about the behaviour of the leading trajectory for small values of λ or/and high values of s is that it has the form of (13) with some x_0.

Comparing the value of l obtained from eq. (9) and (13) we are faced with a striking difference between the behaviour of the trajectory for $\lambda \to -0$ and $\lambda \to +0$. It means that the solution of the integral eq. (4) has a singularity in λ at $\lambda = 0$, namely the bound state trajectory has a jump in λ.

Physically it is a sensible result, because the two cases are quite different in nature: if $\lambda \to -0$ then the diagrams corresponding to the kernel contain a ghost state singularity in contrast to the case $\lambda \to +0$.

If we compare the behaviour of the trajectory given by eq. (13) with trajectories given by a Yukawa potential [5] or by a $g\varphi^3$ type theory [1] it may be seen that our trajectory has a much weaker dependence on s. It is possible, that it can explain the experimental situation in elastic scattering, where the observed limit for the shrinkage of the diffractional peak is much smaller than the value predicted by the aforementioned theories [1,5].

Finally we remark the following: we substituted the simple one bubble kernel of ref. [2] by a more involved kernel which contains them and we obtained that the asymptotic behaviour of the amplitude in the crossed channel is determined by one leading Regge pole, unlike the result in ref. [2] where there was a leading branch point. One can think about the possibility of building up the exact scattering amplitude in steps in which we have always Fredholm type kernels resulting in a solution with a leading pole.

After completing this work we noticed the paper of Pac [6] about the same problem. We remark, however, that in this paper the use of the asymptotic form of Q_l is not allowed, because as we noticed the main contribution to Det$[1 - K_l(1/F)]$ comes from the integration over the region, where the argument of Q_l may be near unity.

The authors are much indebted to Dr. G. Domokos for valuable discussions.

References

1) B.W. Lee and R.F. Sawyer, Phys. Rev. 127 (1962) 2266.
2) R.F. Sawyer, Phys. Rev. 131 (1963) 1384.
P. Suranyi, Physics Letters 6 (1963) 59.
G. Domokos and P. Suranyi, JINR preprint E-1400 and Nuclear Phys., to be published.
3) W. Thirring, Lectures on compound systems in theoretical phys., (IAEA, Vienna, 1963).
4) G.C. Wick, Phys. Rev. 96 (1962) 1124.
5) A. Ahmadzadeh, P.G. Burke and C. Tate, Phys. Rev. 131 (1963) 1315.
6) P.Y. Pac, Progr. Theor. Phys. 30 (1963) 210.

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A MECHANISM OF INDUCTION OF SYMMETRIES AMONG THE STRONG INTERACTIONS

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Received 13 March 1964

Recently Cutkosky has presented a model of N vector mesons interacting amongst themselves in such a manner that each vector meson may be considered as a bound state of pairs of vector mesons. He then proceeds to show that N must be equal to the dimension of some Lie algebra and that the (renormalized) coupling constants must be proportional to the structure constants of the group. Amongst the postulates involved is the conservation of electric charge (or, more generally, any additive quantum number which is not the same for all members of the multiplet). Cutkosky raised the possibility that this postulate may be redundant, but left the question open. It is the purpose of this note to present a discussion of the model, very similar to the work of Cutkosky, in which the charge conservation need to be explicitly imposed, but may be derived as a consequence.

We follow Cutkosky and consider only such trilinear vertices for which the effective interaction may be written in the form

$$g\, F_{abc}\, \sigma A_a A_b A_c \,,$$

where σ is some function of the momenta and F_{abc} is antisymmetric under the interchange of any two of its labels. We use hermitian vector fields so that F_{abc} is purely imaginary. According to the model, the wavefunction of the particle r considered as being composed of the two particles, a, b may be written in the form

$$\psi^{(r)}_{a,b}(\underset{\sim}{x}) = F_{abr}\, \psi(x) \,.$$

The orthonormal property of the wavefunctions $\psi(r)$ yields

$$F^*_{abr} F_{abs} = [\int |\psi(x)|^2 d^3x]^{-1}\, \delta_{rs}$$

By choosing $\psi(\underset{\sim}{x})$ to be normalized and remembering the properties of F_{abc} we have

$$F_{abr} F_{abs} = -\delta_{rs} \,. \qquad (1)$$

The one-meson exchange potential may be written in the form

$$V_{ab,cd} = (F_{adr} F_{bcr} - F_{acr} F_{bdr})\, v = M_{ab,cd}\, v \,, \qquad (2)$$

where the dependence of $V_{ab,cd}$ on the momenta is solely contained in the momentum dependence of the function v.

