do the calculations himself, without graduate students.

MESHKOV: Another permutation differing by mass. In order to get the symmetry properties out its very handy to write the octets and decuplets and such things the way George has done and then sort of "stand them on their side."

Appendix II: Octet + Octet Decuplet + Octet Reactions Catalogue of Reactions and Charge Independence Relations

(1)	$N^+K^+ \to N_*^{++}K^0$	$N^0K^0 \to N_*^-K^+$	a = 3b
	$N^+K^+ \to N_*^+K^+$	$N^0K^0 \to N_*^0K^0$	b
	$N^+K^0 \to N_*^+K^0$	$N^0K^+ \to N_*^0K^+$	c = b
	$N^+K^0 \to N_*^0K^+$	$N^0K^+ \to N_*^+K^0$	d = b
(2)	$\Sigma^+K^+ \to N_*^{++}\pi^0$	$\Sigma^-K^0 \to N_*^-\pi^0$	a
	$\Sigma^+K^+ \to N_*^+\pi^+$	$\Sigma^-K^0 \to N_*^0\pi^-$	b
	$\Sigma^+K^0 \to N_*^{++}\pi^-$	$\Sigma^-K^+ \to N_*^-\pi^+$	c
	$\Sigma^+K^0 \to N_*^+\pi^0$	$\Sigma^-K^+ \to N_*^0\pi^0$	d
	$\Sigma^+K^0 \to N_*^0\pi^+$	$\Sigma^-K^+ \to N_*^+\pi^-$	e
	$\Sigma^0K^+ \to N_*^{++}\pi^-$	$\Sigma^0K^0 \to N_*^-\pi^+$	f
	$\Sigma^0K^+ \to N_*^+\pi^0$	$\Sigma^0K^0 \to N_*^0\pi^0$	g
	$\Sigma^0K^+ \to N_*^0\pi^+$	$\Sigma^0K^0 \to N_*^+\pi^-$	h

a+b+c+d+e = 2f+2g+2h

a+c+f = b+d+e+g+h

b+c+e+f+h = 2a+2d+2g

2a = 3b (since Σ^+K^+ is pure T = 3/2)

c = 3g (from crossed reaction)

(3)	$\Sigma^+K^+ \to N_*^{++}\eta$	$\Sigma^-K^0 \to N_*^-\eta$	a=3b
	$\Sigma^+K^0 \to N_*^+\eta$	$\Sigma^-K^+ \to N_*^0\eta$	b
	$\Sigma^0K^+ \to N_*^+\eta$	$\Sigma^0K^0 \to N_*^0\eta$	c=2b
(4)	$\Sigma^+K^+ \to Y_*^+K^+$	$\Sigma^-K^0 \to Y_*^-K^0$	a
	$\Sigma^+K^0 \to Y_*^+K^0$	$\Sigma^-K^+ \to Y_*^-K^+$	b=c+2e-a
	$\Sigma^+K^0 \to Y_*^0K^+$	$\Sigma^-K^+ \to Y_*^0K^0$	c
	$\Sigma^0K^+ \to Y_*^+K^0$	$\Sigma^0K^0 \to Y_*^-K^+$	d=e
	$\Sigma^0K^+ \to Y_*^0K^+$	$\Sigma^0K^0 \to Y_*^0K^0$	e
(5)	$\Lambda K^+ \to N_*^{++}\pi^-$	$\Lambda K^0 \to N_*^-\pi^+$	a=3c
	$\Lambda K^+ \to N_*^+\pi^0$	$\Lambda K^0 \to N_*^0\pi^0$	b=2c
	$\Lambda K^+ \to N_*^0\pi^+$	$\Lambda K^0 \to N_*^+\pi^-$	c

Reactions (6) do not exist.

(7)	$\Lambda K^+ \to Y_*^+K^0$	$\Lambda K^0 \to Y_*^-K^+$	a=2b
	$\Lambda K^+ \to Y_*^0K^+$	$\Lambda K^0 \to Y_*^0K^0$	b
(8)	$N^+\pi^+ \to N_*^{++}\pi^0$	$N^0\pi^- \to N_*^-\pi^0$	a
	$N^+\pi^+ \to N_*^+\pi^+$	$N^0\pi^- \to N_*^0\pi^0$	b
	$N^+\pi^0 \to N_*^{++}\pi^-$	$N^0\pi^0 \to N_*^-\pi^+$	c
	$N^+\pi^0 \to N_*^+\pi^0$	$N^0\pi^0 \to N_*^0\pi^0$	d
	$N^+\pi^0 \to N_*^0\pi^+$	$N^0\pi^0 \to N_*^+\pi^-$	e
	$N^+\pi^- \to N_*^+\pi^-$	$N^0\pi^+ \to N_*^0\pi^+$	f
	$N^+\pi^- \to N_*^0\pi^0$	$N^0\pi^+ \to N_*^+\pi^0$	g
	$N^+\pi^- \to N_*^-\pi^+$	$N^0\pi^+ \to N_*^{++}\pi^-$	h

a+b+f+g+h = 2c+2d+2e 2a = 3b

b+c+e+f+h = 2a+2d+2g h = 3d

(9)	$N^+\pi^+ \to N_*^{++}\eta$	$N^0\pi^- \to N_*^-\eta$	a=3c
	$N^+\pi^0 \to N_*^+\eta$	$N^0\pi^0 \to N_*^0\eta$	b=2c
	$N^+\pi^- \to N_*^0\eta$	$N^0\pi^+ \to N_*^+\eta$	c
(10)	$N^+\pi^+ \to Y_*^+K^+$	$N^0\pi^- \to Y_*^-K^0$	a
	$N^+\pi^0 \to Y_*^+K^0$	$N^0\pi^0 \to Y_*^-K^+$	b
	$N^+\pi^0 \to Y_*^0K^+$	$N^0\pi^0 \to Y_*^0K^0$	c
	$N^+\pi^- \to Y_*^0K^0$	$N^0\pi^+ \to Y_*^0K^+$	d=b
	$N^+\pi^- \to Y_*^-K^+$	$N^0\pi^+ \to Y_*^+K^0$	e=b+2c-a
(11)	$N^+\eta \to N_*^{++}\pi^-$	$N^0\eta \to N_*^-\pi^+$	a=3c
	$N^+\eta \to N_*^+\pi^0$	$N^0\eta \to N_*^0\pi^0$	b=2c
	$N^+\eta \to N_*^0\pi^+$	$N^0\eta \to N_*^+\pi^-$	c
(12)	$N^+\eta \to Y_*^+K^0$	$N^0\eta \to Y_*^-K^+$	a=2b
	$N^+\eta \to Y_*^0K^+$	$N^0\eta \to Y_*^0K^0$	b
(13)	$N^+\bar{K}^0 \to N_*^{++}K^-$	$N^0K^- \to N_*^-\bar{K}^0$	a=3b
	$N^+\bar{K}^0 \to N_*^+\bar{K}^0$	$N^0K^- \to N_*^0K^-$	b
	$N^+K^- \to N_*^+K^-$	$N^0K^0 \to N_*^0\bar{K}^0$	c=b
	$N^+K^- \to N_*^0\bar{K}^0$	$N^0K^0 \to N_*^+K^-$	d=b
(14)	$N^+K^0 \to Y_*^+\pi^0$	$N^0K^- \to Y_*^-\pi^0$	a
	$N^+K^0 \to Y_*^0\pi^+$	$N^0K^- \to Y_*^0\pi^-$	b=a
	$N^+K^- \to Y_*^+\pi$	$N^0K^0 \to Y_*^-\pi^+$	c
	$N^+K^- \to Y_*^0\pi^0$	$N^0K^0 \to Y_*^0\pi^0$	d
	$N^+K^- \to Y_*^-\pi^+$	$N^0K^0 \to Y_*^+\pi^-$	e=a+2d-c
(15)	$N^+\bar{K}^0 \to Y_*^+\eta$	$N^0K^- \to Y_*^-\eta$	a=2b
	$N^+K^- \to Y_*^0\eta$	$N^0\bar{K}^0 \to Y_*^0\eta$	b
(16)	$N^+\bar{K}^0 \to \Xi_*^0K^+$	$N^0K^- \to \Xi_*^-\bar{K}^0$	a
	$N^+K^- \to \Xi_*^0\bar{K}^0$	$N^0\bar{K}^0 \to \Xi_*^-K^+$	b
	$N^+K^- \to \Xi_*^-K^+$	$N^0\bar{K}^0 \to \Xi_*^0K^0$	c
(17)	$\Sigma^+\pi^+ \to N_*^{++}\bar{K}^0$	$\Sigma^-\pi^- \to N_*^-K^-$	a
	$\Sigma^+\pi^0 \to N_*^{++}K^-$	$\Sigma^-\pi^0 \to N_*^-\bar{K}^0$	b
	$\Sigma^+\pi^0 \to N_*^+\bar{K}^0$	$\Sigma^-\pi^0 \to N_*^0K^-$	c
	$\Sigma^+\pi^- \to N_*^+K^-$	$\Sigma^-\pi^+ \to N_*^0\bar{K}^0$	d
	$\Sigma^+\pi^- \to N_*^0\bar{K}^0$	$\Sigma^-\pi^+ \to N_*^+K^-$	e
	$\Sigma^0\pi^+ \to N_*^{++}K^-$	$\Sigma^0\pi^- \to N_*^-K^0$	f
	$\Sigma^0\pi^+ \to N_*^+K^0$	$\Sigma^0\pi^- \to N_*^0K^-$	g
	$\Sigma^0\pi^0 \to N_*^+K^-$	Σ [illegible] $\to N_*^0K^0$	h

a+b+c+d+e = 2f+2g+2h

a+d+e+f+g = 2b+2c+2h

a+b+f = c+d+e+g+h

a = 3h (since $\Sigma^+\pi^+$, $\Sigma^0\pi^0$ are pure j=2 amplitudes)

