elements is

$$A(\pi^+\pi^-) = g_1 + 5(g_2+g_3),$$
$$A(K^+K^-) = -g_1 + 4(g_2+g_3),$$
$$A(K^0\bar{K}^0) = -2g_1 - (g_2+g_3) \quad (6)$$

(apart from an irrelevant factor), where $A(\pi^+\pi^-)$ stands for the matrix element between the initial $\bar{p}p$ and the final $\pi^+(1)\pi^-(2)-\pi^-(1)\pi^+(2)$ state, etc. [see Eq. (2)]. Of course, $A(\pi^0\pi^0) = A(\eta\eta) = A(\pi^0\eta) = 0$, as it should be.

From (6) we obtain the triangular relation[7]

$$A(\pi^+\pi^-) - A(K^+K^-) + A(K^0\bar{K}^0) = 0. \quad (7)$$

From the experimental data of reference 5, if we take the rate for Reaction (1) to be proportional to the meson momentum times the squared absolute value of the amplitude, we get, in arbitrary units, the following values:

$$|A(\pi^+\pi^-)| = 1.99 \pm 0.10,$$
$$|A(K^+K^-)| = 1.32 \pm 0.19,$$
$$|A(K^0\bar{K}^0)| = 0.87 \pm 0.06.$$

Equation (7) seems very well satisfied by these quantities.

We conclude that, at least for annihilation at rest into two pseudoscalar mesons, the SU(6)-symmetry model seems to give a rather satisfactory agreement with experiments.

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[6]The terms I_2 and I_4, apart from exchange antisymmetrization, correspond to the two independent terms of Eq. (4) in reference 4.

[7]It should be noted that Eq. (5) is fully symmetrical as far as the SU(3) part is concerned. In SU(3) symmetry alone, however, there is no relation for charged meson production, so Eq. (7) is a pure SU(6) prediction. We recall that the only prediction in SU(3) involves apparently S-wave neutral mesons, and gives no contribution in the low-energy region. See M. Konuma and Y. Tomozawa, Phys. Rev. Letters 12, 425 (1964).

EQUIVALENCE OF THE CURRENT ALGEBRA AND GROUP-THEORETIC FORMULATIONS IN PARTICLE PHYSICS*

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Recent developments in particle physics have led to a phenomenological but quantitatively successful correlation of a variety of properties of strongly interacting particles (including the low-lying resonances). The success of this approach raises fresh problems,[1] since the symmetry groups underlying these systemizations were by no means symmetry groups of the Hamiltonian of the system[2]; the apparent violations of the symmetry were sufficiently large to invalidate any traditional method of computing the effects of symmetry violation. And yet the simpler predictions of these (unitary) symmetry groups have had significant success. With a view to avoiding this embarrassment, several people have attempted to depart from a conventional approach to symmetry and to try to base it on the algebra of certain ("current") operators.[3] The formulation involving the (space)-integrated currents has been used recently[4] to rederive several results of the SU(6) and SU(4) theories in a more direct fashion. It is the purpose of this note to show that insofar as one uses only the integrated currents, the formulation in terms of the algebra of currents is completely equivalent to a (specific!) standard formulation of symmetry groups.

Conventionally, one investigated the symmetries of a Hamiltonian, but to postulate a symmetry group acting on the physical states is by no means necessary. In fact, we should now be convinced about the relevance of symmetry groups which are useful, but which are not invariance groups of the Hamiltonian.[2] The only requirement is that the abstract group G be represented by unitary transformations on the Hilbert space of physical states. Without any loss of generality we can use a discrete (countable) matrix index to refer to these states. We must now identify the physical states in terms of the irreducible representations of G. If the group G is assumed to be compact, it follows that every unitary representation is fully reducible and every irreducible representation is finite dimensional. Given any unitary representation of a (finite-parameter) Lie group, there is automatically a representation of its Lie algebra A by Hermitian operators (which are, in general, unbounded). The reduction of the unitary representation automatically reduces the Hermitian representation of the Lie algebra.

We can now define two associated algebraic structures formally: The enveloping algebra E is obtained by essentially taking all polynomials in the generators modulo the Lie-algebra commutation relations. And the tensor operators with respect to the Lie algebra (or, more properly, with respect to the group G) are defined in a manner familiar from the theory of tensor operators with respect to the three-dimensional rotation group. The elements of the enveloping algebra E furnish special kinds of tensor operators.

