can be generalized in a straightforward manner to vector fields, in which case it takes the form given in a recent paper.[10]

Whether kinematical and nonkinematical gradient terms appear in perturbation theory of the isospin-antisymmetric (or SU_3-antisymmetric) parts of the zero-zero components in (6) is an open question. The existing work on this point does not take explicit renormalization into account and is according to our point of view insufficient. We hope to be able to report results on the equal-time structure of renormalized currents in the near future.

We are indebted to Dr. A. H. Voelkel for helpful discussions.

APPENDIX

Our Statement 1 is quite general in the sense that our class of "nonkinematical" gradient terms contains examples $\psi(u,s)$ with arbitrary high powers in s. It is only this point which has been overlooked in Ref. 7. As an investigation of kinematical gradient terms Ref. 7 is completely correct.

[10] Dietz, K. and J. Kupsch, University of Bonn Report 1967 (unpublished).

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Meson Symmetries Based on Nonchiral $SU(3)\otimes SU(3)$†

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A scheme for meson symmetries based on the $SU(3)\otimes SU(3)$ group is presented. First, the phenomenological nonet model of Okubo is reinterpreted in terms of the representation $(3,3^*)$ of $SU(3)\otimes SU(3)$. More general representations of the form (d,d^*), with d a triangular representation of $SU(3)$, are next considered. It is noted that the assumption of the symmetry violation being generated by a hyperchargelike operator always leads to an equal-spacing rule for the meson masses, and to a degeneracy of several meson components. The origin of this degeneracy is discussed in some detail. The representation $(6,6^*)$ is treated at length. The possibility of realizing meson 36-plet $(6,6^*)$ is discussed with reference to existing experimental information.

I. INTRODUCTION

It appears well-established that the pseudoscalar, vector, and tensor mesons appear in nonets rather than octets.[1] The occurrence of the $SU(3)$ singlets along with the octets appear even more significant when we see evidence for singlet-octet mixing so that the $I=Y=0$ mesons belong neither to an octet nor a singlet but to a suitable linear combination among them. The question of mixing between the $I=Y=0$ vector mesons has been treated by various authors[2] phenomenologically and the same treatment was later extended to pseudoscalar and tensor mesons. Several years ago Okubo introduced an ansatz for the problem of mixing of vector mesons[3]: He suggested that the nine vector mesons be associated with the nine components of a second-rank mixed tensor

$$G_\nu{}^\mu = T_\nu{}^\mu + (1/\sqrt{3})\delta_\nu{}^\mu\phi$$

and required that in no mathematical expressions should the trace $G_\mu{}^\mu(=\sqrt{3}\phi)$ occur explicitly. In terms of such an ansatz Okubo was able to give a simple and satisfactory treatment of vector-meson processes including the mixing angles. Glashow and Socolow later extended this discussion to pseudoscalar and tensor mesons.[4] It has been realized for some time that this ansatz can be based on the nonchiral $SU(3)\otimes SU(3)$ group[5] for the symmetry group of mesons, the nonet being identified with the simplest nontrivial representation $3\otimes 3^*$ of $SU(3)\otimes SU(3)$. If we postulate that

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[1] A. H. Rosenfeld, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High-Energy Laboratory, Harwell, England, 1966).

[2] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); S. L. Glashow, *ibid.* **11**, 48 (1963); J. Schwinger, Phys. Rev. **135**, B816 (1964).

[3] S. Okubo, Phys. Letters **15**, 165 (1963).

[4] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[5] This group was introduced, within the context of definite dynamical schemes, by several authors: J. Schwinger, Phys. Rev. Letters **12**, 237 (1964); Ref. 2; F. Gürsey, T. D. Lee, and M. N. Nauenberg, Phys. Rev. **135**, B467 (1964); A. Salam and J. C. Ward, *ibid.* **136**, B763 (1964).

the symmetry breaking is proportional to the difference of the hypercharge generators of the two $SU(3)$ groups, the resulting mass formula coincides with the octet mass formula supplemented by a suitable mixing of the $I=Y=0$ components.[6] The corresponding physical masses now satisfy an equal spacing rule and also display a degeneracy between the $I=1$, $Y=0$ state with one of the $I=Y=0$ states (ρ, ω degeneracy).

This model can be extended to other representations of $SW(3)=SU(3)\otimes SU(3)$. The nonet representation $(3,3^*)$ is the first nontrivial member of a more general class of $SW(3)$ representations (d,d^*) which have been studied before.[6] With the assumption that the $SW(3)$ breaking is proportional to the hyperchargelike generator of $SW(3)$ the mass-formula for these multiplets have been already derived. In an earlier note, these considerations were applied to the next higher representation $(6,6^*)$ of $SW(3)$; it was found that the mass formula leads to an equal-spacing rule and to degeneracy of several components, in particular to the ρ, ω degeneracy.[7]

It is the purpose of this paper to investigate the application of $SW(3)$ to mesons; we demonstrate that the equal spacing rule and the degeneracy of certain meson masses is a general consequence of the $SW(3)$ violation. We study the question of coupling of $SW(3)$ multiplets and deduce the values of mixing angles for various cases. In particular, we find it necessary to consider two distinct types of trilinear meson couplings. The first type is rigorously forbidden in the $SW(3)$ limit, so that the meson decay occurs entirely via symmetry violation. The $SW(3)$-violating, $SU(3)$-symmetric meson coupling in this case turns out to be identical with Okubo's coupling scheme.[3] The second type of three-meson coupling is allowed in the symmetry-limit and the $SW(3)$ invariant coupling does not reduce to Okubo's scheme. It, however, leads to qualitatively the same conclusions. Examples of these two types of coupling occur, respectively, in the decay of a vector and a tensor meson into two pseudoscalar mesons. The most interesting aspect of this distinction is the fact that the mixing angles corresponding to these two cases are opposite in sign. The physical origin of this phenomenon is traced in detail. Finally, the possibility of realizing a 36-plet $(6,6^*)$ for vector and tensor mesons is discussed at length and their relevance to the existing experimental situation summarized.