2b = 3d (from crossed reaction)

$w(35z) = w(N^+\pi^0 \rightarrow \Sigma^0 K^+) = w(N^0\pi^0 \rightarrow \Sigma^0 K^0)$

$w(N^+\pi^- \rightarrow \Sigma^0 K^0) = w(N^0\pi^+ \rightarrow \Sigma^0 K^+) = w(35y)$

$w(N^+\pi^- \rightarrow \Sigma^- K^+) = w(N^0\pi^+ \rightarrow \Sigma^+ K^0) = w(35y)+2w(35z)-w(35x)$

$w(36x) = w(\Xi^-\pi^- \rightarrow \Sigma^- \bar{K}^-) = w(\Xi^0\pi^+ \rightarrow \Sigma^+ \bar{K}^0)$

$w(36y) = w(\Xi^-\pi^0 \rightarrow \Sigma^- \bar{K}^0) = w(\Xi^0\pi^0 \rightarrow \Sigma^+ \bar{K}^-)$

$w(36z) = w(\Xi^-\pi^0 \rightarrow \Sigma^0 \bar{K}^-) = w(\Xi^0\pi^0 \rightarrow \Sigma^0 \bar{K}^0)$

$w(\Xi^-\pi^+ \rightarrow \Sigma^0 \bar{K}^0) = w(\Xi^0\pi^- \rightarrow \Sigma^0 \bar{K}^-) = w(36y)$

$w(\Xi^-\pi^+ \rightarrow \Sigma^+ \bar{K}^-) = w(\Xi^0\pi^- \rightarrow \Sigma^- \bar{K}^0) = w(36y)+2w(36z)-w(36x)$

$w(37) = w(N^+\pi^0 \rightarrow \Lambda K^+) = w(N^0\pi^0 \rightarrow \Lambda K^0)$

$w(N^+\pi^- \rightarrow \Lambda K^0) = w(N^0\pi^+ \rightarrow \Lambda K^+) = 2w(37)$

$w(38) = w(\Xi^-\pi^0 \rightarrow \Lambda \bar{K}^-) = w(\Xi^0\pi^0 \rightarrow \Lambda \bar{K}^0)$

$w(\Xi^-\pi^+ \rightarrow \Lambda \bar{K}^0) = w(\Xi^0\pi^- \rightarrow \Lambda \bar{K}^-) = 2w(38)$

$w(39) = w(N^+\eta \rightarrow N^+\pi^0) = w(N^0\eta \rightarrow N^0\pi^0)$

$w(N^+\eta \rightarrow N^0\pi^+) = w(N^0\eta \rightarrow N^+\pi^-) = 2w(39)$

$w(40) = w(\Xi^-\eta \rightarrow \Xi^-\pi^0) = w(\Xi^0\eta \rightarrow \Xi^0\pi^0)$

$w(\Xi^-\eta \rightarrow \Xi^0\pi^-) = w(\Xi^0\eta \rightarrow \Xi^-\pi^+) = 2w(40)$

$w(41) = w(N^+\eta \rightarrow N^+\eta) = w(N^0\eta \rightarrow N^0\eta)$

$w(42) = w(\Xi^-\eta \rightarrow \Xi^-\eta) = w(\Xi^0\eta \rightarrow \Xi^0\eta)$

$w(43) = w(N^+\eta \rightarrow \Sigma^0 K^+) = w(N^0\eta \rightarrow \Sigma^0 K^0)$

$w(N^+\eta \rightarrow \Sigma^+ K^0) = w(N^0\eta \rightarrow \Sigma^- K^+) = 2w(43)$

$w(44) = w(\Xi^-\eta \rightarrow \Sigma^0 \bar{K}^-) = w(\Xi^0\eta \rightarrow \Sigma^0 \bar{K}^0)$

$w(\Xi^-\eta \rightarrow \Sigma^- \bar{K}^0) = w(\Xi^0\eta \rightarrow \Sigma^+ \bar{K}^-) = 2w(44)$

$w(45) = w(N^+\eta \rightarrow \Lambda K^+) = w(N^0\eta \rightarrow \Lambda K^0)$

$w(46) = w(\Xi^-\eta \rightarrow \Lambda \bar{K}^-) = w(\Xi^0\eta \rightarrow \Lambda \bar{K}^0)$

$w(47) = w(\Sigma^0\eta \rightarrow N^+\bar{K}^-) = w(\Sigma^0\eta \rightarrow N^0\bar{K}^0)$

$w(\Sigma^+\eta \rightarrow N^+\bar{K}^0) = w(\Sigma^-\eta \rightarrow N^0\bar{K}^-) = 2w(47)$

$w(48) = w(\Sigma^0\eta \rightarrow \Xi^- K^+) = w(\Sigma^0\eta \rightarrow \Xi^0 K^0)$

$w(\Sigma^-\eta \rightarrow \Xi^- K^0) = w(\Sigma^+\eta \rightarrow \Xi^0 K^+) = 2w(48)$

$w(49) = w(\Lambda\pi^0 \rightarrow N^+\bar{K}^-) = w(\Lambda\pi^0 \rightarrow N^0\bar{K}^0)$

$w(\Lambda\pi^+ \rightarrow N^+\bar{K}^0) = w(\Lambda\pi^- \rightarrow N^0\bar{K}^-) = 2w(49)$

$w(50) = w(\Lambda\pi^0 \rightarrow \Xi^- K^+) = w(\Lambda\pi^0 \rightarrow \Xi^0 K^0)$

$w(\Lambda\pi^- \rightarrow \Xi^- K^0) = w(\Lambda\pi^+ \rightarrow \Xi^0 K^+) = 2w(50)$

$w(51) = w(\Lambda\eta \rightarrow N^+\bar{K}^-) = w(\Lambda\eta \rightarrow N^0\bar{K}^0)$

$w(52) = w(\Lambda\eta \rightarrow \Xi^- K^+) = w(\Lambda\eta \rightarrow \Xi^0 K^0)$

$w(53x) = w(\Sigma^+\pi^0 \rightarrow N^+\bar{K}^0) = w(\Sigma^-\pi^0 \rightarrow N^0\bar{K}^-)$

$w(53y) = w(\Sigma^+\pi^- \rightarrow N^+\bar{K}^-) = w(\Sigma^-\pi^+ \rightarrow N^0\bar{K}^0)$

$w(53z) = w(\Sigma^0\pi^0 \rightarrow N^+\bar{K}^-) = w(\Sigma^0\pi^0 \rightarrow N^0\bar{K}^0)$

$w(\Sigma^0\pi^+ \rightarrow N^+\bar{K}^0) = w(\Sigma^0\pi^- \rightarrow N^0\bar{K}^-) = w(53x)$

$w(\Sigma^+\pi^- \rightarrow N^0\bar{K}^0) = w(\Sigma^-\pi^+ \rightarrow N^+\bar{K}^-) = w(53x)-w(53y)+2w(53z)$

$w(54x) = w(\Sigma^-\pi^0 \rightarrow \Xi^- K^0) = w(\Sigma^+\pi^0 \rightarrow \Xi^0 K^+)$

$w(54y) = w(\Sigma^-\pi^+ \rightarrow \Xi^- K^+) = w(\Sigma^+\pi^- \rightarrow \Xi^0 K^0)$

$w(54z) = w(\Sigma^0\pi^0 \rightarrow \Xi^- K^+) = w(\Sigma^0\pi^0 \rightarrow \Xi^0 K^0)$

$w(\Sigma^0\pi^- \rightarrow \Xi^- K^0) = w(\Sigma^0\pi^+ \rightarrow \Xi^0 K^+) = w(54x)$

$w(\Sigma^-\pi^+ \rightarrow \Xi^0 K^0) = w(\Sigma^+\pi^- \rightarrow \Xi^- K^+) = w(54x)-w(54y)+2w(54z)$

