Table I. Calculated and experimental squares of baryon mass ratios.

Ratio	Calculated value	Experimental value
Σ/Λ	1.12	1.15
N/Λ	0.72	0.71
Ξ/Λ	1.38	1.40
N^*/Y^*	0.79	0.80
Ξ^*/Y^*	1.22	1.22
Ω/Y^*	1.46	1.47
Y^*/Λ	1.28	1.53

may play a large role in determining this mass difference.

The eigenvalues of the baryon probability matrix corresponding to the $(1-\alpha)$ terms of Eqs. (4a) through (4c) are 11/15, 17/45, and −1/15. These may be identified with the SU(6) representations of dimensions 35, 405, and 2695, respectively. The fact that 11/15 is close to one leads to enhancement of both the strong and electromagnetic mass splitting that corresponds to the representation $\underline{35}$.

It has been shown in previous references that rough agreement between calculated and experimental baryon mass-splitting values occurs also in the standard reciprocal bootstrap model, in which the vector mesons are not involved.[5,9] For this reason one cannot argue that the observed baryon mass splitting is strong evidence for SU(6). Our principal conclusion is that the Σ-Λ mass difference, which is somewhat of a mystery in a quark model of SU(6),[7] arises naturally in the SU(6)-symmetric bootstrap model.

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[7]The $\underline{35}$ rule for electromagnetic and strong mass splitting may easily be derived from a quark model, since the mass-splitting matrix of the fundamental (quark) representation of SU(6) corresponds entirely to the 35-fold representation.

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LORENTZ-COVARIANT SU(6), PARTICLE-ANTIPARTICLE ALGEBRAS, AND SUPERMULTIPLET STRUCTURE

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With the remarkable success of SU(3),[1] culminating in the prediction and discovery of Ω^- resonance, a considerable effort is being made to search for a higher symmetry to explain certain empirical facts which are not encompassed by SU(3). The effort has been roughly in three directions: The first has been to search for a bigger internal-symmetry group; the examples are SU(4),[2] W(3),[3] and R(8).[4] The second approach consists in looking for the algebra which the currents satisfy; the examples are $SU^+(3) \otimes SU^-(3)$, or $\overline{W}(3)$, which leads to the parity doubling of mesons,[5] and further enlargement of the same to the F- and D-type currents.[6] Already in this case parity, which is normally considered as a property of space, has been included. Third come attempts to combine the internal symmetry with the spin group and also the rotation in the three-dimensional space; the result has been SU(6)[7] and SU(6)⊗O(3),[8] respectively. In these cases the groups are essentially nonrelativistic. There have been efforts to "relativitize" SU(6) in a suitable fashion. These efforts[9,10] have been confined to looking for a group which leaves the (Lorentz-invariant) four-Fermi interactions (FFI) invariant. This has led to various kinds of W(6) groups which are obvious generalizations of $\overline{W}(3)$. In all these attempts projections of SU(6) by the operators $\frac{1}{2}(1\pm\gamma_5)$ and $\frac{1}{2}(1\pm\gamma_4)$ have been used to double the algebra. In the case of γ_5 projection, the vector and axial-vector FFI are left invariant, whereas in the case of γ_4 projection the scalar and pseudoscalar FFI are left invariant. Under the double algebras obtained by the above projection operators, the free Lagrangian is not invariant.[11] It has been suggested that the kinetic term in the free Lagrangian be used as an intrinsically symmetry-breaking mechanism.[12] Because the free Lagrangian is not invariant in the exact symmetry limit, one does not know what one means by a particle belonging to a particular representation of the inhomogeneous Lorentz group. Besides being conceptually bewildering, these double algebras are beset with calculational difficulties.

FW algebra. –We should like to present in this Letter the results of our investigation, paying attention to the relativistic invariance of the free Lagrangian. One of the main interesting results is that we do have an SU(6) under which the free relativistic Lagrangian is invariant. Before we concern ourselves with the group SU(6), we shall precisely identify the unitary group generated by the intrinsic spin. We note that any such group should certainly leave the free (Lorentz-invariant) Lagrangian invariant. For a Dirac field with the free Lagrangian

$$\bar{\psi}(\gamma p - m)\psi,$$

such a group of transformations is given by[13]

$$\psi \to \exp(\tfrac{1}{2}i\vec{\Sigma}^{\sim}\cdot\vec{\theta})\psi, \tag{1}$$

where

$$\vec{\Sigma}^{\sim} = \vec{\Sigma} - \frac{i\beta(\vec{\alpha}\times\vec{p})}{\omega} - \frac{\vec{p}\times(\vec{\Sigma}\times\vec{p})}{\omega(\omega+m)},$$

with $\omega = (\vec{p}^2 + m^2)^{1/2}$ and

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}.$$

It is known from the work of Wigner and Foldy[14] that such spin generators exist for fields of arbitrary spin (and finite mass). We term this spin the canonical spin. Since the canonical spin does not mix particle and antiparticle states, separate spin groups may be defined over positive- and negative-frequency parts of the field operator, ψ_+ and ψ_-, given by[15]

$$\psi_\pm = \tfrac{1}{2}(1 \pm H_0/|H_0|)\psi, \tag{2}$$

H_0 being the free Hamiltonian matrix. Hence the intrinsic spin group is $SU_+(2)\otimes SU_-(2)$. We shall refer to this as FW(2). It should be noted that the elements of FW(2) do not commute with the charge-conjugation operator. (Neither does it commute with a general element of the Lorentz group.) Charge conjugation is an outer automorphism of order two of the group FW(2).