II. $SW(3)$ MASS FORMULA AND $SU(3)$ REPRESENTATION MIXING

We consider a class of $SW(3)$ multiplets of the form (d,d^*), where d is a symmetric tensor (triangular) representation of $SU(3)$ with the dimension

$$d=\tfrac{1}{2}(r+1)(r+2)\,,\quad r=0,\,1,\,2,\,\cdots.$$

[6] S. K. Bose, Phys. Rev. **150**, 1231 (1966).

[7] S. K. Bose, Nuovo Cimento **46**, 419 (1966).

The representation d contains the $SU(2)$ representations with isospins

$$I=0,\,\tfrac{1}{2},\,1,\,\cdots\tfrac{1}{2}r$$

once each. The isospin I states have hypercharges

$$Y=2I-\tfrac{2}{3}r=(2I+1)-\tfrac{1}{3}(1+8d)^{1/2}.$$

For the d^* representation the isospins remain the same and the hypercharges change sign. The $SW(3)$ representation (d,d^*) has the decomposition

$$(d,d^*)\rightarrow \mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27}\oplus\cdots$$

in terms of self-conjugate representations of $SU(3)$ represented by traceless mixed tensors,

$$(0,0)=\mathbf{1}\,,\quad (1,1)=\mathbf{8}\,,\quad (2,2)=\mathbf{27}\,,$$
$$(3,3)=\mathbf{64}\,,\quad (4,4)=\mathbf{125}\,,$$

and so on. The highest $SU(3)$ representation contained in the $SW(3)$ representation is

$$(r,r)=(r+1)^3=\tfrac{1}{8}[(1+8d)^{1/2}-1]^3.$$

Each of these self-conjugate representations occurs only once.

The general mass formula for the $SW(3)$ multiplets is derived by considering the matrix elements of the mass operator

$$\mathfrak{M}=a+b\mathcal{Y} \tag{2.1}$$

to get

$$\langle n|\mathfrak{M}|n\rangle=a+\frac{2b(2r+3)}{(2n+1)(2n+3)}\times[I(I+1)-\tfrac{1}{4}Y^2-\tfrac{1}{3}n(n+2)]\,, \tag{2.2}$$

$$\langle n|\mathfrak{M}|n-1\rangle=\frac{b}{2(2n+1)}\times[(n-\tfrac{1}{2}Y)(n-\tfrac{1}{2}Y+1)-I(I+1)]^{1/2}\times[(n+\tfrac{1}{2}Y)(n+\tfrac{1}{2}Y+1)-I(I+1)]^{1/2}\times\left[\frac{(2r+3)^2-(2n+1)^2}{n(n+1)}\right]^{1/2}, \tag{2.3}$$

where $|n\rangle$ stands for the self-conjugate representation of $SU(3)$ with dimensions $(1+n)^3$. In this mass formula a, b are arbitrary numbers characterizing the average mass (squared) of the multiplet and the level spacing respectively, and r determines the dimensionality $\tfrac{1}{4}(r+1)^2(r+2)^2$ of the $SW(3)$ representation.

It has been shown before[6] that the mass formula leads to the same splitting for a nonet as the Okubo ansatz; the two $I=Y=0$ states together with the $I=\tfrac{1}{2}$, $Y=1$ state obey an equal spacing rule and the $I=1$, $Y=0$ state is degenerate with one of the $I=0$, $Y=0$ states. For the $(6,6^*)$, $r=2$ and we have mixing between the **1**, **8**, **27** contained in the multiplet through $\mathbf{1}\leftrightarrow\mathbf{8}$ and $\mathbf{8}\leftrightarrow\mathbf{27}$ mixing. We may define for the $I=Y=0$

members three physical states L_1, L_2, L_3 obtained as the eigenstates of the mass matrix; the corresponding masses are the eigenvalues obtained by solving a cubic equation. The roots are

$$\lambda_1=a+\tfrac{4}{3}b\,,$$
$$\lambda_2=a-\tfrac{2}{3}b\,,$$
$$\lambda_3=a-(8/3)b\,.$$

The states L_1, L_2, L_3 are mixtures of the $I=Y=0$ members of the **1**, **8**, **27** representations according to

$$L_1=(1/\sqrt{2})A+(\sqrt{\tfrac{2}{5}})B+(\sqrt{\tfrac{1}{10}})C\,,$$
$$L_2=(1/\sqrt{3})A-(1/15)^{1/2}B-(\sqrt{\tfrac{3}{5}})C\,,$$
$$L_3=(1/\sqrt{6})A-2(2/15)^{1/2}B+(\sqrt{\tfrac{3}{10}})C\,,$$

where A, B, and C are the $I=Y=0$ members of **1**, **8**, and **27**, respectively. All the other members of the **8** mix with the corresponding elements of **27**. If we denote the physical states by $m(I,Y)$ and $\tilde{m}(I,Y)$ we have the masses (squared)

$$m(I,Y)=a+\tfrac{2}{3}[I(I+1)-\tfrac{1}{4}Y^2-\tfrac{3}{2}]b+\tfrac{1}{3}\{[I(I+1)-\tfrac{1}{4}Y^2]^2-4I(I+1)-(5/4)Y^2+13\}^{1/2}b\,,$$
$$\tilde{m}(I,Y)=a+\tfrac{2}{3}[I(I+1)-\tfrac{1}{4}Y^2-\tfrac{3}{2}]b-\tfrac{1}{3}\{[I(I+1)-\tfrac{1}{4}Y^2]^2-4I(I+1)-(5/4)Y^2+13\}^{1/2}b\,,$$

and the physical states are mixture of **8** and **27** according to

$$m(I=1,\,Y=0)=(2/\sqrt{5})B(I=1,\,Y=0)+(1/\sqrt{5})C(I=1,\,Y=0)\,,$$
$$m(I=\tfrac{1}{2},\,Y=\pm1)=(\sqrt{\tfrac{3}{5}})B(I=\tfrac{1}{2},\,Y=\pm1)+(\sqrt{\tfrac{2}{5}})C(I=\tfrac{1}{2},\,Y=\pm1)\,,$$
$$\tilde{m}(I=1,\,Y=0)=(1/\sqrt{5})B(I=1,\,Y=0)-(2/\sqrt{5})C(I=1,\,Y=0)\,,$$
$$\tilde{m}(I=\tfrac{1}{2},\,Y=\pm1)=(\sqrt{\tfrac{2}{5}})B(I=\tfrac{1}{2},\,Y=\pm1)-(\sqrt{\tfrac{3}{5}})C(I=\tfrac{1}{2},\,Y=\pm1)\,,$$