$w(55) = w(\Sigma^+\pi^0 \rightarrow \Lambda\pi^+) = w(\Sigma^-\pi^0 \rightarrow \Lambda\pi^-)$

$w(\Sigma^+\pi^- \rightarrow \Lambda\pi^0) = w(\Sigma^-\pi^0 \rightarrow \Lambda\pi^0) = w(55)$

$w(\Sigma^0\pi^+ \rightarrow \Lambda\pi^+) = w(\Sigma^0\pi^- \rightarrow \Lambda\pi^-) = w(55)$

$w(56) = w(\Sigma^+\pi^- \rightarrow \Lambda\eta) = w(\Sigma^-\pi^+ \rightarrow \Lambda\eta)$

$w(\Sigma^0\pi^0 \rightarrow \Lambda\eta) = W(S \qquad = w(56)$

$w(57) = w(\Sigma^+\eta \rightarrow \Sigma^+\pi^0) = w(\Sigma^-\eta \rightarrow \Sigma^-\pi^0)$

$w(\Sigma^+\eta \rightarrow \Sigma^0\pi^+) = w(\Sigma^-\eta \rightarrow \Sigma^0\pi^-) = w(57)$

$w(\Sigma^0\eta \rightarrow \Sigma^+\pi^-) = w(\Sigma^0\eta \rightarrow \Sigma^-\pi^+) = w(57)$

$w(58) = w(\Sigma^+\eta \rightarrow \Sigma^+\eta) = w(\Sigma^-\eta \rightarrow \Sigma^-\eta)$

$w(\Sigma^0\eta \rightarrow \Sigma^0\eta) = w(58)$

$w(59) = w(\Sigma^+\eta \rightarrow \Lambda\pi^+) = w(\Sigma^-\eta \rightarrow \Lambda\pi^-)$

$w(\Sigma^0\eta \rightarrow \Lambda\pi^0) = w(59)$

$w(60) = w(\Lambda\pi^+ \rightarrow \Sigma^+\pi^0) = w(\Lambda\pi^- \rightarrow \Sigma^-\pi^0)$

$w(\Lambda\pi^+ \rightarrow \Sigma^0\pi^+) = w(\Lambda\pi^- \rightarrow \Sigma^0\pi^-) = w(60)$

$w(\Lambda\pi^0 \rightarrow \Sigma^+\pi^-) = w(\Lambda\pi^0 \rightarrow \Sigma^-\pi^+) = w(60)$

$w(61) = w(\Lambda\pi^+ \rightarrow \Sigma^+\eta) = w(\Lambda\pi^- \rightarrow \Sigma^-\eta)$

$w(\Lambda\pi^0 \rightarrow \Sigma^0\eta) = w(61)$

$w(62) = w(\Lambda\pi^+ \rightarrow \Lambda\pi^+) = w(\Lambda\pi^- \rightarrow \Lambda\pi^-)$

$w(\Lambda\pi^0 \rightarrow \Lambda\pi^0) = w(62)$

$w(63) = w(\Lambda\eta \rightarrow \Sigma^+\pi^-) = w(\Lambda\eta \rightarrow \Sigma^-\pi^+)$

$w(\Lambda\eta \rightarrow \Sigma^0\pi^0) = w(63)$

$w(64) = w(\Lambda\eta \rightarrow \Lambda\eta)$

$w(65x) = w(\Sigma^+\pi^+ \rightarrow \Sigma^+\pi^+) = w(\Sigma^-\pi^- \rightarrow \Sigma^-\pi^-)$

$w(65y) = w(\Sigma^+\pi^0 \rightarrow \Sigma^+\pi^0) = w(\Sigma^-\pi^0 \rightarrow \Sigma^-\pi^0)$

$w(65z) = w(\Sigma^+\pi^- \rightarrow \Sigma^+\pi^-) = w(\Sigma^-\pi^+ \rightarrow \Sigma^-\pi^+)$

$w(65u) = w(\Sigma^+\pi^- \rightarrow \Sigma^0\pi^0) = w(\Sigma^-\pi^+ \rightarrow \Sigma^0\pi^0)$

$w(65v) = w(\Sigma^+\pi^- \rightarrow \Sigma^-\pi^+) = w(\Sigma^-\pi^+ \rightarrow \Sigma^+\pi^-)$

$w(65w) = w(\Sigma^0\pi^+ \rightarrow \Sigma^+\pi^0) = w(\Sigma^0\pi^- \rightarrow \Sigma^-\pi^0)$

$w(\Sigma^0\pi^+ \rightarrow \Sigma^0\pi^+) = w(\Sigma^0\pi^- \rightarrow \Sigma^0\pi^-) = w(65y)$

$w(\Sigma^0\pi^0 \rightarrow \Sigma^+\pi^-) = w(\Sigma^0\pi^0 \rightarrow \Sigma^-\pi^+) = w(65u)$

$w(\Sigma^+\pi^0 \rightarrow \Sigma^0\pi^+) = w(\Sigma^-\pi^0 \rightarrow \Sigma^0\pi^-) = w(65w)$

$w(\Sigma^0\pi^0 \rightarrow \Sigma^0\pi^0) = w(65x)-w(65y)+w(65z)-w(65u)+w(65v)-w(65w)$

Note: One additional nonlinear relation between amplitudes exist here.

$w(66) = w(\Sigma^+\pi^0 \rightarrow \Sigma^+\eta) = w(\Sigma^-\pi^0 \rightarrow \Sigma^-\eta)$

$w(\Sigma^+\pi^- \rightarrow \Sigma^0\eta) = w(\Sigma^-\pi^+ \rightarrow \Sigma^0\eta) = w(66)$

$w(\Sigma^0\pi^+ \rightarrow \Sigma^+\eta) = w(\Sigma^0\pi^- \rightarrow \Sigma^-\eta) = w(66)$

All these relations hold independent of unitary symmetry as long as charge independence is valid.

107 linearly independent cross sections.

Weyl Reflection Equations

w(1x) = w(2x) = w(65x)

w(1y) = w(8w) = w(32x)

w(1z) = w(4x) = w(36x)

w(3x) = w(35x) = w(2z)

w(7x) = w(31x) = w(2y)

w(19y) = w(20y) = w(65v)

w(21y) = w(30z) = w(54y)

w(29y) = w(30y) = w(65z)

w(29z) = w(22y) = w(53y)

w(3y)+2w(3z)-w(3x)=w(20x)=w(36y)+2w(36z)-w(36x)

w(7y)+2w(7z)-w(7x)=w(30x)=2w(32y)+w(32z)-w(32x)

w(35y(+2w(35z)-w(35x)=w(19x)=w(4y)+2w(4z)-w(4x)

w(22x)-w(22y)+2w(22z)=w(19z)=w(54x)-w(54y)+2w(54z)

w(21x)-w(21y)+2w(21z)=w(20y)=w(53x)-w(53y)+2w(53z)

2w(31y)+w(31z)-w(31x)=w(29x)=w(8y)+2w(8z)-w(8x)

w(3y)+2w(5)=w(22x)+2w(24)=w(32z)+2w(34)

w(3y)+2w(11)=w(54x)+2w(50)=w(8y)+2w(16)

w(7y)+2w(9)=(w(22x)+2w(26)=w(36y)+2w(38)

w(7y)+2w(15)=w(53x)+2w(49)=w(4y)+2w(12)

w(21x)+2w(23)=w(31z)+2w(33)=w(4y)+2w(6)

w(21x)+2w(25)=w(35y)+2w(37)=w(8y)+2w(10)

w(31z)+2w(39)=w(54x)+2w(48)=w(36y)+2w(44)

w(35y)+2w(43)=w(53x)+2w(47)=w(32z)+2w(40)

w(3z)+w(5)+w(11)+w(13)=w(65w)+w(61)+w(66)=w(4z)+w(6)+w(12)+w(14)

w(72)+w(9)+w(15)+w(17)=w(65y)+w(55)+w(60)+w(62)=w(8z)+w(10)+w(16)+w(18)

w(21z)+w(23)+w(25)+w(27)=w(22z)+w(24)+w(26)+w(28)=w(65u)+w(66)+w(55)+w(56)

w(31y)+w(33)+w(39)+w(41)=w(65y)+w(57)+w(58)+w(66)=w(32y)+w(34)+w(40)+w(42)

w(35z)+w(37)+w(43)+w(45)=w(65w)+w(55)+w(55)+w(57)+w(59)=w(36z)+w(38)+w(44)+w(46)

w(53z)+w(47)+w(49)+w(51)=w(65u)+w(57)+w(60)+w(63)=w(54z)+w(48)+w(50)+w(52)

w(65x)-w(65y)+w(65z)-w(65u)+w(65v)-w(65w)=w(64)

w(56)=w(63) ; w(58) = w(62) ; w(59) = w(61).

