RELATIVE WEIGHTS OF THE DECAYS OF CERTAIN RESONANCES IN THEORIES WITH BROKEN SYMMETRY*

C. Dullemond
Department of Physics, University of Washington, Seattle, Washington

and

A. J. Macfarlane and E. C. G. Sudarshan
Department of Physics and Astronomy, University of Rochester, Rochester, New York

(Received 5 March 1963)

(I) The principal aim of this work is to derive, in illustration of a general procedure,[1] relationships that obtain among the relative weights[2] of the decays of various resonances into baryon-meson states in the Ne'eman–Gell-Mann theory[3] of strong interactions, when exact invariance under SU_3 suffers a first-order perturbation by interactions invariant under only isospin and strangeness transformations. In particular, we deal with those resonances[4] which Glashow and Sakurai[5] have associated with the irreducible representation (IR) of SU_3 with highest weight[6] (3, 0).

(II) We may best explain our procedure by considering first a much simpler situation involving the decay of a particle (of isospin I with z component ν) into two particles (of isospins I_1 and I_2 with z components ν_1 and ν_2) in a theory whose charge independence suffers a first-order perturbation by an interaction (e.g., the electromagnetic interaction) which commutes with the operator I_z but not with $\vec{I}^2$ and hence may be taken to transform under R_3 like the $I_z = 0$ component of a vector operator. We denote the matrix element for the decay by $\langle I_1\nu_1 I_2\nu_2|T|I\nu\rangle$ and consider the sums

$$P_1(\nu) = \sum_{\nu_2} |\langle I_1\nu-\nu_2 I_2\nu_2|T|I\nu\rangle|^2, \tag{1}$$

$$P_2(\nu_1) = \sum_{\nu_2} |\langle I_1\nu_1 I_2\nu_2|T|I\nu_1+\nu_2\rangle|^2. \tag{2}$$

We easily prove that $P_1(\nu)$ is (A_1) independent of ν to zeroth order and (B_1) of the form $(\alpha+\beta\nu)$ to first order in the perturbation, and similarly that P_2 is (A_2) independent of ν_1 and (B_2) of the form $(\gamma+\delta\nu_1)$. The result (A_1) states the equality of the total weight for all decays for different charge states of the decaying particle; result (A_2) is the Shmushkevich theorem[7] for the decay. Results (B_1) and (B_2) are new. The proof of these results involves only simple facts regarding R_3 including properties of Clebsch-Gordan coefficients, the Wigner-Eckart theorem, and the fact that $C(j1j, m0m)$ is proportional to m for fixed j.

We illustrate using the decays of the well-known 3-3 nucleon resonance N^* into nucleon plus pion states. As an aid to the application of the above results, we draw up a Shmushkevich table (Table I) for the allowed processes. We see that results (A_1) and (B_1) give

$$\Gamma_1 = \Gamma_2+\Gamma_3 = \Gamma_4+\Gamma_5 = \Gamma_6, \tag{3.A$_1$}$$

$$\Gamma_1+\Gamma_2+\Gamma_4 = \Gamma_3+\Gamma_5+\Gamma_6, \tag{3.A$_2$)N}$$

$$\Gamma_1+\Gamma_3 = \Gamma_2+\Gamma_5 = \Gamma_4+\Gamma_6, \tag{3.A$_2$)π}$$

and hence we have the complete solution

$$2\Gamma_1 = 3\Gamma_2 = 6\Gamma_3 = 6\Gamma_4 = 3\Gamma_5 = 2\Gamma_6, \tag{4}$$

in the zeroth order of the perturbation. In the first order of the perturbation, we fail to get a complete solution, but only the identities

$$\Gamma_1-(\Gamma_2+\Gamma_3) = (\Gamma_2+\Gamma_3)-(\Gamma_4+\Gamma_5)$$
$$= (\Gamma_4+\Gamma_5)-\Gamma_6, \tag{5.B$_1$}$$

$$(\Gamma_1+\Gamma_3)-(\Gamma_2+\Gamma_5) = (\Gamma_2+\Gamma_5)-(\Gamma_4+\Gamma_6). \tag{5.B$_2$)π}$$

(III) We now generalize the discussion of para-

Table II. Shmushkevich table for $(3,0)\rightarrow(1,1)+(1,1)$.

Decay	N^*	Y_1^*	Ξ^*	Ω	N	Λ	Σ	Ξ	K	η	π	$\bar{K}$
w_1	1				1						1	
w_2	1						1		1			
w_3		1			1							1
w_4		1				1					1	
w_5		1					1				1	
w_6		1					1			1		
w_7		1						1	1			
w_8			1			1						1
w_9			1				1					1
w_{10}			1				1	1		1		
w_{11}			1					1			1	
w_{12}				1				1				1

and, in the first order,

$$\tfrac{1}{4}(2w_1)+\tfrac{1}{2}(2w_8+2w_9)=\tfrac{2}{3}(2w_3+2w_4+w_5)$$
$$=2(2w_8+2w_9)-2w_{12}, \qquad (10.\mathrm{D}_1)$$

$$(w_1+w_3)+(w_3+w_8+w_9+w_{12})=3(w_4+w_8)$$
$$+\tfrac{1}{3}(w_1+w_4+w_5+w_9). \qquad (10.\mathrm{D}_2)$$

A full solution of the zeroth-order problem does not follow from Eqs. (8) and (9). However, we can use the consequences of charge independence [obtained as in paragraph (II)] to tell what fractions of each w_α belong to the different charge complexions of the αth group of decays, and hence use the consequences[11] of invariance under the Weyl reflections of SU_3 to derive the complete solution

$$3w_1=12\,w_3=8w_4=12\,w_5=12\,w_8=12\,w_9=6w_{12}. \qquad (11)$$

In the first-order problem, we have Eqs. (8) and (10) and no others. It is thus possible to express all remaining w_α in terms of the experimentally accessible quantities w_1, w_4, w_5, and w_9, but not possible to obtain any identity involving only these quantities. Hence, although we can make predictions using our broken unitary symmetry theory, we cannot actually test it.

We conclude by remarking that our methods generalize[1] in each of the situations discussed to scattering and production reactions.

---

*Research supported by the U. S. Atomic Energy Commission.

[1]This is discussed in a forthcoming publication by the authors, which contains proof of certain statements made below.

[2]Relative weight converts into relative reduced width by multiplication by appropriate kinematic factors.

[3]Y. Ne'eman, Nucl. Phys. 26, 222 (1961). M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Phys. Rev. 125, 1067 (1962).

[4]For recent information, see S. L. Glashow and A. H. Rosenfeld, University of California Radiation Laboratory Report UCRL 10579, 1962 (unpublished).

[5]S. L. Glashow and J. J. Sakurai, Nuovo Cimento 25, 237 (1962); 26, 622 (1962).

[6]See, e.g., R. E. Behrends et al., Rev. Modern Phys. 34, 1 (1962).

[7]A discussion of Shmushkevich's method is given by R. E. Marshak and E. C. G. Sudarshan, in Introduction to Elementary Particle Physics (Interscience Publishers, Inc., New York, 1961), p. 185. The original paper is I. M. Shmushkevich, Dokl. Akad. Nauk S.S.S.R. 103, 235 (1955).

[8]S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 944 (1962).

[9]B. Diu (unpublished).

[10]J. Ginibre (unpublished).

[11]C. A. Levinson, H. J. Lipkin, and S. Meshkov, Nuovo Cimento 23, 236 (1962).