where B, C denote pure **8** and **27** type states. The $(0,\pm2)$, $(2,0)$, and $(\tfrac{3}{2},\pm1)$ states evidently do not mix with any other. We get the mass spectrum

$$m(2,0)=m(1,0)=L_1=a+\tfrac{4}{3}b\,, \tag{2.4}$$

$$\tilde{m}(1,0)=m(1,\,\pm2)=L_2=a-\tfrac{2}{3}b\,, \tag{2.5}$$

$$m(\tfrac{1}{2},\,\pm1)=m(\tfrac{3}{2},\,\pm1)=a+\tfrac{1}{3}b\,, \tag{2.6}$$

and the equal-spacing rule

$$L_3-\tilde{m}(\tfrac{1}{2},\,\pm1)=\tilde{m}(\tfrac{1}{2},\,\pm1)-L_2=L_2-m(\tfrac{1}{2},\,\pm1)\quad m=(\tfrac{1}{2},\,\pm1)-L_1 \tag{2.7}$$

can be easily verified.

A similar calculation can be carried out for the spectrum of the $SW(3)$ representation $(10,10^*)$. In this case we have four $I=Y=0$ states. Most other states also mix. The physical states have the following spectrum:

$(3,0)$, $(2,0)$, $(1,0)$, $(0,0)$	at $m=a+2b$,
$(\tfrac{5}{2},\pm1)$, $(\tfrac{3}{2},\pm1)$, $(\tfrac{1}{2},\pm1)$	at $m=a+b$,
$(2,0)$, $(1,0)$, $(0,0)$, $(2,\pm2)$, $(1,\pm2)$	at $m=a$,
$(\tfrac{3}{2},\pm1)$, $(\tfrac{3}{2},\pm1)$, $(\tfrac{1}{2},\pm1)$	at $m=a-b$,
$(1,0)$, $(0,0)$, $(1,\pm2)$	at $m=a-2b$,
$(\tfrac{1}{2},\pm1)$	at $m=a-3b$,
$(0,0)$	at $m=a-4b$.

It is clear that the degeneracy and equal spacing rules that were valid for the $(3,3^*)$ and $(6,6^*)$ representations of $SW(3)$ continue to be valid for the $(10,10^*)$ representation. Similar construction can be carried out for any representation (d,d^*).

III. ORIGIN OF THE REPRESENTATION MIXING

As long as the $SW(3)$ symmetry is broken by a term proportional to the hypercharge, a degeneracy in the resulting masses could have been anticipated: This is based on the observation that the symmetry violation of $SU(3)\otimes SU(3)$ leaves the $SU(2)\otimes SU(2)$ subgroup as an invariance group. Hence, the $SW(2)$ multiplets continue to be degenerate in energy. All these states have the same hypercharge but the isospins differ from each other in integral steps. Each isospin I_1 from $SU_1(2)$ and I_2 from $SU_2(2)$ in the structure $SW(2)=SU_1(2)\otimes SU_2(2)$ would lead to all isospins I satisfying $I_1+I_2\geq I\geq|I_1-I_2|$ each isospin occurring only once. Strictly speaking, the invariance groups are really $SU_1(2)\otimes U_1(1)\otimes SU_2(2)\otimes U_2(1)$ which implies $Y=Y_1+Y_2$ also. The degeneracy so predicted would be true not only for the class (d,d^*) of representations of $SW(3)$ with d being a triangular representation of $SU(3)$ but in the general case of $SW(3)$ representations. It is also valid for symmetry violations more general than the one we have considered.

The spacing rule as well as the complete degeneracy do depend, however, on the precise nature of the symmetry violation. Thus, for example, for the $(6,6^*)$ representation, the $Y=\pm2$ states are degenerate with two $Y=0$ states. For the $(10,10^*)$ case this degeneracy is augmented by other degeneracies between members with different hypercharges. These "accidental" degeneracies as well as the equal-spacing rule are consequences of the requirement that the symmetry violation be proportional to $\mathcal{Y}=Y_1-Y_2$. If we had chosen it to contain another term we could generate a mass spectrum in which these "accidental" degeneracies would be lifted and for which the simple equal spacing rule would not hold. But all the consequences of $SW(2)$ invariance would continue to be valid.

An arithmetical computational scheme can be used to get the physical states and the mass spectrum. One proceeds as follows: Decompose d into isotopic multiplets (I_1,Y_1). In the tensor notation these are distinguished by the number of "3" indices occurring in it. Do the same with d^*. Then the hypercharge difference is given by the sum of the number of upper "3" indices (from d) and the number of lower "3" indices (from d^*). Thus for the $(3,3^*)$ nonet the representation $G_\nu{}^\mu$ breaks up according to

$$G_\nu{}^\mu \to G_j{}^j \oplus G_3{}^3 \quad 1\le\mu,\nu\le 3$$
$$\oplus G_3{}^j \oplus G_k{}^3 \quad 1\le j,k\le 2\,.$$

This shows that the physical states with $I=Y=0$ are $(1/\sqrt{2})G_j{}^j$ and $G_3{}^3$ rather than a pure singlet and octet. We can write

$$(1/\sqrt{2})G_j{}^j=(\sqrt{\tfrac{2}{3}})A+(1/\sqrt{3})B\,,$$
$$G_3{}^3=(1/\sqrt{3})A-(\sqrt{\tfrac{2}{3}})B\,.$$

For the vector mesons this corresponds to the familiar relation

$$|\omega\rangle=(\sqrt{\tfrac{2}{3}})|\omega_0\rangle+(1/\sqrt{3})|\phi_0\rangle\,,$$
$$|\phi\rangle=(1/\sqrt{3})|\omega_0\rangle-\sqrt{\tfrac{2}{3}}|\phi_0\rangle\,.$$

We note also that in this scheme the $I=1$, $Y=0$ and $I=0$, $Y=0$ members of the quartet representation of $SW(2)$ are degenerate implying the degeneracy of ω and ρ.

