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Theory of Universal Primary Interactions

by

E. C. G. SUDARSHAN

Syracuse University,
New York

This theory originates from observations that the theory of particle interactions becomes much simpler and more capable of correlating apparently unrelated phenomena if it is postulated that electromagnetism and weak interactions are not primary properties of the nucleons but are acquired by virtue of their interaction with vector and axial vector fields.

TEN years ago, Marshak and I analysed the then existing experimental data on weak interactions and concluded that all the experiments could not be consistently interpreted in terms of a general theoretical scheme[1]. We singled out four crucial experiments which should be remeasured with different results if a satisfactory theory of weak interactions was to be developed. We also showed that the only possible universal theory would be a vector-axial vector interaction. This was satisfying because the principle of chirality invariance[1] for the interaction, already valid for electromagnetism, unambiguously led to this interaction. Almost immediately Feynman and Gell-Mann[2] developed the same theoretical scheme and re-introduced the notion of a conserved vector current[3] to show why the vector coupling constant was not renormalized. The chirality invariant theory, which we formulated a decade ago[1,2], has been accepted as the theory of universal Fermi interaction. Even the small departure of the ratio of the vector and axial vector coupling constants from unity by a factor of approximately $\sqrt{(25/18)}$ can be computed on the basis of the original formulation of the theory[4].

Several new experimental developments[5], however, make it desirable to re-examine the theoretical basis for the universal four-fermion interaction. Within the realm of weak interactions, it has been found that the leptonic decays of kaons and hyperons are slower, by a factor of about ten, than the predictions of the universal four-fermion interaction extended to strange particles. There is now unmistakable evidence for a small violation of *CP* invariance in weak interactions leading to the neutral long lived kaon. Among strong interactions, the existence of a whole collection of vector mesons has been experimentally established. These mesons are strongly coupled to baryons and their contribution to low energy pion–nucleon scattering and the electromagnetic properties of the nucleons make them essential to an understanding of particle interactions. In fact, soon after the first vector meson resonances were experimentally identified, following a method suggested by us[6], Sakurai pointed out the qualitative explanation of some features of strong interactions based on the hypothesis of a universal vector meson coupling to hadrons (for discussion see refs. 7–10). Evidence has been accumulating for the validity of such a universal coupling. Finally, there has been some evidence for axial vector meson resonances, although they cannot yet be considered experimentally established[5].

Interaction Types and Families of Particles

In the present theory, I distinguish four families of particles and their associated fields. These four families have their characteristic interaction properties and are shown in Table 1.

Table 1. PARTICLE FAMILIES

Family	Particles	Interactions
Lepton	e, μ, ν_e, ν_μ	Weak, electromagnetic
Photon	γ	Electromagnetic
Baryon	$p, \quad n$	Strong
Meson	$\pi, \eta, A_1, \varrho, \omega, \varphi, E, D$	Strong, electromagnetic, weak

The pseudoscalar and pseudovector mesons are both quanta of the same axial vector field, and may be written

$$A_\lambda = B_\lambda + \xi \frac{1}{m\pi} \partial_\lambda \varphi_\pi; \quad \partial^\lambda B_\lambda = 0 \tag{1}$$

where φ_π is the pion field and B_λ is the transverse part associated with the pseudovector particles. As a consequence

$$\partial^\lambda A_\lambda = \xi \,.\, \frac{1}{m_\pi} \,.\, \Box^2 \varphi_\pi = -\xi \,.\, m_\pi \,.\, \varphi_\pi \tag{2}$$

where the second member is strictly valid only when the pion is on the "mass-shell". On the other hand, the vector field has only a vector particle because it is required that

$$\partial^\lambda V_\lambda = 0 \tag{3}$$

The nucleon field is denoted by N and the electromagnetic field by a_λ. It is now possible to formulate the fundamental primary interactions. For simplicity, in the first instance, the strange particles are ignored.

