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Consequences of Symmetry Groups for Electromagnetic Properties

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We illustrate a general and rapid approach to electromagnetic (EM) matrix elements in the unitary symmetry theory. The basic idea exploits the relationship of the EM charge-current density to the (postulated) term of the strong interactions that breaks their $SU3$ invariance. The two quantities are tensor components under $SU3$ which can be transformed into each other by a certain Weyl reflection W_3 of $SU3$. It follows that using the unitary transformation W_3, from a theory whose $SU3$ invariance is perturbed by charge-independent (CI) interactions, we may formally generate a theory whose $SU3$-invariance is perturbed by EM interactions. In this way, we translate known results in the former into non-trivial results in the latter theory, which we separate into two categories. Electromagnetic results of the first kind arise from the equalities of the masses of all members of an isomultiplet which hold to *all* orders in the CI perturbation of $SU3$ invariance and thus hold in identical form for all EM properties. If we use the Okubo mass formula, which holds when the CI perturbation of $SU3$ invariance is first order or second order, we are led to EM relations of the second kind that hold in addition for the linear and quadratic EM properties, respectively. We illustrate this result with results for the octet that have been derived before by methods much less amenable to generalization. New EM relations of both kinds are also given for radiative decuplet to octet decays. Our work is closely related to that of Levinson *et al.*

Denote the components of unitary multiplets by $|I\nu Y\rangle$, $I(I+1)$, ν and Y being the total isospin, z-component, and hypercharge labels. We use the Weyl reflection W_3 of $SU3$, which obeys

$$W_3 Y W_3 = -(I_z + \tfrac{1}{2}Y) = -Q, \qquad W_3^2 = 1,$$

and affects octet components as follows:

$$W_3|1-10\rangle = |\tfrac{1}{2}-\tfrac{1}{2}1\rangle;\ W_3|110\rangle = |\tfrac{1}{2}\tfrac{1}{2}-1\rangle;\ W_3|\tfrac{1}{2}\tfrac{1}{2}1\rangle = |\tfrac{1}{2}-\tfrac{1}{2}-1\rangle,$$

$$W_3|100\rangle = \tfrac{1}{2}|100\rangle + \frac{\sqrt{3}}{2}|000\rangle;\ W_3|000\rangle = \frac{\sqrt{3}}{2}|100\rangle - \tfrac{1}{2}|000\rangle.$$

Denote tensor components that transform under $SU3$, such as the generators I_z and Y, as $\mathbf{I}_z$, $\mathbf{Y}$; then

$$W_3 \mathbf{Y} W_3 = -(\mathbf{I}_z + \tfrac{1}{2}\mathbf{Y}) = -\mathbf{Q}.$$

If $\Phi(\mathbf{Q})$ describes any EM property, its matrix elements satisfy

$$\langle\Psi_f|\,\Phi(\mathbf{Q})\,|\Psi_i\rangle = \langle\chi_f|\,\Phi(-\mathbf{Y})\,|\chi_i\rangle,$$

where $|\chi_{i,f}\rangle = W_3\,|\Psi_{i,f}\rangle$. To evaluate the right-hand side we can use

$$\langle I'\nu'Y'|\,\Phi(-\mathbf{Y})\,|I\nu Y\rangle = \Phi(I, Y)\,\delta(II')\,\delta(\nu\nu')\,\delta(YY'), \qquad (1)$$

with $\Phi(I, Y)$ dependent on the functional form Φ. Electromagnetic relations that hold for unknown $\Phi(I, Y)$ are those of the first kind. For the baryon octet, setting $\phi(p) = \langle \frac{1}{2}\,\frac{1}{2}\,1|\,\Phi(\mathbf{Q})\,|\frac{1}{2}\,\frac{1}{2}\,1\rangle$ and so forth, we obtain

$$\begin{aligned} \phi(\Sigma^+) &= \phi(p); \\ \phi(n) = \phi(\Xi^0) &= \tfrac{3}{2}\phi(\Lambda) - \tfrac{1}{2}\phi(\Sigma^0); \\ 2\phi_T(\Sigma^0, \Lambda) &= \sqrt{3}\,\phi(\Lambda) - \sqrt{3}\,\phi(\Sigma^0); \end{aligned} \qquad (2)$$

these relations are valid for any EM property. Turning to EM results of the second kind for the baryon octet, we consider first magnetic moments that are matrix elements of quantities which transform in the same manner as **Q** itself. Here we can use a specialization of (1):

$$\langle I\nu Y|\,(-Y)\,|I\nu Y\rangle = F(I, Y) = BY + c[I(I+1) - \tfrac{1}{4}Y^2 - \tfrac{1}{9}(\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu)],$$

which is Okubo's first-order mass formula with "tracelessness" built in. For the baryon octet, we then obtain two results of the second kind:

$$\mu(\Lambda) = -\mu(\Sigma^0), \quad \text{and} \quad \mu(\Sigma^+) + \mu(\Sigma^-) = 2\mu(\Sigma^0), \qquad (3)$$

of which the latter follows from CI alone. Equations (2) and (3) are the usual set of results.

Analogous discussion is possible for any unitary multiplet. Similarly we can treat EM self-energies; only here we use Okubo's second-order mass formula as a specialization of (1) to derive results of the second kind. As a final (and more interesting) example consider radiative decays of decuplet baryon resonances into baryons. Relations of the first kind, which hold for an *arbitrary but fixed* number of photons in the final state, are the following:

$$M(N_*^0 \to n) = 2M(Y_*^0 \to \Sigma^0) = -M(\Xi_*^0 \to \Xi^0) = -\frac{2}{\sqrt{3}}\,M(Y_*^0 \to \Lambda),$$

$$M(Y_*^- \to \Sigma^-) = M(\Xi_*^- \to \Xi^-) = 0, \quad M(N_*^+ \to p) = M(Y_*^+ \to \Sigma^+),$$

in an obvious notation. For one-photon decays we have an additional relation $M(N_*^+ \to p) = M(N_*^0 \to n)$ of the second kind.