No relations in first order breakdown of unitary symmetry.

w(59)=w(61)

Final Baryon:

A_3 = w(1x)+w(1y)+w(1z)+3w(3y)+3w(3z)+3w(5)+3w(11)+w(19x)+w(19y)+w(19z)
+ 3w(31y)+3w(31z)+3w(33)+3w(39)+w(41)+3w(47)+3w(49)+w(51)+w(53x)+3w(5

= B_3 = 2w(7y)+2w(7z)+2w(15)+2w(21r)+2w(21z)+2w(23)+2w(35y)+2w(35z)
+ 2w(43)+2w(57)+w(58)+2w(60)+w(61)+w(63)+w(65x)+w(65y)+w(65z)
+ w(65u)+w(65v)+w(65w)+2w(66)+2w(44)+2w(36y)+2w(36z)+2w(24)
+ 2w(22x)+2w(22z)+2w(16)+2w(8y)+3w(8z).

= C_3 = 6w(9)+2w(17)+6w(25)+2w(27)+6w(37)+2w(45)+6w(55)+3w(56)
+ 3w(59)+3w(62)+w(64)+2w(46)+6w(38)+2w(28)+6w(26)+2w(28)+6w(10).

= D_3 = w(29x)+w(29y)+w(29z)+3w(54x)+3w(54x)+3w(54z)+w(52)+3w(50)+3w(48)
+ w(20x)+w(20y)+w(20z)+w(14)+3w(12)+3w(6)+3w(4y)+3w(4z)
+ w(2x)+w(2y)+w(2z).

Smushkevich Equations

Initial Baryon:

w(1x)+w(1y)+w(1z)+3w(31y)+2w(31y)+3w(31z)+3w(33)+3w(35y)+3w(35z)+3w(37)+3w(39
+w(41)+3w(43)+w(45)+w(30x)+w(30y)+w(30z)+w(28)+w(3w(26)+3w(24)
+3w(22x)+3w(22z)+w(20x)+w(20y)+w(20z). = A_1

= 2w(3y)+2w(3z)+2w(5)+2w(7y)+2w(7z)+2w(9)+2w(47)+2w(53x)
+2w(53z)+2w(55)+w(56)+2w(57)+w(58)+w(59)+w(65x)+w(65y)
+w(65z)+w(65u)+w(65v)+w(65w)+2w(66)+2w(54x)+2w(54z)
+2w(48)+2w(10)+2w(8y)+2w(8z)+2w(6)+2w(4y)+2w(4z). = B_1

= 6w(11)+2w(13)+6w(15)+2w(17)+6w(49)+2w(51)+6w(60)+3w(61)
+ 3w(62)+3w(63)+w(64)+2w(52)+6w(50)+2w(18)+6w(16)+2w(14)+6w(12). = C_1

= w(19x)+w(19y)+w(19z)+3w(21x)+3w(21z)+3w(23)+3w(25)+w(27)
+w(29x)+w(29y)+w(29z)+w(46)+3w(44)+w(42)+3w(40)+3w(38)
+3w(36y)+3w(36z)+3w(34)+3w(32y)+3w(32z)+w(2x)+w(2y)+w(2z). = D_1

Initial Meson:

w(1x)+w(1y)+w(1z)+3w(3y)+ew(3z)+3w(5)+3w(7y)+3w(7z)+3w(9)+3w(11)+w(13)
+3w(15)+w(17)+w(19x)+w(19z)+3w(21x)+3w(21z)+3w(23)+3w(25)+w(27)
+w(29x)+w(29y)+w(29z). = A_2

= 2w(31y)+2w(31z)+2w(33)+2w(35y)+2w(35z)+2w(37)+2w(49)
+2w(53x)+2w(53z)+2w(55)+w(56)+2w(60)+w(61)+w(62)+w(65x)+w(65y)
+w(65z)+w(65u)+w(65v)+w(65w)+2w(66)+2w(54x)+2w(54z)+2w(50)
+2w(38)+2w(36y)+2w(36z)+2w(34)+2w(34)+2w(32z). = B_2

= 6w(39)+2w(41)+6w(43)+2w(45)+6w(47)+2w(51)+6w(57)+3w(58)
+3w(59)+3w(63)+w(64)+2w(52)+6w(48)+2w(46)+6w(44)+2w(42)+6w(40). = C_2

= w(30x)+w(30y)+w(30z)+w(28)+3w(26)+3w(24)+3w(22x)+3w(22z)
+ w(20x)+w(20y)+w(20z)+w(18)+3w(16)+w(14)+3w(12)+3w(10)
+3w(8y)+3w(8z)+3w(6)+3w(4y)+3w(4z)+w(2x)+w(2y)+w(2z). = D_2

In first order breakdown of unitary symmetry:

$2A_1 + 2D_1 = B_1 + 3C_1$; $2A_2 + 2D_2 = B_2 + 3C_2$

No relation in second order.

Final Meson:

A_4 = w(1x)+w(1y)+w(1z)+3w(7y)+3w(7z)+3w(9)+3w(15)+w(17)+w(29x)+w(29y)
+ w(29z)+3w(35y)+3w(35z)+3w(37)+3w(43)+w(45)+3w(54x)+3w(54z)
+ w(52)+3w(50)+3w(48)+w(w(20x)+w(20y)+w(20z).

=B_4= 2w(3y)+2w(3z)+2w(11)+2w(21x)+2w(21z)+2w(25)+2w(31y)+2w(31z)
+2w(39)+2w(55)+2w(57)+w(59)+2w(60)+w(62)+w(63)+w(65x)+w(65y)+w(65z)
+w(65u)+w(65v)+w(65w)+2w(40)+2w(32y)+2w(32z)+2w(26)
+2w(22x)+2w(22z)+2w(12)+2w(4y)+2w(4z).

=C_4= 6w(5)+2w(13)+6w(23)+2w(27)+6w(33)+2w(41)+3w(56)+3w(58)
+3w(61)+w(64)+6w(66)+w(42)+6w(34)+6w(24)+w(14)+6w(6).

=D_4= w(19x)+w(19y)+w(19z)+3w(47)+3w(49)+w(51)+3w(53x)43w(53z)
+w(46)+3w(44)+3w(38)+3w(36y)+3w(36z)+w(30x)+w(30y)+w(30z)
+2w(28)+w(18)+3w(16)+3w(10)+3w(8y)+3w(8z)+w(2x)+w(2y)+w(2z)

In first order breakdown of unitary symmetry:

$2A_3 + 2D_3 = B_3 + 3C_3$; $2A_4 + 2D_4 = B_4 + 3\ C_4$

No relation in second order.

Appendix I: Octet + Octet Octet + Octet Reactions   Reaction Cross Sections And Relations Induced by Charge Independence

$w(1x) = w(N^+K^+ \to N^+K^+) = w(N^0K^0 \to N^0K^0)$

$w(1y) = w(N^+K^0 \to N^+K^0) = w(N^0K^+ \to N^0K^+)$

$w(1z) = w(N^+K^0 \to N^0K^+) = w(N^0K^+ \to N^+K^0)$

$w(2x) = w(\Xi^-K^- \to \Xi^-\bar{K}^-) = w(\Xi^0K^0 \to \Xi^0K^0)$

$w(2y) = w(\Xi^-\bar{K}^0 \to \Xi^-\bar{K}^0) = w(\Xi^0\bar{K}^0 \to \Xi^0\bar{K}^0)$

$w(2z) = w(\Xi^-\bar{K}^0 \to \Xi^0\bar{K}^-) = w(\Xi^0\bar{K}^- \to \Xi^-\bar{K}^0)$

$w(3x) = w(\Sigma^+K^+ \to N^+\pi^{+}) = w(\Sigma^-K^0 \to N^0\pi^-)$

$w(3y) = w(\Sigma^+K^0 \to N^+\pi^0) = w(\Sigma^-K^+ \to N^0\pi^0)$

$w(3z) = w(\Sigma^0K^+ \to N^+\pi^0) = w(\Sigma^0K^0 \to N^0\pi^0)$

$w(\Sigma^0K^+ \to N^0\pi^+) = w(\Sigma^0K^0 \to N^+\pi^-) = w(3y)$

$w(\Sigma^+K^0 \to N^0\pi^+) = w(\Sigma^-K^+ \to N^+\pi^-) = w(3y) + 2w(3z) - w(3x)$

$w(4x) = w(\Sigma^-K^- \to \Xi^-\pi^-) = w(\Sigma^+\bar{K}^0 \to \Xi^0\pi^+)$

$w(4y) = w(\Sigma^-\bar{K}^0 \to \Xi^-\pi^0) = w(\Sigma^+\bar{K}^- \to \Xi^0\pi^0)$

$w(4z) = w(\Sigma^0\bar{K}^- \to \Xi^-\pi^0) = w(\Sigma^0\bar{K}^0 \to \Xi^0\pi^0)$