So far we have discussed the particle-antiparticle doubling of the intrinsic spin group. Similar considerations also apply to any internal-symmetry group. In particular, for the SU(3) group we have $FW(3) = SU_+(3)\otimes SU_-(3)$ defined by the generators

$$\tfrac{1}{2}(1 \pm H_0/|H_0|)\lambda_i, \tag{3}$$

where λ_i, with $i = 1,\cdots,8$ are the SU(3) generators as defined by Gell-Mann.[1] In the framework of FW(3), the mesons have to be assigned to the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ in view of the charge-conjugation invariance of the strong interactions. This leads to two nonets of mesons, degenerate in mass, spin, and parity, which may be chosen to be even and odd under charge conjugation. When the invariance under FW(3) is reduced to the invariance under SU(3), the mass degeneracy between the two nonets can be lifted. It might be remarked here that the presence of the abnormal octet $(C = -1)$ could be used for explaining the CP nonconservation in $K_2 \to 2\pi$ decay.[16]

Since the FW(3) as well as the SU(3) commute with the (improper) Lorentz group, no restriction is placed on the spin-parity assignment of the multiplets. If we choose to identify the mesons with the particle-antiparticle part of the local vector and axial-vector currents associated with the SU(3) generators, in the low relative-momentum limit, we obtain a 1^- multiplet with $C=-1$ and a 0^- multiplet with $C=+1$.

FW(6) and the multiplet structure.—The canonical spin group defined above can be combined in a nontrivial fashion with an internal-symmetry group to give rise to a larger group. If we choose the internal-symmetry group to be the SU(3), the larger group is SU(6).[17,18] The particle-antiparticle doubling applied to the SU(6) leads to the group FW(6) characterized by the 70 generators which transform like the matrices

$$\tfrac{1}{2}(1\pm H_0/|H_0|)\Sigma_j^-,\ \tfrac{1}{2}(1\pm H_0/|H_0|)\lambda_i,\quad \tfrac{1}{2}(1\pm H_0/|H_0|)\Sigma_j^-\lambda_i. \tag{4}$$

The free Lagrangian is invariant under this group, whereas a general local interaction is not. This is due to the inherent nonlocality in the Foldy-Wouthuysen projection. Before concerning ourselves with the details of the structure of the interaction, let us consider the assignment of physical particles to the representations of FW(6). The representation $(\underline{1},\underline{56})\oplus(\underline{56},\underline{1})$ corresponds to the baryons and the antibaryons. The mesons are to be assigned to the representation $(\underline{6},\underline{6}^*)\oplus(\underline{6}^*,\underline{6})$. Thus we have C (charge-conjugation) doublets of mesons.

Recently Michel and Sakita[19] have argued, starting from a set of plausible postulates, that as long as only finite-dimensional unitary representations of the "little group" are considered, it is not possible for structures like SU(6) to arise. The case of free particles that we have discussed above is in apparent contradiction with this conclusion. We note, however, that this is due to one of their postulates being too stringent.

Interaction structure.—Consider a general current-current interaction of the relativistically covariant fields in the form

$$\sum_n G_n(\bar\psi O_n\psi)(\bar\psi O_n\psi)^\dagger, \tag{5}$$

where O_n is any set of operators, say $\lambda_\alpha\gamma_\mu$, etc. This interaction consists of a variety of interactions between particles and antiparticles, which may be symbolically represented by the expression

$$(\{a^\dagger+b\}O\{a+b^\dagger\})\cdot(\{a^\dagger+b\}O\{a+b^\dagger\})^\dagger.$$

We may choose to isolate the pure "particle-particle" terms in this: $(a^\dagger Oa)\cdot(a^\dagger Oa)^\dagger$, or the "particle-antiparticle" terms: $(a^\dagger Oa)\cdot(bOb^\dagger)^\dagger$. The relativistic interactions rewritten for the pure particle-particle and particle-antiparticle interactions are thus represented in terms of the two-component Foldy-Wouthuysen-Tani amplitudes. We may write down the apparently FW(6)-invariant interaction

$$\begin{aligned}&G_{11}(a^\dagger\sigma_\mu\lambda_\alpha a)(a^\dagger\sigma_\mu\lambda_\alpha a)^\dagger\\&+G_{110}(a^\dagger a)(a^\dagger a)^\dagger+G_{12}(a^\dagger\sigma_\mu\lambda_\alpha a)(b^\dagger\sigma_\mu\lambda_\alpha b)^\dagger\\&+G_{120}(a^\dagger a)(b^\dagger b)^\dagger+G_{21}(b^\dagger\sigma_\mu\lambda_\alpha b)(a^\dagger\sigma_\mu\lambda_\alpha a)^\dagger\\&+G_{210}(b^\dagger b)(a^\dagger a)^\dagger+G_{22}(b^\dagger\sigma_\mu\lambda_\alpha b)(b^\dagger\sigma_\mu\lambda_\alpha b)^\dagger\\&+G_{220}(b^\dagger b)(b^\dagger b)^\dagger,\end{aligned} \tag{6}$$

