

SU(6)⊗O(3) STRUCTURE OF STRONGLY INTERACTING PARTICLES

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Recently there has been considerable interest in the possibility of an SU(6) structure of the strongly interacting particles. Significant success has been achieved in the application of the SU(6) structure to multiplet assignments,¹ mass spectra,² meson-baryon coupling structure,¹ and electromagnetic^{3,4} and weak interaction⁵ properties of baryons. The group SU(6), as defined in the present context, contains the intrinsic spin group SU(2)_S and the internal symmetry group SU(3). The theory is thus naturally a nonrelativistic one. The basic group of a nonrelativistic theory (as contrasted with a Galilean or a Lorentz group) is the Newtonian group of space and time translations and rota-

tions in the three-dimensional space. It is thus natural to combine the SU(6) structure with this Newtonian group to construct the relevant combined space-time and internal symmetry group. As the first step in this direction, we postulate the invariance of the strong interactions under SU(6)⊗O(3), where O(3) is a group of rotations in the three-dimensional space, independent of the spin group SU(2)_S contained in SU(6). We then examine the various consequences of this postulate. We find the following results:

(i) A unique SU(6)⊗O(3)-invariant (parity-conserving) Yukawa coupling, bilinear in the baryon supermultiplet components, can be con-

structed.

(ii) There exist a 0^- nonet, a 1^- nonet, and a 2^- nonet of mesons. The octet part of the 0^- nonet is identified with π , K , \bar{K} , and η . The 1^- nonet is identified with ρ , K^* , \bar{K}^* , φ , and ω .

(iii) In addition to the above submultiplets, there exists a second 1^- octet.

(iv) The vector-nonet decay into two pseudoscalar mesons is no longer forbidden in the limit of exact symmetry.

(v) Within the framework of our mass formula the 0^- singlet and the 2^- singlet are degenerate in mass among themselves and with the (unphysical) degenerate mass of φ and ω . This predicted mass differs by 3 percent from the observed mass of the X_0 (960 MeV) meson.

No appreciable η - X_0 mixing is expected.

Multiplet assignments and coupling structure. - We shall denote the irreducible representations of $SU(6) \otimes O(3)$ by (a_6, b) , where a_6 is the dimensionality of the representation of $SU(6)$ and $b = 2l + 1$ is the dimensionality of the representation of $O(3)$. As the assignment of

the baryons and baryon resonances according to $SU(6)$ is reasonable, we assign these to $(56, 1)$. For the mesons, we require an assignment such that we can construct $SU(6) \otimes O(3)$ -invariant (parity-conserving) Yukawa interactions. The simplest such assignment is $(35, 3)$. This supermultiplet decomposes under the $SU(3) \otimes SU(2)_J$ classification [in which the spin group $SU(2)_S$ and the $O(3)$ group structures are identified] into a vector (1^-) octet and the three nonets (octet + singlet), respectively pseudoscalar (0^-), vector (1^-), and pseudoscalar (2^-). In the symmetric limit all these particles are degenerate in mass. Now the meson matrix can be written as

$$\Phi_{\beta, m}^{\alpha} \equiv \Phi_{sB, m}^{rA} = (\sigma^k)_s^r R_{B, m}^{A, k} + \delta_s^r \{ S_{B, m}^A - \frac{1}{3} \delta_B^A S_{C, m}^C \}, \quad (1)$$

where r, s are two-valued spinor indices; A, B, C are three-valued $SU(3)$ indices; and k, m are three-valued vector indices; summation over repeated indices is understood. We can indicate the $SU(3) \otimes SU(2)_J$ reduction by writing

$$R_{B, m}^{A, k} = \frac{1}{3} \delta_m^k R_{B, n}^{A, n} + \frac{1}{2} (R_{B, m}^{A, k} - R_{B, k}^{A, m}) + \frac{1}{2} (R_{B, m}^{A, k} + R_{B, k}^{A, m} - \frac{2}{3} \delta_m^k R_{B, n}^{A, n}), \quad (2)$$

which displays the nonet of 0^- , 1^- , and 2^- unnormalized meson wave functions. The parity of the particle states is entirely due to the property of the three-dimensional space.⁶ The baryon-meson interaction could now be immediately written down:

$$\frac{G}{i} \bar{B}_{\alpha\gamma\delta} \nabla_m B^{\beta\gamma\delta} \Phi_{\beta, m}^{\alpha}, \quad (3)$$

which is seen to be $SU(6) \otimes O(3)$ invariant. Using the expressions (1) and (2), we see that 0^- , 1^- , and 2^- mesons are, respectively, coupled to baryons via $\vec{\sigma} \cdot \nabla$, $\vec{\sigma} \times \nabla$, and $(\vec{\sigma} \nabla)$. Thus the vector nonet is coupled to baryons through a magnetic-type coupling, whereas the vector octet is coupled through an electric-type coupling.⁷ The coupling structure (3) leads to two immediate results:

(a) In the coupling of the nonets, the D/F ratio for the octet coupling to baryons is unique and is given by $1 \frac{3}{2}$. In contrast, the second vector octet is coupled via pure F -type coupling.

(b) Assuming π - N coupling $g_{pS}^2/4\pi \sim 15$, one obtains the N^* width to be ~ 75 MeV.¹

In analogy with (3) we may write down the trilinear meson coupling

$$\frac{\lambda}{i} \Phi_{\gamma, l}^{\beta} \nabla_m \Phi_{\alpha, l}^{\gamma} \Phi_{\beta, m}^{\alpha}, \quad (4)$$

which is also $SU(6) \otimes O(3)$ invariant and allows the decays $\rho \rightarrow \pi + \pi$, $K^* \rightarrow K + \pi$, and $\omega \rightarrow K + \bar{K}$.

Mass formula. - The highly symmetric $SU(6) \otimes O(3)$ discussed so far is broken, as evidenced by the lifting of the mass degeneracy between the vector and the pseudoscalar particles prior to the stage at which $SU(3)$ invariance is broken. Before deriving a mass formula from general considerations, we obtain a suggestive mass formula for mesons with the following reasoning: The symmetry could be broken by invoking the spin-orbit coupling term $\vec{L} \cdot \vec{S}$. Also, the symmetry breaking could split a nonet into an octet and a singlet through the appearance of $C_2^{(3)}$, the quadratic Casimir operator of $SU(3)$, but we must guarantee that the φ and the ω of the vector nonet remain degenerate. The simplest way to accomplish

this gives rise to the following mass formula involving a single symmetry-breaking parameter⁸:

$$m(a_3, a_2, a_1, b, J) = m_0(a_3, a_2, b) + m_1(1 + L \cdot S)C_2^{(3)}, \quad (5)$$

where a_3 and a_2 are, respectively, the dimensionalities of $SU(3)$ and $SU(2)_S$ contained in $SU(6)$. The symmetry-breaking chain is $SU(6) \otimes O(3) \supset SU(3) \otimes SU(2)_J$. Further breaking of $SU(3)$ to $U(2)$ by a neutral octet tensor gives a Gell-Mann-Okubo splitting of the $SU(3)$ multiplets and the standard ω - ϕ mixing. The mass formula (5) gives rise to a large separation of the respective octet and singlet members in the 0^- and the 2^- nonets. In fact the degenerate mass of the 1^- nonet, 0^- singlet, and 2^- singlet is ~ 930 MeV; the mass of the 0^- octet is ~ 550 MeV and that of the 2^- octet is ~ 1400 MeV.¹⁰ Hence there is no analog of the ϕ - ω mixing for either the 0^- or the 2^- nonet. We note that the mass of the second 1^- octet is not related by the mass formula to the nonets, as m_0 is dependent on the intrinsic spin which is unity for the nonets and zero for the octet. Under the $SU(3)$ breaking, the known relation

$$m_{K^*}^2 - m_\rho^2 = m_K^2 - m_\pi^2$$

is valid in this model also; it can be generalized to the prediction that the difference in (mass)² values of the isodoublet and isotriplet of all the meson octets is the same.

The Casimir operators $C_2^{(3)}$ and $(\vec{L} \cdot \vec{S})C_2^{(3)}$ in the mass formula (5) are contained in $(35, 1) \otimes [(35, 1) \oplus (35, 3)]$. Under the assumption that the symmetry-breaking interaction transforms like $(35, 1) \otimes [(35, 1) \oplus (35, 3)]$, we can get a general mass formula which will be discussed in detail elsewhere. We note here that for the baryons which have been assigned to $(56, 1)$ there is no contribution from $(35, 1) \otimes (35, 3)$, and the general mass formula reduces to the one already obtained.²

So far we have discussed only $(56, 1)$ and $(35, 3)$ for baryons and mesons. In addition to the various other representations with different values of a_6 to which they could be assigned,¹ we could assign them to representations with any arbitrary value of $b = 2l + 1$.