$w(4z) = w(\Sigma^0\bar{K}^- \to \Xi^-\pi^0) = w(\Sigma^0\bar{K}^0 \to \Xi^0\pi^0)$

$w(\Sigma^0\bar{K}^0 \;\; \Xi^0\pi^-) = w(\Sigma^+\bar{K}^- \to \Xi^-\pi^+) = w(4y)$

$w(\Sigma^-\bar{K}^0 \to \Xi^0\pi^-) = w(\Sigma^+\bar{K}^- \to \Xi^-\pi^+) = w(4y) + 2w(4z) - w(4x)$

$w(5) = w(\Sigma^0K^+ \to N^+\eta) = w(\Sigma^0K^0 \to N^0\eta)$

$w(\Sigma^+K^0 \to N^+\eta) = w(\Sigma^-K^+ \to N^0\eta) = 2w(5)$

$w(6) = w(\Sigma^0\bar{K}^- \to \Xi^-\eta) = w(\Sigma^0\bar{K}^0 \to \Xi^0\eta)$

$w(\Sigma^-\bar{K}^0 \to \Xi^-\eta) = w(\Sigma^+\bar{K}^- \to \Xi^0\eta) = 2w(6)$

$w(7x) = w(\Sigma^+K^+ \to \Sigma^+K^+) = w(\Sigma^-K^0 \to \Sigma^-K^0)$

$w(7y) = w(\Sigma^+K^0 \to \Sigma^0K^+) = w(\Sigma^-K^+ \to \Sigma^0K^0)$

$w(7z) = w(\Sigma^0K^+ \to \Sigma^0K^+) = w(\Sigma^0K^0 \to \Sigma^0K^0)$

$w(\Sigma^0K^+ \to \Sigma^+K^0) = w(\Sigma^0K^0 \to \Sigma^-K^+) = w(7y)$

$w(\Sigma^+K^0 \to \Sigma^+K^0) = w(\Sigma^-K^+ \to \Sigma^-K^+) = w(7y) + 2w(7z) = w(7x)$

$w(8x) = w(\Sigma^-\bar{K}^- \to \Sigma^-\bar{K}^-) = w(\Sigma^+\bar{K}^0 \to \Sigma^+\bar{K}^0)$

$w(8y) = w(\Sigma^-\bar{K}^- \to \Sigma^-\bar{K}^-) = w(\Sigma^+\bar{K}^- \to \Sigma^0\bar{K}^0)$

$w(8z) = w(\Sigma^0\bar{K}^- \to \Sigma^0\bar{K}^-) = w(\Sigma^0\bar{K}^0 \to \Sigma^0\bar{K}^0)$

$w(\Sigma^0\bar{K}^- \to \Sigma^-K^0) = w(\Sigma^0\bar{K}^0 \to \Sigma^+\bar{K}^-) = w(8y)$

$w(\Sigma^-\bar{K}^0 \to \Sigma^-\bar{K}^0) = w(\Sigma^+\bar{K}^- \to \Sigma^+\bar{K}^-) = w(8y) + 2w(8z) - w(8x)$

$w(9) = w(\Sigma^0K^+ \to \Lambda K^+) = w(\Sigma^0K^0 \to \Lambda K^0)$

$w(\Sigma^+K^0 \to \Lambda K^+) = w(\Sigma^-K^+ \to \Lambda K^0) = 2w(9x)$

$w(10) = w(\Sigma^0\bar{K}^- \to \Lambda\bar{K}^-) = w(\Sigma^0\bar{K}^0 \to \Lambda\bar{K}^0)$

$w(\Sigma^-\bar{K}^0 \to \Lambda\bar{K}^-) = w(\Sigma^+\bar{K}^- \to \Lambda\bar{K}^0) = 2w(10)$

$w(11) = w(\Lambda K^+ \to N^+\pi^0) = w(\Lambda K^0 \to N^0\pi^0)$

$w(\Lambda K^+ \to N^0\pi^+) = w(\Lambda K^0 \to N^+\pi^-) = 2w(11)$

$w(12) = w(\Lambda\bar{K}^- \to \Xi^-\pi^0) = w(\Lambda\bar{K}^0 \to \Xi^0\pi^0)$

$w(\Lambda\bar{K}^- \to \Xi^0\pi^-) = w(\Lambda\bar{K}^0 \to \Xi^0\pi^+) = 2w(12)$

$w(13) = w(\Lambda K^+ \to N^+\eta) = w(\Lambda K^0 \to N^0\eta)$

$w(14) = w(\Lambda\bar{K}^- \to \Xi^-\eta) = w(\Lambda\bar{K}^0 \to \Xi^0\eta)$

$w(15) = w(\Lambda K^+ \to \Sigma^0K^+) = w(\Lambda K^0 \to \Sigma^0K^0)$

$w(\Lambda K^+ \to \Sigma^+K^0) = w(\Lambda K^0 \to \Sigma^-K^+) = 2w(15)$

$w(16) = w(\Lambda\bar{K}^- \to \Sigma^0\bar{K}^-) = w(\Lambda\bar{K}^0 \to \Sigma^0\bar{K}^0)$

$w(\Lambda\bar{K}^- \to \Sigma^-\bar{K}^0) = w(\Lambda\bar{K}^0 \to \Sigma^+\bar{K}^-) = 2w(16)$

$w(17) = w(\Lambda K^+ \to \Lambda K^+) = w(\Lambda K^0 \to \Lambda K^0)$

$w(18) = w(\Lambda\bar{K}^- \to \Lambda\bar{K}^-) = w(\Lambda\bar{K}^0 \to \Lambda\bar{K}^0)$

$w(19x) = w(\Xi^0K^+ \to N^+\bar{K}^0) = w(\Xi^-K^0 \to N^0\bar{K}^-)$

$w(19y) = w(\Xi^0K^0 \to N^0\bar{K}^0) = w(\Xi^0K^+ \to N^+\bar{K}^-)$

$w(19z) = w(\Xi^0K^0 \to N^+\bar{K}^-) = w(\Xi^-K^+ \to N^0\bar{K}^0)$

$w(20x) = w(N^0\bar{K}^- \to \Xi^-K^0) = w(N^+\bar{K}^0 \to \Xi^0K^+)$

$w(20y) = w(N^0\bar{K}^0 \to \Xi^0K^0) = w(N^+\bar{K}^- \to \Xi^-K^+)$

$w(20z) = w(N^0\bar{K}^0 \to \Xi^-K^+) = w(N^+\bar{K}^- \to \Xi^0K^0)$

$w(21x) = w(\Xi^0K^+ \to \Sigma^+\pi^0) = w(\Xi^-K^0 \to \Sigma^-\pi^0)$

$w(21y) = w(\Xi^0K^0 \to \Sigma^+\pi^-) = w(\Xi^-K^+ \to \Sigma^-\pi^+)$

$w(21z) = w(\Xi^0K^0 \to \Sigma^0\pi^0) = w(\Xi^-K^+ \to \Sigma^0\pi^0)$

$w(\Xi^0K^+ \to \Sigma^0\pi^+) = w(\Xi^-K^0 \to \Sigma^0\pi^-) = w(21x)$

$w(\Xi^0K^0 \to \Sigma^-\pi^+) = w(\Xi^-K^+ \to \Sigma^+\pi^-) = w(21x) - w(21y) + 2w(21z)$

$w(22x) = w(N^0\bar{K}^- \to \Sigma^-\pi^0) = w(N^+\bar{K}^0 \to \Sigma^+\pi^0)$

$w(22y) = w(N^0\bar{K}^0 \to \Sigma^-\pi^+) = w(N^+\bar{K}^- \to \Sigma^+\pi^-)$

$w(22z) = w(N^0\bar{K}^0 \to \Sigma^0\pi^0) = w(N^+\bar{K}^- \to \Sigma^0\pi^0)$

$w(N^0\bar{K}^- \to \Sigma^0\pi^-) = w(N^+\bar{K}^0 \to \Sigma^0\pi^+) = w(22x)$

$w(N^0\bar{K}^0 \to \Sigma^+\pi^-) = w(N^+\bar{K}^- \to \Sigma^-\pi^+) = w(22x) - w(22y) + 2w(22z)$