(*i*) *Electromagnetism*

The electrons and muons are coupled to the Maxwell field, a_λ, according to the usual form

$$-\mathrm{e}\,(\bar{\mu}\gamma_\lambda\,\mu + \bar{\mathrm{e}}\,\gamma_\lambda\,\mathrm{e})\,a^\lambda \tag{4}$$

The nucleons are not directly coupled to the Maxwell field, but the neutral vector fields ρ and ω are coupled according to

$$-\,(\mathrm{e}/g)\,(m_\varrho^2 \,.\, \rho_\lambda + m_\omega^2 \,.\, \omega_\lambda) \tag{5}$$

where g is a strong coupling constant, the value of which is specified later.

(*ii*) *Strong Interactions*

The leptons do not have any strong interactions. The strong interactions are completely specified by the Yukawa coupling

$$\begin{aligned} \tfrac{1}{2}\, g\, \overline{N}\, \{ & \gamma^\lambda\, \boldsymbol{\tau}.\boldsymbol{\rho}_\lambda + (g'/g)\, \sigma^{\lambda\nu}\, \tfrac{1}{2}\, \boldsymbol{\tau}.\boldsymbol{\rho}_{\lambda\nu} \\ & + (g_0/g)\, \gamma^\lambda\, \omega_\lambda + (g'_{00}/g)\, \sigma^{\lambda\nu}\, \tfrac{1}{2}\, \varphi_{\lambda\nu} \\ & + (f/g)\, \gamma^\lambda\gamma_5\, \boldsymbol{\tau}.\mathbf{A}_\lambda + (f'/g)\, \sigma^{\lambda\nu}\, \gamma_5\, \tfrac{1}{2}\, \boldsymbol{\tau}.\mathbf{A}_{\lambda\nu} \\ & + (f_0/g)\, \gamma^\lambda\gamma_5\, E_\lambda + (f'_{00}/g)\, \sigma^{\lambda\nu}\, \gamma_5\, \tfrac{1}{2}\, D_{\lambda\nu} \}\, N \end{aligned} \tag{6}$$

where the symbols stand for the respective vector and axial vector fields. By an argument related to the symmetry properties of the non-relativistic limit of this interaction[11] the ratios

$$(g'/g) = 5/3;\ (g'_{00}/g) = (g_0/g) = 1;$$
$$(f/g) = (f'/g) = (5/3\sqrt{2}); \tag{7}$$
$$(f_0/g) = (f'_{00}/g) = 1$$

are derived.

Given these ratios, by virtue of the coupling of expression (5) the electric charge and magnetic moment of the proton and neutron can be calculated. It is found that the proton charge is +e and the neutron charge is zero. For the proton and neutron magnetic moments

$$(\mu_p/\mu_n) = -\{1+(g/g')\,(m_\varrho/m)\} = -1{\cdot}49;$$
$$(\mu_p - \mu_n) = 1+(g'/g)\,(2m/m_\varrho) = 5{\cdot}1 \tag{8}$$

are obtained.

The observed values are −1·46 and 4·7. The predicted electric and magnetic form factors will have a fall-off with momentum transfer governed by the vector meson mass[12] and will have a pole at the vector meson mass. This is in qualitative agreement with experimental results.

(*iii*) *Weak Interactions*

The purely leptonic weak interactions are of the form

$$\frac{G}{\sqrt{2}}\,(\bar{e}\,\gamma_\lambda\,(1+\gamma_5)\,\nu_e)\,(\bar{\mu}\,\gamma^\lambda\,(1+\gamma_5)\,\nu_\mu)^\dagger \tag{9}$$

with possible terms involving the electron covariant or the muon covariant quadratically. The hadrons do not couple directly to leptons or to each other. The hadron weak interactions are the consequence of the vector–axial vector field coupling to lepton covariants. This interaction is

$$(-G/g)\,.\,(m_\rho\,\rho^\lambda + m_A^2\,A^\lambda)\,(\bar{\mu}\,\gamma_\lambda\,(1+\gamma_5)\nu_\mu + \bar{e}\,\gamma_\lambda\,(1+\gamma_5)\nu_e) \tag{10}$$

By virtue of the strong interaction, expression (6), there is an effective nucleon–lepton four-fermion coupling. For small momentum transfers it can be approximated by the familiar beta decay interaction[1,2]

$$\frac{G}{\sqrt{2}}\,(\bar{N}\,\gamma_\lambda\,(1+g_A\,\gamma_5)\,\tau^+ N)\,(\bar{e}\,\gamma^\lambda\,(1+\gamma_5)\nu_e) \tag{11}$$