Electromagnetic properties.—The structure of the nonrelativistic electromagnetic interaction consists of the electric charge and current interactions which transform, respectively, like $(35, 1)$ and $(35, 3)$ belonging to the spin

singlet term in its $SU(2)_S \otimes SU(3) \otimes O(3)$ reduction; and the magnetic interaction which transforms like $(35, 3)$ but belongs to the spin triplet term under this reduction. In the limit of exact $SU(6) \otimes O(3)$, we observe that the lowest order matrix elements of the electromagnetic interactions between the states of $(56, 1)$ baryon multiplet, which would define their magnetic moment and electric and magnetic form factors (and transition moments and form factors), vanish, with the exception of the charge form factor, which is universal and proportional to electric charge. We have to consider then the interference between the term which breaks the $SU(6) \otimes O(3)$ symmetry and the primitive electromagnetic interactions. We have already invoked the mechanism of symmetry breaking by the spin-orbit interaction in connection with the mass formula. A more primitive symmetry breaking is provided by the recoil of the (spinning) baryon. The generator K_j of the transformation to a frame moving with an increase of velocity in the j th direction (for both the Galilei and the Lorentz systems) transforms as $(1, 3)$ for free spinless particles, and as $(1, 3) \oplus (35, 3)$ for free spinning particles⁹ (in Lorentz systems). In any case, by virtue of the commutation relation

$$(1/i)[K_j, H] = P_j$$

being valid for arbitrary interacting Lorentz or Galilei systems, the generator K_j or the Hamiltonian H (or both!) has parts which transform as the three-component (vector) representation of $O(3)$. Assuming that this transforms as $(1, 3)$, we deduce in a very transparent fashion that the leading contributions to the linear electromagnetic properties transform as the $(35, 1)$ representation. This yields the results already obtained³ for the ratio of the neutron and proton magnetic moments, $\mu_n/\mu_p = -2/3$, and also gives rise to similar relations for the baryon magnetic moments and transition moments. We also get the result that all the magnetic form factors are multiples of a common magnetic form factor, and that the electric (current) form factors are simple multiples of a common electric (current) form factor, the electric charge being the multiplicative factor. The latter result implies that the electric (current) form factor of a neutron vanishes. It is to be noted that the nonrelativistic magnetic interaction gets contributions from

both the relativistic electric and magnetic form factors. In the present model we note that the magnetic contribution comes mainly from the coupling of baryons to the vector nonet, which is magnetically coupled, in contrast to the vector octet, which is electrically coupled.

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⁴B. Sakita (reference 3); C. H. Chan and A. Q. Sarker, Phys. Rev. Letters 13, 731 (1964); V. Singh, to be published. The last-named author has taken into account both 35 and 405 breaking to get electromagnetic mass shifts of baryons.

⁵P. Babu, this issue [Phys. Rev. Letters 14, (1965)]. Added in proof: See also S. P. Rosen and S. Pakvasa, Phys. Rev. Letters 13, 773 (1964); M. Suzuki, Phys. Letters 14, 64 (1965); K. J. Barnes, P. Carruthers, and F. von Hippel, Phys. Rev. Letters 14, 82 (1965).

⁶In the symmetry scheme under consideration, we do not have a meson as a bound state of a quark and an antiquark. This point will be discussed elsewhere from the point of view of the current-current picture.

⁷This is to be contrasted with the conventional SU(3) model in which the vector nonet has a relativistic vector coupling which contributes to both electric and magnetic couplings in the nonrelativistic limit.

⁸Here it is understood that we use $(\text{mass})^2$ for mesons.

⁹L. L. Foldy, Phys. Rev. 87, 688 (1952); *ibid.* 87, 693 (1952).

¹⁰We take our complete mass formula to be $m = m_0 + m_1(1 + L \cdot S)C_2^{(3)} + m_3[I(I+1) - (Y^2/4)]$, where $m_0 \sim (930)^2$, $m_1 \sim (430)^2$, and $m_3 \sim -(376)^2$. The quoted mass values correspond to the contributions from the first two terms.