$w(23) = w(\Xi^0K^0 \to \Sigma^0\eta) = w(\Xi^-K^+ \to \Sigma^0\eta)$

$w(\Xi^0K^+ \to \Sigma^+\eta) = w(\Xi^-K^0 \to \Sigma^-\eta) = 2w(23)$

$w(24) = w(N^0\bar{K}^0 \to \Sigma^0\eta) = w(N^+\bar{K}^- \to \Sigma^0\eta)$

$w(N^0\bar{K}^- \to \Sigma^-\eta) = w(N^+\bar{K}^0 \to \Sigma^+\eta) = 2w(24)$

$w(25) = w(\Xi^0K^0 \to \Lambda\pi^0) = w(\Xi^-K^+ \to \Lambda\pi^0)$

$w(\Xi^0K^+ \to \Lambda\pi^+) = w(\Xi^-K^0 \to \Lambda\pi^-) = 2w(25)$

$w(26) = w(N^0\bar{K}^0 \to \Lambda\pi^0) = w(N^+\bar{K}^- \to \Lambda\pi^0)$

$w(N^0\bar{K}^- \to \Lambda\pi^-) = w(N^+\bar{K}^0 \to \Lambda\pi^+) = 2w(26)$

w(2[illegible]) $= w(\Xi^0K^0 \to \Lambda\eta) = w(\Xi^-K+ \to \Lambda\eta)$

$w(28) = w(N^0\bar{K}^0 \to \Lambda\eta) = w(N^+\bar{K}^- \to \Lambda\eta)$

$w(29x) = w(\Xi^0K^+ \to \Xi^0K^+) = w(\Xi^-K^0 \to \Xi^-K^0)$

$w(29y) = w(\Xi^0K^0 \to \Xi^0K^0) = w(\Xi^-K^+ \to \Xi^-K^+)$

$w(29z) = w(\Xi^0K^0 \to \Xi^-K^+) = w(\Xi^-K^+ \to \Xi^0K^0)$

$w(30x) = w(N^0\bar{K}^- \to N^0\bar{K}^-) = w(N^+\bar{K}^0 \to N^+\bar{K}^0)$

$w(30y) = w(N^0\bar{K}^0 \to N^0\bar{K}^0) = w(N^+\bar{K}^0 \to N^+\bar{K}^0)$

$w(30z) = w(N^0\bar{K}^0 \to N^+K^-) = w(N^+\bar{K}^- \to N^0\bar{K}^0)$

$w(31x) = w(N^+\pi^{+\,+} \to N^+\pi^+) = w(N^0\pi^- \to N^0\pi^-)$

$w(31y) = w(N^+\pi^0 \to N^+\pi^0) = w(N^0\pi^0 \to N^0\pi^0)$

$w(31z) = w(N^+\pi^0 \to N^0\pi^+) = w(N^0\pi^0 \to N^+\pi^-)$

$w(N^+\pi^- \to N^0\pi^0) = w(N^0\pi^+ \to N^+\pi^0) = w(31z)$

$w(N^+\pi^- \to N^+\pi^-) = w(N^0\pi^+ \to N^0\pi^+) = 2w(31y) + w(31z) - w(31x)$

$w(32x) = w(\Xi^-\pi^- \to \Xi^-\pi^-) = w(\Xi^0\pi^+ \to \Xi^0\pi^+)$

$w(32y) = w(\Xi^-\pi^0 \to \Xi^-\pi^0) = w(\Xi^0\pi^0 \to \Xi^0\pi^0)$

$w(32z) = w(\Xi^-\pi^0 \to \Xi^0\pi^-) = w(\Xi^0\pi^0 \to \Xi^-\pi^+)$

$w(\Xi^-\pi^+ \to \Xi^0\pi^0) = w(\Xi^0\pi^- \to \Xi^-\pi^0) = w(32z)$

$w(\Xi^-\pi^+ \to \Xi^-\pi^+) = w(\Xi^0\pi^- \to \Xi^0\pi^-) = 2q(32y) + w(32z) - w(32x)$