Here the Fermi coupling constant is the same as the one occurring in the muon decay interaction in expression (9). According to expression (7) (for alternative derivation see refs. 13 and 14), the ratio of Gamow-Teller to Fermi interaction is

$$g_A = (-G_A/G_V) = (f/g) = (5/3\sqrt{2}) \tag{12}$$

The effective *V–A* four-fermion interaction is thus recovered as in the original formulation[1,2] but with the correct ratio of the Gamow-Teller and Fermi coupling constants. When the momentum-dependent terms are considered the familiar induced pseudoscalar term[15] and the weak magnetism term[16,17] with the usual values are obtained.

Because of the electromagnetic coupling in expression (5), the fundamental principle of electric charge-current conservation demands that the neutral components of ρ and ω remain divergence-free. Thus no neutral lepton currents are expected to be present in weak interactions. None is, of course, found.

There is a degree of time reversal and *CP* violation in the strong interaction in expression (6) and thus in the effective nuclear beta decay interaction. This effect seems to be beyond the present experimental accuracy; in a typical beta transition the energy release is a fraction of an MeV. But even for 1 MeV energy release, the amplitude of the *CP* violating term is

$$(f'/f)\,(m_e/m_A) \simeq 10^{-3} \tag{13}$$

of the normal (*CP* conserving) amplitude.

Using the observed value of the charged pion and muon lifetimes and by virtue of expressions (1) and (10), a relation for the absolute value of the parameter ξ and the strong vector coupling constant g can be deduced. From expression (10), the effective pion decay interaction is

$$\frac{G\,m_A^2\,\xi}{g}\,.\,\frac{1}{m_\pi}\,.\,\partial^\lambda\varphi_\pi\,\bar{\mu}\,\gamma_\lambda\,(1+\gamma_5)\,\nu_\mu \tag{14}$$

From the muon lifetime of $2{\cdot}198\times 10^{-6}$ sec

$$G = 2{\cdot}43\times 10^{-7}\,m_\pi^{-2} \tag{15}$$

From the pion lifetime of $2{\cdot}551\times 10^{-8}$ sec and expression (14)

$$(\xi/g) = 1{\cdot}02\times 10^{-2} \tag{16}$$

Using the divergence relation of equation (2) and the fact that the pion–nucleon coupling implied by expressions (1) and (6) is

$$\tfrac{1}{2}\,f\,\xi\,\bar{N}\,\gamma^\lambda\gamma_5\,\partial_\lambda\,(\tau\,.\,\varphi_\pi)\,N \tag{17}$$

which leads to[15]

$$\xi^2 = \tfrac{1}{2}\,(m_\pi/m_A)^2 = 8\times 10^{-3} \tag{18}$$

These give

$$\xi = 0{\cdot}09$$
$$g = 9{\cdot}0 \tag{19}$$

The value $g = 9{\cdot}0$ is in essential agreement with the phenomenology of strong interactions

Strange Particle Weak Interactions

For the leptonic weak decays of strange particles we extend the scheme by considering the charged strange vector and axial vector fields V'_λ and A'_λ on the same footing as the charged non-strange vector and axial vector fields V_λ and A_λ by the replacement

$$V_\lambda \rightarrow V_\lambda + V'_\lambda \tag{20}$$
$$A_\lambda \rightarrow A_\lambda + A'_\lambda$$

in the primary weak interaction coupling of expression (10). According to equation (1) this implies a coupling of the kaon to the charged lepton currents similar to expression (14). The ratio of kaon and pion transition rates can now be calculated to obtain

$$\frac{\Gamma\,(\kappa\rightarrow\mu\nu)}{\Gamma\,(\pi\rightarrow\mu\nu)} = \frac{m_\pi}{m_\kappa}\left[\frac{1-(m_\mu/m_\kappa)^2}{1-(m_\mu/m_\pi)^2}\right]^2 \simeq 1{\cdot}4$$