For the $(6,6^*)$ representation we have the $SW(3)$ breaking according to

$$G_{\nu\lambda}{}^{\mu\sigma}\to G_{km}{}^{jl}\oplus\{G_{k3}{}^{jl}+G_{km}{}^{j3}\}\oplus\{G_{33}{}^{jl}+G_{km}{}^{33}+G_{k3}{}^{j3}\}$$
$$\oplus\{G_{k3}{}^{33}+G_{33}{}^{j3}\}\oplus G_{33}{}^{33}\,.$$

Re-expressing these components in terms of the **1**, **8**, **27** representations, we can explicitly display the mixing among these representations.

A similar computational scheme can be worked out for other (not necessarily self-conjugate) representations $SW(3)$. We have yet to consider the question of particle assignments and the question of coupling between particles. These questions are obviously related. We start by considering the simple case where mesons are assigned to nonets.

IV. STRUCTURE OF THREE-MESON COUPLINGS

The $SW(3)$ invariant interaction for the reaction

$$A\to B+C \tag{4.1}$$

is essentially unique and is given by

$$gA_\alpha{}^\lambda\{B_\mu{}^\beta C_\nu{}^\gamma\epsilon_{\alpha\beta\gamma}\epsilon^{\lambda\mu\nu}\}^*+\text{H. c.} \tag{4.2}$$

This may be rewritten in the form

$$g[A_\lambda{}^\lambda\{B_\mu{}^\mu C_\nu{}^\nu-B_\nu{}^\mu C_\mu{}^\nu\}^*-A_\mu{}^\lambda\{B_\mu{}^\lambda C_\nu{}^\nu+B_\nu{}^\nu C_\mu{}^\lambda\}^*$$
$$+A_\nu{}^\lambda\{B_\nu{}^\mu C_\mu{}^\lambda+B_\mu{}^\lambda C_\nu{}^\mu\}^*]+\text{H. c.} \tag{4.3}$$

This equation shows that the $SW(3)$ invariant 3-nonet coupling (4.2) does not reduce to the coupling scheme which is obtained by the Okubo ansatz (in which the traces $A_\lambda{}^\lambda$, etc., are not allowed to occur explicitly). It may, however, happen that such a coupling may be forbidden for certain transitions. We examine the reaction (4.1) for various physical processes.

A. Decay of Vector Mesons into Two Pseudoscalar Mesons

In this case A is a vector meson and B, C are pseudoscalar mesons, so that we may put

$$A_\alpha{}^\lambda=V_\alpha{}^\lambda+(1/\sqrt{3})V\delta_\alpha{}^\lambda\,,$$
$$B_\alpha{}^\lambda=C_\alpha{}^\lambda=P_\alpha{}^\lambda+(1/\sqrt{3})P\delta_\alpha{}^\lambda\,. \tag{4.4}$$

In this case the entire 3-nonet coupling vanishes in the $SW(3)$ limit because of charge-conjugation (C) invariance. To see this it is sufficient to note that the first term of (4.3) contains the unitary singlet vector meson decaying into two pions, which is forbidden. More generally, we may observe that C invariance require an F-type coupling of the pseudoscalar octets while $SW(3)$ invariance leads to a pure D-type coupling. Hence, the entire decay transition must be through $SW(3)$ violating interactions which involve an $SU(3)$ invariant C-invariant interaction. For the vector octet part such an interaction is essentially unique and given by

$$fV_\nu{}^\lambda\{P_\nu{}^\mu(1)P_\mu{}^\lambda(2)-P_\mu{}^\lambda(1)P_\nu{}^\mu(2)\}^*+\text{H. c.} \tag{4.5}$$

This expression in turn could be rewritten in the form

$$fA_\nu{}^\lambda\{B_\nu{}^\mu C_\mu{}^\lambda-B_\mu{}^\lambda C_\nu{}^\mu\}^*+\text{H. c.} \tag{4.6}$$

which is exactly what we would have obtained using the Okubo ansatz. In particular, the mixing angle given by our broken $SW(3)$ model would be the same as the value given already by Okubo.

B. Decay of Tensor Mesons into Vector Plus Pseudoscalar Mesons

Because of the particular charge-conjugation properties (all octets being normal octets) this case is identical with the above case. The decay proceeds via $SW(3)$-violating couplings in accordance with the Okubo scheme.

C. Decay of Tensor Mesons into Two Pseudoscalar Mesons

In this case the $SW(3)$-invariant interaction is also charge-conjugation invariant. The coupling is not the same as given by the Okubo ansatz, but is rather given by (4.3). Rewriting it in an obvious notation we get

$$g\,\text{Tr}(T_8[P_8(1)P_8(2)]_+{}^*)$$
$$-(1/\sqrt{3})T_1\,\text{Tr}(P_8(1)P_8(2))^*$$
$$+(2/\sqrt{3})T_1(P_1(1)P_1(2))^*$$
$$-(2/\sqrt{3})P_1\,\text{Tr}(T_8P_8)^*\,. \tag{4.7}$$

Comparing (4.7) with the coupling scheme of Glashow and Sokolow,[4] we see that we can still obtain suppression of the rate for the decay

$$f' \rightarrow 2\pi,$$

but the mixing angle be chosen negative.[8] In a purely phenomenological treatment there is no difficulty with a negative mixing angle since the masses determine the mixing angle up to a sign.