where $\sigma_\mu=(1,\vec\Sigma^-)$. This interaction is only symbolic in the sense that for the complete specification we have to specify the momenta of the various positive- and negative-frequency components, a, b, $a^\dagger$, and $b^\dagger$, and assure the conservation of momentum in the interaction. Let us then consider a typical term, say,

$$G_{12}(a^\dagger\sigma_\mu\lambda_\alpha a)(b^\dagger\sigma_\mu\lambda_\alpha b)^\dagger,$$

which must be specified more precisely in the form (in the interaction representation)

$$\int d^3p_1d^3p_2d^3p_3d^3p_4\,\delta(\vec p_1-\vec p_2-\vec p_3+\vec p_4)G_{12}(\vec p_1,\vec p_2,\vec p_3,\vec p_4)\times[a^\dagger(\vec p_1)\sigma_\mu\lambda_\alpha a(\vec p_2)][b^\dagger(\vec p_3)\sigma_\mu\lambda_\alpha b(\vec p_4)]^\dagger. \tag{7}$$

Written in this form we see that the system is invariant for the general values of the momenta under the Newtonian group (i.e., space-time translations and space rotations). But under a pure Lorentz transformation, the various amplitudes [represented symbolically here by $a(\vec p)$, $b(\vec p)$, etc.] undergo a spin "rotation" proportional to the momentum in view of the structure[20]

$$\vec K=\tfrac{1}{2}(\omega\vec q+\vec q\omega)+[\vec p\times\vec s/(m+\omega)],$$

$$\vec q=i(\partial/\partial\vec p),$$

for the Lorentz generator. Hence to have SU(6) invariance in every Lorentz frame we must have equal values for $\vec p_1$, $\vec p_2$, $\vec p_3$, and $\vec p_4$. Stated somewhat differently, the interaction is approximately invariant under SU(6) if the form factor $G(\vec p_1,\vec p_2,\vec p_3,\vec p_4)$ tends to be appreciable only for the region where the differences of these momenta (measured in units of $m+\omega$)

are small; to restore the Lorentz invariance it will be necessary to supplement this with "small" additional terms. Such a form factor corresponds to a nonlocal interaction. It is to be noted that no restriction is placed on the absolute momentum of any of the operators as it should be in a proper relativistic theory.

The interactions of the type

$$\int d^3p_1 d^3p_2 d^3p_3 d^3p_4\, \delta^3(\vec{p}_1-\vec{p}_2-\vec{p}_3+\vec{p}_4) G_{120}(\vec{p}_1\vec{p}_2\vec{p}_3\vec{p}_4) \times [a^\dagger(\vec{p}_1)a(\vec{p}_2)][b^\dagger(\vec{p}_3)b(\vec{p}_4)]^\dagger \quad (8)$$

have weaker restrictions on the form factor since it suffices to have $(\vec{p}_1-\vec{p}_2)$ and $(\vec{p}_3-\vec{p}_4)$, i.e., momentum transfer, be small.

We note also that the interactions with coefficients G_{11}, G_{110}, G_{120}, G_{210}, G_{22}, and G_{220} in Eq. (6) admit the larger group FW(6), since the particle and antiparticle can be transformed independently. The terms with the coefficients G_{12} and G_{21} break this symmetry; but it is precisely these terms which arise from crossing. Hence crossing symmetry and FW(6) are not compatible. The symmetry reduces to SU(6).

So far we have not considered the mixed terms of the type

$$(a^\dagger Oa)(aOb)^\dagger,$$

which contribute to Yukawa vertices. These are not invariant under FW(6), nor FW(3), but are invariant under SU(3). With respect to FW(6), this transforms as a $(\underline{6}, \underline{6}^*)$, so that the long-range force contains $(\underline{6}, \underline{6}^*) \otimes (\underline{6}^*, \underline{6})$ which breaks under SU(6) as $\underline{35} \otimes \underline{35} \oplus \underline{35} \oplus \underline{35} \oplus \underline{1}$. Hence the very existence of the Yukawa coupling requires the symmetry to be broken at most like $\underline{35} \otimes \underline{35}$.

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[15]This double algebra, though reminiscent of the one obtained from $\frac{1}{2}(1 \pm \gamma_4)$(see reference 10), restricted to the spin-half field, coincides with it only in the zero velocity limit. H_0 projection can be used for any field of arbitrary spin.

[16]Such a possibility also exists for the case of the existence of the parity doublets of reference 5.

[17]Under the present SU(6) the free Lagrangian is invariant, in contrast to the SU(6) of reference 7.

[18]SU(3) and SU(2) could be embedded in SU(5), but this leads to multiplets which contain both fermions and bosons and does not seem to be of interest at present.

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