$w(33) = w(N^+\pi^0 \to N^+\eta) = w(N^0\pi^0 \to N^0\eta)$

$w(N^+\pi^- \to N^0\eta) = w(N^0\pi^+ \to N^+\eta) = 2w(33)$

$w(34) = w(\Xi^-\pi^0 \to \Xi^-\eta) = w(\Xi^0\pi^0 \to \Xi^0\eta)$

$w(\Xi^-\pi^+ \to \Xi^0\eta) = w(\Xi^0\pi^- \to \Xi^-\eta) = 2w(34)$

$w(35x) = w(N^+\pi^+ \to \Sigma^+K^+) = w(N^0\pi^- \to \Sigma^-K^0)$

$w(35y) = w(N^+\pi^0 \to \Sigma^+K^0) = w(N^0\pi^0 \to \Sigma^-K^+)$

(18)	$\Sigma^+\pi^+ \to Y_*^+\pi^+$	$\Sigma^-\pi^- \to Y_*^-\pi^-$	a
	$\Sigma^+\pi^0 \to Y_*^+\pi^0$	$\Sigma^-\pi^0 \to Y_*^-\pi^0$	b
	$\Sigma^+\pi^0 \to Y_*^0\pi^+$	$\Sigma^-\pi^0 \to Y_*^0\pi^-$	c
	$\Sigma^+\pi^- \to Y_*^+\pi^-$	$\Sigma^-\pi^+ \to Y_*^-\pi^+$	d
	$\Sigma^+\pi^- \to Y_*^0\pi^0$	$\Sigma^-\pi^+ \to Y_*^0\pi^0$	e
	$\Sigma^+\pi^- \to Y_*^-\pi^+$	$\Sigma^-\pi^+ \to Y_*^+\pi^+$	f
	$\Sigma^0\pi^+ \to Y_*^+\pi^0$	$\Sigma^0\pi^- \to Y_*^-\pi^0$	g=c
	$\Sigma^0\pi^+ \to Y_*^0\pi$	$\Sigma^0\pi^- \to Y_*^0\pi^-$	h=b
	$\Sigma^0\pi^0 \to Y_*^+\pi^-$	$\Sigma^0\pi^0 \to Y_*^-\pi^+$	j=e
	$\Sigma^0\pi^0 \to Y_*^0\pi^0$	k = a-b-c+d-e+f	
(19)	$\Sigma^+\pi^0 \to Y_*^+\eta$	$\Sigma^-\pi^0 \to Y_*^-\eta$	a
	$\Sigma^+\pi^- \to Y_*^0\eta$	$\Sigma^-\pi^+ \to Y_*^0\eta$	b=a
	$\Sigma^0\pi^+ \to Y_*^+\eta$	$\Sigma^0\pi^- \to Y_*^-\eta$	c=a
(20)	$\Sigma^+\pi^0 \to \Xi_*^0K^+$	$\Sigma^-\pi^0 \to \Xi_*^-K^0$	a
	$\Sigma^+\pi^- \to \Xi_*^0K^0$	$\Sigma^-\pi^+ \to \Xi_*^-K^+$	b
	$\Sigma^+\pi^- \to \Xi_*^-K^+$	$\Sigma^-\pi^+ \to \Xi_*^0K^0$	c=a+2e-b
	$\Sigma^0\pi^+ \to \Xi_*^0K^+$	$\Sigma^0\pi^- \to \Xi_*^-K^0$	d=a
	$\Sigma^0\pi^0 \to \Xi_*^0K^0$	$\Sigma^0\pi^0 \to \Xi_*^-K^+$	e
(21)	$\Sigma^+\eta \to N_*^{++}K^-$	$\Sigma^-\eta \to N_*^-\bar{K}^0$	a=3b
	$\Sigma^+\eta \to N_*^+\bar{K}^0$	$\Sigma^-\eta \to N_*^0K^-$	b
	$\Sigma^0\eta \to N_*^+K^-$	$\Sigma^0\eta \to N_*^0\bar{K}^0$	c=2b
(22)	$\Sigma^+\eta \to Y_*^+\pi^0$	$\Sigma^-\eta \to Y_*^-\pi^0$	a
	$\Sigma^+\eta \to Y_*^0\pi^+$	$\Sigma^-\eta \to Y_*^0\pi^-$	b = a
	$\Sigma^0\eta \to Y_*^+\pi^-$	$\Sigma^0\eta \to Y_*^-\pi^+$	c = a
(23)	$\Sigma^+\eta \to Y_*^+\eta$	$\Sigma^-\eta \to Y_*^-\eta$	a
		$\Sigma^0\eta \to Y_*^0\eta$	b = a
(24)	$\Sigma^+\eta \to \Xi_*^0K^+$	$\Sigma^-\eta \to \Xi_*^-K^0$	a=2b
	$\Sigma^0\eta \to \Xi_*^0K^0$	$\Sigma^0\eta \to \Xi_*^-K^+$	b
(25)	$\Lambda\pi^+ \to N_*^{++}K^-$	$\Lambda\pi^- \to N_*^-\bar{K}^0$	a=3b
	$\Lambda\pi^+ \to N_*^+\bar{K}^0$	$\Lambda\pi^- \to N_*^0K^-$	b
	$\Lambda\pi^0 \to N_*^+K^-$	$\Lambda\pi^0 \to N_*^0\bar{K}^0$	c=2b
(26)	$\Lambda\pi^+ \to Y_*^+\pi^0$	$\Lambda\pi^- \to Y_*^-\pi^0$	a
	$\Lambda\pi^+ \to Y_*^0\pi^+$	$\Lambda\pi^- \to Y_*^0\pi^-$	b=a
	$\Lambda\pi^0 \to Y_*^+\pi^-$	$\Lambda\pi^0 \to Y_*^-\pi^+$	c=a
(27)	$\Lambda\pi^+ \to Y_*^+\eta$	$\Lambda\pi^- \to Y_*^-\eta$	a
		$\Lambda\pi^0 \to Y_*^0\eta$	b=a
(28)	$\Lambda\pi^+ \to \Xi_*^0K^+$	$\Lambda\pi^- \to \Xi_*^-K^0$	a=2b
	$\Lambda\pi^0 \to \Xi_*^0K^0$	$\Lambda\pi^0 \to \Xi_*^-K^+$	b
(29)	$\Lambda\eta \to Y_*^+\pi^-$	$\Lambda\eta \to Y_*^-\pi^+$	a
		$\Lambda\eta \to Y_*^0\pi^0$	b=a
(30)	$\Lambda\eta \to \Xi_*^0K^0$	$\Lambda\eta \to \Xi_*^-K^+$	
(31)	$\Xi^0K^+ \to N^{++}K^-$	$\Xi^-K^0 \to N_*^-\bar{K}^0$	a=3b
	$\Xi^0K^+ \to N_*^+\bar{K}^0$	$\Xi^-K^0 \to N_*^0K^-$	b
	$\Xi^0K^0 \to N_*^+K^-$	$\Xi^-K^+ \to N_*^0\bar{K}^0$	c=b
	$\Xi^0K^0 \to N_*^0\bar{K}^0$	$\Xi^-K^+ \to N_*^+K^-$	d=b
(32)	$\Xi^0K^+ \to Y_*^+\pi^0$	$\Xi^-K^0 \to Y_*^-\pi^0$	a
	$\Xi^0K^+ \to Y_*^0\pi^+$	$\Xi^-K^0 \to Y_*^0\pi^-$	b = a
	$\Xi^0K^0 \to Y_*^+\pi^-$	$\Xi^-K^+ \to Y_*^-\pi^+$	c
	$\Xi^0K^0 \to Y_*^0\pi^0$	$\Xi^-K^+ \to Y_*^0\pi^0$	d
	$\Xi^0K^0 \to Y_*^-\pi^+$	$\Xi^-K^+ \to Y_*^-\pi^-$	e=a+2d-c
(33)	$\Xi^0K^+ \to Y_*^+\eta$	$\Xi^-K^0 \to Y_*^-\eta$	a=2b
	$\Xi^0K^0 \to Y_*^0\eta$	$\Xi^-K^+ \to Y_*^0\eta$	b
(34)	$\Xi^0K^+ \to \Xi_*^0K^+$	$\Xi^-K^0 \to \Xi_*^-K^0$	a
	$\Xi^0K^0 \to \Xi_*^0K^0$	$\Xi^-K^+ \to \Xi_*^-K^+$	b
	$\Xi^0K^0 \to \Xi_*^-K^+$	$\Xi^-K^+ \to \Xi_*^0K^0$	c
(35)	$\Sigma^+K^0 \to Y_*^+K^0$	$\Sigma^-K^- \to Y_*^-K^-$	a
	$\Sigma^+K^- \to Y_*^+K^-$	$\Sigma^-\bar{K}^0 \to Y_*^-K^0$	b=c+2e-a
	$\Sigma^+K^- \to Y_*^0K^0$	$\Sigma^-\bar{K}^0 \to Y_*^0K^-$	c
	$\Sigma^0K^0 \to Y_*^+K^-$	$\Sigma^0K^- \to Y_*^-K^0$	d=e
	$\Sigma^0K^0 \to Y_*^0K^0$	$\Sigma^0K^- \to Y_*^0K^-$	e
(36)	$\Sigma^+\bar{K}^0 \to \Xi_*^0\pi^+$	$\Sigma^-K^- \to \Xi_*^-\pi^-$	a
	$\Sigma^+K^- \to \Xi_*^0\pi^0$	$\Sigma^-\bar{K}^0 \to \Xi_*^-\pi^0$	b
	$\Sigma^+K^- \to \Xi_*^-\pi^+$	$\Sigma^-\bar{K}^0 \to \Xi_*^0\pi^-$	c=b+2d-a
	$\Sigma^0\bar{K}^0 \to \Xi_*^0\pi^0$	$\Sigma^0K^- \to \Xi_*^-\pi^0$	d
	$\Sigma^0\bar{K}^0 \to \Xi_*^-\pi^+$	$\Sigma^0K^- \to \Xi_*^0\pi^-$	e
(37)	$\Sigma^+K^- \to \Xi_*^0\eta$	$\Sigma^-\bar{K}^0 \to \Xi_*^-\eta$	a=2b
	$\Sigma^0\bar{K}^0 \to \Xi_*^0\eta$	$\Sigma^0K^- \to \Xi_*^-\eta$	b
(38)	$\Sigma^+K^- \to \Omega^-K^+$	$\Sigma^-\bar{K}^0 \to \Omega^-K^0$	a
	$\Sigma^0\bar{K}^0 \to \Omega^-K^+$	$\Sigma^0K^- \to \Omega^-K^0$	b
(39)	$\Lambda\bar{K}^0 \to Y_*^+K^-$	$\Lambda K^- \to Y_*^-\bar{K}^0$	a=2b
	$\Lambda\bar{K}^0 \to Y_*^0\bar{K}^0$	$\Lambda K^- \to Y_*^0K^-$	b
(40)	$\Lambda\bar{K}^0 \to \Xi_*^0\pi^0$	$\Lambda K^- \to \Xi_*^-\pi^0$	a
	$\Lambda\bar{K}^0 \to \Xi_*^-\pi^+$	$\Lambda K^- \to \Xi_*^0\pi^-$	b=2a
(41)	$\Lambda\bar{K}^0 \to \Xi_*^0\eta$	$\Lambda K^- \to \Xi_*^-\eta$	
(42)	$\Lambda\bar{K}^0 \to \Omega^-K^+$	$\Lambda K^- \to \Omega^-K^0$	
(43)	$\Xi^0\pi^+ \to Y_*^+\bar{K}^0$	$\Xi^-\pi^- \to Y_*^-K^-$	a
	$\Xi^0\pi^0 \to Y_*^+K^-$	$\Xi^-\pi^0 \to Y_*^-\bar{K}^0$	b
	$\Xi^0\pi^0 \to Y_*^0\bar{K}^0$	$\Xi^-\pi^0 \to Y_*^0K^-$	c
	$\Xi^0\pi^- \to Y_*^0K^-$	$\Xi^-\pi^+ \to Y_*^0\bar{K}^0$	d=b
	$\Xi^0\pi^- \to Y_*^-\bar{K}^0$	$\Xi^-\pi^+ \to Y_*^+K^-$	e=b+2c-a
(44)	$\Xi^0\pi^+ \to \Xi_*^0\pi^+$	$\Xi^-\pi^- \to \Xi_*^-\pi^-$	a
	$\Xi^0\pi^0 \to \Xi_*^0\pi^0$	$\Xi^-\pi^0 \to \Xi_*^-\pi^0$	b
	$\Xi^0\pi^0 \to \Xi_*^-\pi^+$	$\Xi^-\pi^0 \to \Xi_*^0\pi^-$	c
	$\Xi^0\pi^- \to \Xi_*^0\pi^-$	$\Xi^-\pi^+ \to \Xi_*^-\pi^+$	d=2b+e-a
	$\Xi^0\pi^- \to \Xi_*^-\pi^0$	$\Xi^-\pi^+ \to \Xi_*^0\pi^0$	e=c
(45)	$\Xi^0\pi^0 \to \Xi_*^0\eta$	$\Xi^-\pi^0 \to \Xi_*^-\eta$	a
	$\Xi^0\pi^- \to \Xi_*^-\eta$	$\Xi^-\pi^+ \to \Xi_*^0\eta$	b
(46)	$\Xi^0\pi^0 \to \Omega^-K^+$	$\Xi^-\pi^0 \to \Omega^-K^0$	a
	$\Xi^0\pi^- \to \Omega^-K^0$	$\Xi^-\pi^+ \to \Omega^-K^+$	b=2a
(47)	$\Xi^0\eta \to Y_*^+K^-$	$\Xi^-\eta \to Y_*^-K^0$	a=2b
	$\Xi^0\eta \to Y_*^0\bar{K}^0$	$\Xi^-\eta \to Y_*^0K^-$	b
(48)	$\Xi^0\eta \to \Xi_*^0\pi^0$	$\Xi^-\eta \to \Xi_*^-\pi^0$	a
	$\Xi^0\eta \to \Xi_*^-\pi^+$	$\Xi^-\eta \to \Xi_*^0\pi^-$	b
(49)	$\Xi^0\eta \to \Xi_*^0\eta$	$\Xi^-\eta \to \Xi_*^-\eta$	
(50)	$\Xi^0\eta \to \Omega^-K^+$	$\Xi^-\eta \to \Omega^-K^0$	
(51)	$\Xi^0K^0 \to \Xi_*^0K^0$	$\Xi^-K^- \to$ [illegible]	a
	$\Xi^0K^- \to \Xi_*^0K^-$	$\Xi^-K^0 \to \Xi_*^-K^0$	b
	$\Xi^0K^- \to \Xi_*^-K^0$	$\Xi^-K^0 \to \Xi_*^0K^-$	c
(52)	$\Xi^0K^0 \to \Omega^-\pi^+$	$\Xi^-K^- \to \Omega^-\pi^-$	a=2b
	$\Xi^0K^- \to \Omega^-\pi^0$	$\Xi^-K^0 \to \Omega^-\pi^0$	b
(53)	$\Xi^0K^- \to \Omega^-\eta$	$\Xi^-K^0 \to \Omega^-\eta$	

There are 247 equations. In addition there is one nonlinear equation connecting the amplitudes of of the reaction (18). The total number of linearly independent cross sections is given by 334 - 247 = 87.