To understand the marked difference between the $SW(3)$-breaking interaction for vector-meson decays and the $SW(3)$ invariant interaction for tensor meson decays into two pseudoscalar mesons as to the sign of the mixing angle, we proceed as follows: for the $SW(3)$-breaking coupling (4.5) we have

$$V_\beta{}^\alpha[P_\nu{}^\alpha(1)P_\beta{}^\nu(2)-P_\beta{}^\nu(1)P_\nu{}^\alpha(2)]^*,$$

and hence for the decay of the *physical state* of the ϕ meson we have $\alpha=\beta=3$ so that we must have two kaons in the final state since both the pseudoscalar mesons must have an index "3." Similarly, the physical state of the ω meson has $\alpha, \beta=1, 2$ so that ω is forbidden to decay into kaons. The important observation is that according to (4.5) the meson $A_3{}^3$ is the linear combination

$$A_3{}^3=V_3{}^3+(1/\sqrt{3})V, \tag{4.8}$$

so that according to the definition

$$\phi=\omega_0\cos\theta-\phi_0\sin\theta,$$
$$\omega=\omega_0\sin\theta+\phi_0\cos\theta,$$

we have the mixing angle

$$\theta=\sin^{-1}(1/\sqrt{3}). \tag{4.9}$$

For the $SW(3)$ invariant coupling (4.3)

$$T_\alpha{}^\lambda[P_\mu{}^\beta(1)P_\nu{}^\gamma(2)\epsilon_{\alpha\beta\gamma}\epsilon^{\lambda\mu\nu}]^*,$$

the situation is quite different. Because of the two factors $\epsilon_{\alpha\beta\gamma}$ and $\epsilon^{\lambda\mu\nu}$, the physical state $T_3{}^3$ cannot decay into kaons and must therefore decay into pions. At the same time the components $T_k{}^j$ must decay into kaons. This situations is the opposite of what happened with $SW(3)$-breaking couplings. We have therefore to identify the f meson which decays predominantly into pions with $T_3{}^3$ and the f' meson with $T_k{}^k$. Thus f and f' have the same $SW(3)$ transformation properly as ϕ and ω. This interchange in the roles of f and f' corresponds to the redefinition

$$\sin\theta \rightarrow -\cos\theta, \quad \cos\theta \rightarrow \sin\theta$$

which is the same as

$$\theta \rightarrow \theta-\tfrac{1}{2}\pi.$$

Hence with $T_3{}^3$ being identified with f, the customary definition of the mixing angle is

$$\theta_2=-90°+\tan^{-1}(1/\sqrt{2})=-\tan^{-1}\sqrt{2}. \tag{4.10}$$

We also note that the mass differences are inverted with this identification: $T_k{}^k$ corresponds to the heavier meson and $T_3{}^3$ to the lighter one. If one identifies the A_2 resonance with $T_l{}^k$ we have violation of the simple mass formula. If we need to use a more general mass formula, the physical states would be different and the selection rules just outlined will not hold without modification.

D. Decay of a Vector (Tensor) Meson into a Vector (Tensor) Plus Pseudoscalar Meson

These decays are allowed in $SW(3)$ limit and the discussion proceeds exactly as in the case of a tensor decay into two pseudoscalars. The rates of the decays

$$f' \rightarrow A_2+\pi,$$
$$\phi \rightarrow \rho+\pi$$

are suppressed in an analogous manner.

Thus it turns out that while the $SW(3)$ invariant interactions do not generate the couplings in accordance with the Okubo ansatz, it gives qualitatively the same predictions like suppression of $f' \rightarrow 2\pi$, $\phi \rightarrow \rho\pi$, etc. This also reveals that $SW(3)$-breaking interactions are not small compared with $SW(3)$-invariant interactions; it has been known that Yukawa couplings of mesons could not be $SW(3)$ invariant because of the absence of a baryon nonet. The Yukawa couplings and vector meson decays proceed through similar interactions.

Finally, the following should be reiterated: One is traditionally used to identifying the physical meson states with the components of a mixed tensor without any reference to the type of interaction in which the meson participates. Our discussion has shown that this need not, in fact, be unique. The identification of the physical meson state is to be done only with reference to the (strong) decay state of the meson, i.e., with reference to the two-particle channel in which the meson can show up as a resonance. Thus the identification of the physical ω and ϕ states get interchanged while discussing $\phi \rightarrow \rho+\pi$ decays as compared to $\phi \rightarrow 2\pi$, $K\bar{K}$ decays. This interchange corresponds to the reversal of the sign of the mixing angle, which is brought about by $SW(3)$ violation. This is a noval situation which baffles our customary intuition; but is intimately related to the structure of the $SW(3)$ group. In fact, the $SU(3)$-invariant coupling (4.5), supplemented by the identification of physical meson states, contain information about representation mixing.[9] Thus symmetric coupling contain the seeds of symmetry violation. This situation was foreseen by Sakurai,[2] who suggested that $SU(3)$ violation be brought about by ω-ϕ mixing.

[8] A similar situation has been noticed in the *Chiral* $SU(3)\otimes SU(3)$ symmetry. See W. P. Moran and R. E. Marshak, Progr. Theoret. Phys. (Kyoto), supplement in honor of Professor S. Tomonaga (to be published).

[9] For general discussion of representation mixing in $SU(3)$, see N. Mukunda, A. J. Macfarlane, and E. C. G. Sudarshan, Phys. Rev. 138, B665 (1965).

V. POSSIBLE MESON ASSIGNMENTS ACCORDING TO $SW(3)$

Let us take pseudoscalar mesons in (3,3*) with vector mesons and tensor mesons in (6,6*). The decay vector → pseudoscalar+pseudoscalar cannot go through the $SW(3)$ invariant coupling since it violates C invariance. This can be seen by recalling that C invariance requires an F-type coupling of the octet part of the vector meson 36-plet with the pseudoscalar mesons. But since the coupling

$$A_{\mu\nu}{}^{\alpha\beta}\{B_\mu{}^\alpha C_\nu{}^\beta+B_\nu{}^\beta C_\mu{}^\alpha\}^*+\text{H. c.} \tag{5.1}$$

is essentially unique and obviously symmetric in B and C, it cannot yield such an F-type coupling. This decay should then proceed by $SW(3)$-breaking coupling. This coupling also happens to be unique: The $SU(3)$-invariant coupling of the nonets B and C which is antisymmetric in B and C gives two octets

$$\{B_\sigma{}^\mu C_\nu{}^\sigma-B_\nu{}^\sigma C_\sigma{}^\mu\}\,,\quad \{B_\nu{}^\mu C_\sigma{}^\sigma-B_\sigma{}^\sigma C_\nu{}^\mu\}$$

and a **10** and **10***. However, in the decomposition of the 36-plet of $SW(3)$ with respect to $SU(3)$ we get **27**, **8**, **1**; the octet part is given by

$$A_{\nu\tau}{}^{\mu\tau}-\tfrac{1}{3}A_{\sigma\tau}{}^{\sigma\tau}\delta_\nu{}^\mu.$$