Given the potential (2) we can proceed to compute the scattering amplitude T. The result of such a computation can be written in the form of a matrix powerseries in the matrix M with coefficients which are functions of momenta in the form

$$T_{ab,cd} = a_1 M_{ab,cd} + a_2 M_{ab,cy} M_{xy\ cd} + \dots$$

or, more succinctly,

$$T = \sum_{\nu=1}^{\infty} a_\nu M^\nu \,. \qquad (3)$$

The precise dependence of the coefficient functions a_ν on the momenta would involve the specific form of v as well as on the cutoff used. We now observe that self consistency demands that the amplitude T must exhibit N poles corresponding to the N vector meson bound states. It must then have the form

$$T_{ab,cd} = F_{abr} F_{dcr}\, \tau + C_{ab,cd} \,,$$

where $C_{ab,cd}$ corresponds to the continuum contributions and τ has the standard dependence on the momenta for one-particle exchanges. Since we have already neglected the possible contributions from such two-particle exchanges in computing the potential (2), for consistency we must neglect them here also. We are thus led to the (approximate) form

$$T_{ab,cd} = F_{abr} F_{dcr}\, \tau \,. \qquad (4)$$

Such a scattering amplitude satisfies the relation

$$T^2 = \tau T \,. \qquad (5)$$

We now wish to find the condition under which the matrix power series (3) for T can satisfy this relation.

A sufficient condition for this to be satisfied is to have M satisfy the relation

$$M^2 = m M \,. \qquad (6)$$

From the relation

$$\left(\sum_{\nu=1}^{\infty} a_\nu M^\nu\right)^2 = \tau\left(\sum_{\nu=1}^{\infty} a_\nu M^\nu\right)$$

we cannot immediately conclude that M must satisfy the relation (6). If we consider the summation over ν to be restricted to be over a finite set, say $1 \leq \nu \leq n$, then M need satisfy only a characteristic equation of degree $2n$ and may, in general, have $2n$ distinct roots rather than the two roots implied by (6). Now zero is always an eigenvalue but the other eigenvalues are in general dependent on the momenta since the coefficients a_ν are functions of the momenta. But this cannot be, since M is a numerical matrix. It thus appears very plausible that the relation (6) is also a necessary condition; it, of course, implies that τ can be expressed in terms of the coefficient functions a_ν.

But if (6) is satisfied it follows that T has the same ab, cd dependence as M so that we may write,

$$\mu F_{abr} F_{dcr} = M_{ab,cd} = F_{adr} F_{bcr} - F_{acr} F_{bdr} \,.$$

Since this relation must hold for all abcd, we get by cycle permutation of abc and addition

$$(\mu + 2)\{F_{adr} F_{bcr} + F_{bdr} F_{car} + F_{cdr} F_{abr}\} = 0 \,.$$

Since μ is a function of momenta it follows that

$$F_{adr} F_{bcr} + F_{bdr} F_{car} + F_{cdr} F_{abr} = 0 \,. \qquad (7)$$

This relation together with the antisymmetry of the F_{abc} in its indices shows that F_{abc} are the structure constants of a Lie algebra. This is Cutkosky's result [1]).

It is important to note that while this derivation is very similar to the one given by Cutkosky it does not make use of electric charge conservation; but once the Jacobi identity (7) is demonstrated, we may choose a maximal commuting set of generators of the Lie algebra to define the conserved charges (additive quantum numbers).

The author is pleased to thank R. Acharva and M. Grisaru, L. O'Raifeartaigh, K. Raman and H. Schnitzer for encouragement and critical remarks.

Reference

1) R. E. Cutkosky, Phys. Rev. 131 (1963) 1888 ; Proc. Seminar on Unified Theories of Elementary Particles, ed. D. Lurié and N. Mukunda, University of Rochester Report URPA 11 (1963).

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