<u>Weyl Reflection Equations</u>

(1) w(1a) = w(17a) = w(52a)

(2) w(1b) = w(18a) = w(51a)

(3) w(1c) = w(35a) = w(44a)

(4) w(1d) = w(36a) = w(43a)

(5) w(2b) = w(10a) = w(51c)

(6) w(2c) = w(13a) = w(46b)

(7) w(2e) = w(16a) = w(43e)

(8) w(4a) = w(8b) = w(51b)

(9) w(4b) = w(13b) = w(44d)

(10) w(4c) = w(14b) = w(43d)

(11) w(8f) = w(34a) = w(35b)

(12) w(8h) = w(31a) = w(38a)

(13) w(10d) = w(32b) = w(35c)

(14) w(10e) = w(31b) = w(36c)

(15) w(13c) = w(18d) = w(34b)

(16) w(13d) = w(20b) = w(32c)

(17) w(14c) = w(17d) = w(34c)

(18) w(14e) = w(20c) = w(31c)

(19) w(16b) = w(17e) = w(32e)

(20) w(16c) = w(18f) = w(31d)

(21) w(2a)+w(3a)=w(8a)+w(9a)=(52b)+w(53)

(22) w(2d)+w(3b)=w(14a)+w(15a)=w(44e)+w(45b)

(23) w(2f)+w(5a)=w(17f)+w(25a)=w(38b)+w(42)

(24) w(2h)+w(50)=w(20d)+w(28a)=w(35d)+w(39a)

(25) w(4d)+w(7a)=w(17g)+w(25b)=w(36e)+w(40b)

(26) w(4e)+w(7b)=w(18h)+w(26b)=w(35a)+w(39b)

(27) w(8c)+w(11a)=w(17b)+w(21a)=w(46a)+w(50)

(28) w(8e)+w(11c)=w(20a)+w(24a)=w(43b)+w(47a)

(29) w(8g)+w(9c)=w(36b)+w(37a)=w(32a)+w(33a)

(30) w(10b)+w(12a)=w(17c)+w(21b)=w(44c)+w(48b)

(31) w(10c)+w(12b)=w(18c)+w(22b)=w(43c)+w(47b)

(32) w(14d)+w(15b)=w(18e)+w(19b)=w(32d)+w(33b)

(33) w(2g)+w(30)+w(5b)=w(18g)+w(19c)+w(26a)+w(27)
=w(30d)+w(37b)+w(40a)+w(40)

(34) w(8d)+w(9b)+w(11b)=w(18b)+w(19a)+w(22a)+w(23)
=w(44b)+w(45a)+w(48a)+w(49)

(35) w(17h)+w(21c)+w(25c)=w(18j)+w(22c)+w(26c)+w(29a)
=w(20e)+w(24b)+w(28b)+w(30)

(36) w(18k)=w(23)=w(27)=w(29).

There are five Smushkevich equations in the first order

(1) 2 w(1a-d)+w(8a-h)+w(9a-c)+w(10a-e)+w(11a-c)+w(12a-b)
+w(13a-d)+w(14a-e)+w(15a-b)+w(16a-c)
\- w(2a-e)+w(3a-b)+w(4a-c)+w(17a-e)+w(18a-f)+w(19a-c)+w(20a-c)
+w(21a-b)+w(22a-b)+w(23a)+w(24a)+w(35a-c)+w(36a-c)
+w(37a)+w(38a)
-3 2w(5a-c)+2w(7a-b)+2w(25a-c)+2w(26a-c)+2w(27a)+w(27b)
+2w(28a-b)+2w(29a)+w(29b)+2w(30)+2w(39a-b)
+2w(40a-b)+2w(41)+2w(42)
+2 w(31a-d)+w(32a-e)+w(33a-b)+w(34a-c)+w(43a-e)
+w(44a-e)+w(45a-b)+w(46a-b)+w(47a-b)+w(48a-b)
+w(49)+w(50)+w(51a-c)+w(52a-b)+w(53) = 0

(2) 2 w(1a-d)+w(2a-h)+w(3a-c)+w(4a-e)+w(5a-c)+w(7a-b)
+w(31a-d)+w(32a-e)+w(33a-b)+w(34a-c)
\- w(8a-b)+w(8f-h)+w(9a)+w(9c)+w(10a)+w(10d-e)
+w(17a)+w(17d-g)+w(18a)+w(18d-h)+w(19b-c)+w(20b-d)
+w(25a-b)+w(26a-b)+w(27a)+w(28a)+w(42a)+w(42d-e)
+w(44a)+w(44a-e)+w(45b)+w(46b)
-3 2w(11a-c)+2w(12a-b)+2w(21a-c)+2w(22a-c)+2w(23a)+w(23b)
+2w(24a-b)+2w(29a)+w(29b)+2w(30)+2w(47a-b)+2w(48a-b)+w
+2w(49)+2w(50)
+2 w(13a-d)+w(14a-e)+w(15a-b)+w(16a-c)+w(35a-e)+w(36a-e)
+w(37a-b)+w(38a-b)+w(39a-b)+w(40a-b)+w(41)+w(42)
+w(51a-c)+w(52a-b)+w(53) = 0

(3) 2 w(1a-d)+w(4a-e)+w(7a-b)+w(10a-e)+w(12a-b)+w(16a-c)+w(24a-b)
+w(28a-b)+w(30)+w(34a-c)+w(38a-b)+w(42)+w(46a-b)
+w(47a-b)+w(50)
\- w(2b-c)+w(2e-f)+w(2h)+w(5a)+w(5c)+w(8b-c)+w(8e-f)
+w(8h)+w(11a)+w(11c)+w(14b-c)+w(14e)+w(18a)+w(18c-d)
+w(18f)+w(18h)+w(18j)+w(22b-c)+w(26b-c)+w(29a)+w(32b-c)
+w(32e)+w(36a)+w(36c)+w(36e)+w(40b)+w(44a)+w(44c-d)
+w(48b)+w(52a)
-3 2w(3a-c)+2w(9a-c)+2w(15a-b)+2w(19a-c)
+2w(23a)+w(23b)+2w(27a)+w(27b)+2w(33a-b)+2w(37a-b)
+2w(41)+2w(45a-b)+2w(49)+2w(53)
+2 w(13a-d)+w(17a-h)+w(20a-e)+w(21a-c)+w(25a-c)
+w(31a-d)+w(35a-e)+w(39a-b)+w(43a-e)+w(51a-c) = 0

(4) Write

A = w(1a)+w(2a)+w(2c)+w(2f)+w(3a)+w(5a)+w(8a)+w(8c)+w(8b)
\+ w(9a)+w(11a)+w(13a)+w(17a-b)+w(17b)+w(21a)+w(25a)+w(31a)

B = w(4a-b)+w(4d)+w(7a)+w(8a)+w(10a-b)+w(10e)+w(12a)+w(14a)
+w(14c)+w(14e)+w(15a)+w(18b)+w(18d)+w(18f-g)+w(18j)
+w(19a)+w(19c)+w(22a)+w(22c)+w(23a)+w(26a)+w(26c)
+w(27a)+w(29a)+w(32a)+w(32c)+w(32e)+w(33a)
+w(35a-b)+w(35d)+w(39a)+w(43a-c)+w(47a) ;

C = 2 w(16a-c)+w(20a-e)+w(24a-b)+w(28a-b)+w(30)+w(34a-c)
+w(36a-e)+w(37a-b)+w(40a-b)+w(41)+w(44a-e)+w(45a-b)
+w(48a-b)+w(49)+w(51a-c)

D = 2 w(38a-b)+w(42)+w(46)+w(50)+w(52a-b)+2w(53)

Then A-B = B-C = C-D

(5) In the second order only one equation remains, namely,

A + 3c = 3 B + D