Hence the $SW(3)$-breaking $SU(3)$-invariant, C-invariant coupling is given by

$$f_1A_{\nu\tau}{}^{\mu\tau}\{B_\sigma{}^\mu C_\nu{}^\sigma-B_\nu{}^\sigma C_\sigma{}^\mu\}^* +f_2A_{\nu\tau}{}^{\mu\tau}\{B_\nu{}^\mu C_\sigma{}^\sigma-B_\sigma{}^\sigma C_\nu{}^\mu\}^*. \tag{5.2}$$

If we appeal to Okubo's prescription for the pseudoscalar nonet we should put $f_2=0$.

The coupling (5.2) has the very interesting property that the *vector* meson 27-plet would not be coupled to pairs of pseudoscalar mesons and cannot therefore be found by searching for two meson resonances. In the physical case we have to consider the effect of $SW(3)$ breaking leading to mixing of $SU(3)$ representations, but they mix only states with the same I, I_3, and Y in the different representations. The splitting of the 36-plet has already been discussed. For the pseudoscalar nonet it is appropriate to neglect the $SU(3)$ mixing in the first approximation and treat η and η^* to belong to **8** and **1**, respectively.

We can now consider the assignment of vector mesons to the 36-plet; a possible choice is

$$A_{kl}{}^{kl}=\omega(782)\,,\quad A_{k3}{}^{k3}=\phi(1020)\,,\quad A_{kl}{}^{k3}=K^*(890)$$
$$A_{kl}{}^{jl}-\tfrac{1}{3}A_{ml}{}^{ml}\delta_k{}^j=\rho(769)\,.$$

Other possible candidates are

$$A_{3l}{}^{33}=K^{*\prime}\sim 1100\text{ MeV}\,,\quad A_{33}{}^{33}=\phi'\sim 1200\text{ MeV}\,,$$
$$A_{k3}{}^{j3}-\tfrac{1}{3}\delta_k{}^iA_{33}{}^{33}=\rho'\sim 1000\text{ MeV}\,,$$

and the other states with $(I,Y)=(2,0)$, $(\frac{3}{2},\pm1)$, $(1,\pm2)$ which cannot show up as a two-meson resonance. In fact, the entire 27-plet of $SU(3)$ cannot show up in C-invariant, $SW(3)$-violating decays of vector mesons since $(8\times8)_{\text{antisymmetric}}$ does not contain 27. There is some evidence for a second ρ meson at a mass of about 965 MeV; this evidence together with indirect theoretical evidence from an analysis of π-p charge-exchange polarization for a second ρ meson has been recently summarized by Logan, Beaupre, and Sertorio.[10] There is at present no clear evidence for the existence of $K^{*\prime}$ or ϕ'.

Turning to the decay of tensor mesons in a (6,6*), we find the decay

$$\text{tensor}\to\text{vector}+\text{pseudoscalar}$$

is now forbidden by $SW(3)$ invariance. So we have to again appeal to $SW(3)$ breaking. We shall not consider this decay further here, but instead study the more interesting decay

$$\text{tensor}\to\text{pseudoscalar}+\text{pseudoscalar}\,,$$

which involves the coupling

$$(6,6^*)\to(3,3^*)\times(3,3^*)\,.$$

A unique C-invariant $SW(3)$-symmetric coupling can be written down in the form

$$gA_{\mu\nu}{}^{\alpha\beta}[B_\mu{}^\alpha C_\nu{}^\beta+B_\nu{}^\beta C_\mu{}^\alpha]^*. \tag{5.3}$$

In the presence of symmetry violation in accordance with our ansatz (that is, through $\mathcal{Y}$) we would have the 36-plet break up as discussed in Sec. II above. The coupling (5.3) can now be seen to lead to the following consequences:

$A_{33}{}^{33}$ can only decay into $B_3{}^3$ and $C_3{}^3$,
$A_{33}{}^{k3}$ can only decay into $B_3{}^k$ and $C_3{}^3$,
$A_{33}{}^{kl}$ can only decay into $B_3{}^k$ and $C_3{}^l$,
$A_{k3}{}^{j3}$ can only decay into $B_k{}^3$ and $C_3{}^j$,
$A_{k3}{}^{jl}$ can only decay into $B_k{}^j$ and $C_3{}^l$,
$A_{km}{}^{jl}$ can only decay into $B_k{}^j$ and $C_m{}^l$.

This means that there is an $I=Y=0$ particle that can decay only into $\eta\eta$, $\eta^*\eta$, or $\eta\eta^*$ final states; another $I=Y=0$ particle then can decay into $K\bar{K}$ final states or into $\eta\eta$, $\eta\eta^*$, or $\eta^*\eta^*$ final states but *not* into pions; and a third $Y=I=0$ particle which cannot decay into kaons. Among the $I=1$, $Y=0$ states there is a particle that can decay into $K\bar{K}$ or $\eta\eta^*$, $\eta\eta$, $\eta^*\eta^*$ but *not* into pions; and another $I=1$, $Y=0$ particle that can decay only into nonstrange mesons but not into $K\bar{K}$.

For the tensor mesons there are two possibilities. The better known spin-2+ mesons are $A_2(1300)$,

[10] R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters **18**, 259 (1967). Further references on ρ' are cited in this paper.