So far we have required no property of the Hamiltonian; to make the symmetry group and its associated algebras physically relevant, we have to make specific identification of the physical states furnishing the representation vectors for the unitary representation of the group G as well as of the dynamical variables with various classes of tensor operators. For example, in the SU(6) theory we would identify the components of the 56-component baryon multiplet and the 35-component meson multiplet suitably, and at the same time specify the transformation properties of, say, the magnetic moment operator. We stress here the irrelevance of any special properties of the Hamiltonian. In particular, there is nothing approximate about the structure of the representation or the geometric (Wigner-Eckart) dependences of the matrix elements of tensor operators. Thus, for example, we get the now-familiar $-\frac{3}{2}$ ratio between proton and neutron magnetic moments simply from the assumption that the magnetic moment operator transforms as a tensor operator. It is also known that the familiar D/F ratio for the SU(6) model is deduced by considering the tensor transformation properties of the current to which the mesons are coupled, the coupling itself violating the SU(6) symmetry![5]

From force of habit we may now assert that the Hamiltonian be "approximately invariant" under this symmetry group and that mass splittings within particle multiplets be "small." We can, however, dispense with this assumption; from the postulated unitary representation furnished by the physical states we have a set of Hermitian current operators with the prescribed Lie-algebra commutation relations. If we believe that the symmetry group commutes with, say, spatial momenta, the "current" elements of it will connect only states with the same spatial momenta. These operators, in other words, are translation invariant. One possible realization of such translation-invariant operators is by integration over all space of a suitable "current-density" operator (distribution). If the unitary transformations associated with the symmetry group G take local canonical fields into local canonical fields, we can (formally) construct the elements of the algebra in terms of current density operators bilinear in the canonical fields. We shall, however, dispense with this luxury. The more physically relevant assumption is that there exists a subset of one-particle states which themselves furnish an irreducible representation of G. We thus see that, given the symmetry group, we have the appropriate current algebra (with no restrictions on the Hamiltonian!).

We now show the converse to be true. For this purpose we start from the current algebra and the physical identification of the dynamical variables. It is unnecessary to specialize either to canonical fields or to (formal) commutation relations between current-density operators; nor is it necessary to make detailed assumptions about the structure of, say, the magnetic moment operator in terms of the current-density operators. The important point is the identification of the tensor character of the relevant dynamical variable, and the particle multiplet furnishing the representation. In such a case we have, in fact, the Hermitian representation of a (finite-dimensional) Lie algebra A. We could, therefore, equally well define the unitary representation of the Lie group G, by exponentiating the representation for A. And in the case of a compact Lie algebra the Hermitian representations are completely reducible into finite-dimensional Hermitian representations.

Given the current algebra and the multiplet we will, in general, still have ambiguities about the identification of their transformation properties. For example,[6] the SU(4) current algebra acting on a multiplet containing the nucleon can treat the nucleon as 4 or $\bar{4}$ or as part of the 20 or $\overline{20}$. We could eliminate the first two alternatives, if we so choose, by normalizing the isotopic spin current matrix elements of the nucleon resonances. Even then the last two alternatives are equally good and give an ambiguous prediction for the ratio of proton and neutron magnetic moments.[7] It is easy to show that this ambiguity is associated with the (outer) automorphism of the SU(4) current algebra generated by the extended charge-conjugation operator. The relevant observation is that this ambiguity is equally well present in the equivalent group-theoretic formulation.

We conclude that the formulation in terms of the algebra of currents is equivalent to the specific formulation of the group-theoretic scheme discussed above. In neither case is the invariance of the Hamiltonian relevant, and in either case the necessary physical assumptions are the same. It is perhaps appropriate to seek further elaborations of the group-theoretic formulation in the language of the algebra of the currents.

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[1]For a progress report, see Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January 1965 (W. H. Freeman & Company, San Francisco, California, to be published).

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[5]For some reason, this aspect of the SU(6) couplings is rarely mentioned!

[6]I thank Dr. C. H. Woo for a very helpful correspondence.

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SUPERMULTIPLET SCHEMES AND MESON POLE MODELS FOR ELECTROMAGNETIC FORM FACTORS

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Supermultiplet schemes combining spin and internal symmetries generally lead to rather specific predictions for electromagnetic[1] and weak-interaction form factors. Some of these predictions depend only upon the assumed transformation properties of the electromagnetic or weak couplings. These are the predictions

$$G_M^p(q^2) = -\tfrac{3}{2}G_M^n(q^2), \quad G_E^n(q^2) \equiv 0, \qquad (1)$$

where G_M and G_E are the Sachs form factors. Other predictions depend upon specific meson pole-dominance models:

$$\mu_p \approx \left(1+\frac{2m}{\mu}\right), \quad \frac{G_M^p(q^2)}{G_E^p(q^2)} \approx \frac{1}{2m}\,\frac{1+2m/\mu}{1-q^2/2m\mu}, \qquad (2)$$

where m and μ are the central baryon and odd-parity meson masses, and μ_p is the total magnetic moment of the proton. We wish to point out a special feature of the pole-dominance model, which results in the possibility of one