$f(1250)$, $K^*(1400)$, $f'(1525)$. There are some indications of a strange resonance with $I=\frac{3}{2}$ at about 1170 MeV [11] and a nonstrange resonance at about 1670 MeV.[12] Several new resonances should be discovered and their spin-parity established before a 36-plet can be completed. We present two provisional assignment of particles:

Scheme (i)

$$f(1250)=A_{kl}{}^{kl},\quad f'(1525)=A_{k3}{}^{k3},\quad K^*(1400)=A_{33}{}^{3j},$$
$$A_2(1300)=A_{k3}{}^{j3}-\tfrac{1}{3}\delta_k{}^jA_{l3}{}^{l3}.$$

The remaining states are

$$f''(I=0,\,Y=0)=A_{33}{}^{33}\qquad(\sim1000),$$

$$A_2'(I=1,\,Y=0)=A_{jm}{}^{jl}-\tfrac{1}{3}\delta_m{}^lA_{jk}{}^{jk}\qquad(\sim1000),$$

$$J(I=2,\,Y=0)=A_{km}{}^{jl}-\tfrac{1}{5}(\delta_k{}^jA_{mi}{}^{li}+\delta_k{}^lA_{mi}{}^{ji}+\delta_m{}^jA_{ki}{}^{li}+\delta_m{}^lA_{ki}{}^{ji})+(1/20)(\delta_k{}^j\delta_m{}^l+\delta_k{}^l\delta_m{}^j)A_{in}{}^{in}\qquad(\sim1000),$$

$$K^{*\prime}(I=\tfrac{3}{2},\,Y=1)=A_{km}{}^{j3}-\tfrac{1}{5}(\delta_k{}^jA_{ml}{}^{3l}+\delta_m{}^jA_{kl}{}^{3l})\qquad(\sim1150),$$

$$K^{*\prime\prime}(I=\tfrac{1}{2},\,Y=1)=A_{km}{}^{k3}\qquad(\sim1150),$$

$$L(I=1,\,Y=2)=A_{33}{}^{kl}\qquad(\sim1275).$$

In this scheme $f'\to2\pi$ would be suppressed at the same time as $f'\to K\bar{K}$ both of which are desirable. But $A_2\to\rho+\pi$ is also suppressed which is undesirable, as experimentally this decay has a partial width of about 70 MeV. We may therefore consider another possible assignment.

Scheme (ii)

$$f(1250)=A_{jk}{}^{jk},$$
$$A_2(1300)=A_{kl}{}^{jl}-\tfrac{1}{3}\delta_k{}^jA_{lm}{}^{lm},$$

$$J(I=2,\,Y=0)=A_{km}{}^{jl}-\tfrac{1}{5}(\delta_k{}^jA_{mn}{}^{ln}+\delta_k{}^lA_{mn}{}^{jn}+\delta_m{}^jA_{kn}{}^{ln}+\delta_m{}^lA_{kn}{}^{jn})+(1/20)(\delta_k{}^j\delta_m{}^l+\delta_k{}^l\delta_m{}^j)A_{in}{}^{in}\qquad(\sim1275),$$

$$K^*(1400)=A_{kl}{}^{k3},\quad f'(1525)=A_{jk}{}^{jk},$$
$$K^{*\prime}(I=\tfrac{3}{2},\,Y=1)=A_{km}{}^{j3}-\tfrac{1}{5}(\delta_k{}^jA_{ml}{}^{3l}+\delta_m{}^jA_{kl}{}^{3l})\qquad(\sim1400),$$

$$A_2'(I=1,\,Y=0)=A_{l3}{}^{j3}-\tfrac{1}{3}\delta_l{}^jA_{k3}{}^{k3},\qquad(\sim1525),$$

$$L(I=1,\,Y=2)=A_{33}{}^{kl}\qquad(\sim1525),$$

$$K^{*\prime}(I=\tfrac{1}{2},\,Y=1)=A_{33}{}^{k3}\qquad(\sim1600),$$

$$f''(I=0,\,Y=0)=A_{33}{}^{33}\qquad(\sim1700).$$

We see that $f'\to2\pi$, $f\to K\bar{K}$ are suppressed and the difficulty with $A_2\to\rho+\pi$ removed. Some evidence for an $I=2$, $Y=0$ state at about 1300 MeV has been presented very recently.[13] This could be identified with J. The 2π resonance at 1670 MeV could be identified with our f'' only if we assign this decay to proceed via $SW(3)$ breaking. In such a case we would expect a reasonable amount of $\eta\eta$ and $\eta\eta^*$ decay modes for these particles.

For the vector mesons, we have already discussed the assignments.

VI. CONCLUDING REMARKS

The discussion in this paper has dealt more or less exclusively with $SW(3)=SU(3)\otimes SU(3)$ with no parity interchange. We could speculate on the origin of such a group by imagining the two $SU(3)$ groups being associated with the positive and negative frequency parts (particle and antiparticle parts) of the primitive ("quark") fields; such a splitting of the field is invariant under the $SU(3)$ group but is not local; it is covariant under charge-conjugation provided we require that the representation (d_1,d_2) of $SW(3)$ goes into (d_2^*,d_1^*) under charge-conjugation.[14] The particular representations (d,d^*) are self-conjugate; these are the representations we have chosen. The symmetry-breaking term $\mathcal{Y}$ is unchanged under charge conjugation in accordance with the invariance of the mass under charge conjugation. On the other hand, we would expect difficulties with crossing symmetry with such a nonlocal group: This is in fact true and we find that the coupling according to $SW(3)$ is not crossing-symmetric.[15] As a consequence we would expect that in any process where exchange of particles off mass-shell is important the symmetry is violated. The large amount of $SW(3)$ violation as seen from vector meson decay is then not very surprising. The baryons themselves cannot be assigned to a multiplet under $SW(3)$ unless we choose (8,1); but in this case, the Yukawa coupling is forbidden since it requires $(8,1)\to(8,1)\otimes(3,3^*)$. We observe, however, that this coupling requires at least one of the particles to be considerably off the mass-shell, so that $SW(3)$ violation is to be expected. It is also perhaps worthwhile to recall that at present, there exists no conclusive evidence against the pres-

[11] J. M. Bishop, A. T. Goshaw, A. R. Erwin, M. A. Thompson, W. D. Walker, and A. Weinberg, Phys. Rev. Letters **16**, 1069 (1966). Earlier reference is cited in this paper.

[12] CERN-Ecole Polytechnique-Orsay-Milan-Saclay collaboration. Phys. Letters **17**, 354 (1965); Aachen-Berlin-CERN collaboration, *ibid.* **18**, 351 (1965); Saclay-Orsay-Bologna collaboration, *ibid.* **19**, 65 (1965); B. C. Maglić *et al.*, *ibid.* **19**, 712 (1966); D. J. Crennell *et al.*, Phys. Rev. Letters **18**, 323 (1967).

[13] R. Vanderhagen, J. Huc, P. Fleury, J. Duboc, R. George, M. Goldberg, B. Makowski, N. Armenise, B. Ghidini, V. Picciarelli, A. Romano, A. Forino, R. Gessaroli, G. Quareni, and A. Quareni-Vignudelli, Phys. Letters **24B**, 493 (1967).

[14] For a discussion of the general problem of adjoining charge-conjugation to internal symmetry groups, see L. C. Biedenharn, J. Nuyts, and H. Ruegg, Commun. Math. Phys. **2**, 231 (1966); T. D. Lee and G. C. Wick, Phys. Rev. **148**, 1385 (1966).

[15] In this connection see also P. Carruthers, Phys. Rev. Letters **18**, 353 (1967); Y. S. Jin, Phys. Letters **24B**, 411 (1967); G. Fleming and E. Kazes, Phys. Rev. Letters **18**, 764 (1967).

ence of large $SU(3)$-breaking in meson-baryon coupling.[16]

The question of particle assignment and mass spectra would apply equally well for the *chiral* $SU(3)\otimes SU(3) = S\bar{W}(3)$ group, which may be thought of as being generated by the decomposition of the primitive (quark) fields into their chiral projections.[17] The mesons in this scheme have to be paired with regard to parity, though the breaking of $S\bar{W}(3)$ could lift the degeneracy between opposite parity mesons. Evidence for parity-$SU(3)$ mixing is not conclusive at the moment, although this might eventually turn out to be the case. It has been speculated that the useful symmetry group for hadrons could be as large as $SW(3)\otimes SW(3)$, with the factor $SW(3)$'s related by parity operation.[18] In such a case our discussion of meson symmetries is to be understood as being restricted to a useful subgroup of such a large symmetry (restricted to meson states with the same parity).

The interesting point that emerges from our study is the possible existence of mesons or even entire multiplets (like a vector 27-plet) which may not appear in two-meson channels. Multiplet assignments may have to await a more complete study of the boson mass spectrum.

Note added in proof. Since the submission of this paper Wu and Tuan have published a paper wherein the connection between isospin degeneracies for mesons and $SW(2)$ symmetry has been noted independently. S. F. Tuan and T. T. Wu [Phys. Rev. Letters **18**, 349 (1967)].

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[16] For the meson coupling to $J^P=\frac{1}{2}^+$ baryons it has been suspected for a long time that kaon couplings are systematically much smaller than pion couplings. This is supported by a determination of (ΛNK) coupling constant by forward dispersion relations due to M. Lusignoli, M. Restignoli, G. Snow, and G. Violini [Phys. Letters **21**, 229 (1966)]. More recently, Yodh has analyzed the consequences of a singlet assignment for the $Y_0^*(1520)$ and found substantial deviation from $SU(3)$ predictions in the branching ratio for $\Sigma\pi$ versus $N\bar{K}$ decays of this particle [G. Yodh, Phys. Rev. Letters **18**, 810 (1967)]. As for the meson couplings connecting $J^P=\frac{3}{2}^+$ baryons with $J^P=\frac{1}{2}^+$ baryons only the pion couplings are known at present (from observed decays of $\frac{3}{2}^+$ baryons) and these are in agreement with $SU(3)$ predictions. However, it is *possible* for this situation to co-exist with sizable $SU(3)$-violations; see S. K. Bose and Y. Hara, Phys. Rev. Letters **17**, 409 (1966).

[17] R. E. Marshak, N. Mukund, and S. Okubo, Phys. Rev. **137** B698 (1965); R. E. Marshak, S. Okubo, and J. Wojtaszek, Phys. Rev. Letters **15**, 463 (1965). This is also the group of vector and axial-vector currents. M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[18] A. Salam and J. C. Ward, Ref. 5; P. G. O. Freund and Y. Nambu, Ann. Phys. (N.Y.) **32**, 201 (1965).

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Superconvergence Relations for Pion-Baryon Scattering

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Superconvergence relations are discussed for the spin-flip amplitudes in pion-baryon scattering at $t=0$ and at $u=0$. These sum rules are evaluated in the resonance saturation approximation, and in this case an extension of the derivation is given. This allows the sum rules to be discussed even if the relevant leading Regge trajectory lies above zero at $t=0$ for mesons or above $-\frac{1}{2}$ at $u=0$ for baryons. We consider all 14 possible sum rules, as well as the moment sum rules, which require stronger assumptions. The spin-flip sum rules are well satisfied for $\pi N\to\pi N$, and in the other cases the ρ and pion couplings to Σ, Λ, and Ξ required to satisfy the relations are in agreement with other models. The moment sum rules are less convergent and so more difficult to test, but for πN the moment sum rule for $B^{(-)}$ at $t=0$ is not satisfied with resonance saturation, and this fact can be understood.

I. INTRODUCTION

ASSUMING that the asymptotic behavior of an invariant amplitude is given by the leading Regge poles, together with possible kinematic factors if there is helicity flip, the amplitude will decrease sufficiently fast in certain cases to allow a dispersion sum rule or superconvergence relation (SCR) to be derived. Such sum rules have been discussed in strong interactions for the systems $\rho\pi$,[1] $\Sigma\pi$,[2] $N^*\pi$,[3,4] NN,[5] and pion-baryon

[1] V. de Alfaro, S. Fubini, G. Furlan, and G. Rossetti, Phys. Letters **21**, 576 (1966) and Ann. Phys. (to be published).

[2] P. Babu, F. T. Gilman, M. Suzuki, Phys. Letters **24B**, 65 (1967).

[3] H. F. Jones and M. D. Scadron, Nuovo Cimento **48A**, 546 (1967).

[4] P. H. Frampton, Nucl. Phys. **B2**, 518 (1967).

[5] R. D'Auria and V. de Alfaro, Nuovo Cimento **48A**, 284 (1967).