Using an experimental value of the pion lifetime and the value of the branching ratio $R = 0{\cdot}65 \pm 0{\cdot}02$ for the two-body leptonic mode of the kaon, for the kaon lifetime

$$\tau(\kappa^+) = \frac{\Gamma\,(\pi\rightarrow\mu\nu)}{\Gamma\,(\kappa\rightarrow\mu\nu)}\,.\,R\,.\,\tau(\pi^+) = 1{\cdot}17\times 10^{-8}\ \text{sec}$$

is obtained which is to be compared with the experimental value[5]

$$\tau(\kappa^+) = 1{\cdot}22\times 10^{-8}\ \text{sec}$$

It is to be emphasized that this rate is predicted without making use of any new smallness parameters (contrast with ref. 18).

For the axial vector decays of hyperons it can be shown that, making use of the divergence relation of equation (2), the effective four-fermion interaction for the leptonic decays of baryons can be computed. It turns out that they are uniformly smaller than the nucleon beta decay coupling by a factor

$$\tan\theta_A = \frac{m_\pi}{m_\kappa}\left(1+\frac{M-m}{2m}\right)^{-1} \simeq 0{\cdot}26 \tag{21}$$

where M/m is the ratio of the hyperon mass to the nucleon mass. If the requirement that there are no strange scalar particles is used it can be demanded that the strange vector fields be free of divergence

$$\partial^\lambda\,V'_\lambda = 0$$

This implies, in turn, that (the leading terms in) the vector coupling between the baryons of different mass should vanish. Thus the vector decays of hyperons and the three-body leptonic decays of kaons both proceed by the smaller momentum dependent couplings; it is not possible to make a simple prediction for the apparent suppression factors. Experimentally, the vector decays have an even smaller ratio to the vector beta decay coupling by a factor of 0·6 as compared with the ratio of the axial vector strengths[5].

Non-leptonic weak decays of baryons and mesons require a self-coupling of the vector–axial vector strange fields with the corresponding non-strange fields.

Implications for Strong Interactions

It is possible to use the present theory to compute the low energy pion–nucleon interactions (paper in preparation). The dominant contributions come from nucleon and nucleon resonance exchanges and the exchange of the vector meson. A simple calculation for the *s*-wave scattering lengths yields the values

$$a_1 = +0{\cdot}160\, m_\pi^{-1};\ a_3 = -0{\cdot}080\, m_\pi^{-1}$$

in good agreement with the values obtained from experiment

$$a_1 = +0{\cdot}183\, m_\pi^{-1};\ a_3 = -0{\cdot}109\, m_\pi^{-1}$$

Similarly, for the *p*-waves it can be predicted that

$$a_{11} = -0{\cdot}115\, \mathrm{m}_\pi^{-1};\ a_{33} = +0{\cdot}117\, m_\pi^{-1}$$
$$a_{13} = a_{31} = \tfrac{1}{4}\, a_{11} = -0{\cdot}029\, m_\pi^{-1}$$

These are in good agreement with the experimental values

$$a_{11} = -0{\cdot}101\, m_\pi^{-1};\ a_{33} = +0{\cdot}215\, m_\pi^{-1};$$
$$a_{13} = -0{\cdot}029\, m_\pi^{-1};\ a_{31} = -0{\cdot}038\, m_\pi^{-1}$$

For inelastic pion–nucleon resonance the present theory leads to the prediction that the $I = 1/2$ amplitude should be $\sqrt{10}$ times the $I = 3/2$ amplitude[19]. The experimental data have been analysed to yield[20]

$$(A_1/A_3) = +\ 3{\cdot}34$$

in excellent agreement.

Mesons were suggested by Yukawa to account for nuclear forces and nuclear beta decay[21]. Within my present scheme there are many mesons, comprising the pseudoscalar, vector and pseudovector mesons. Consequently, the nuclear forces which are obtained have three characteristic ranges; the largest range is the result of pion exchange, and it has been known for quite some time that the "tail" of the nuclear force is consistent with this picture. The vector and pseudovector mesons lead to shorter range potentials; but they also lead to spin-orbit forces which are essential to an understanding of the nucleon–nucleon interaction. Both in complex nuclei and for nucleon–nucleon scattering at higher energies they were phenomenologically introduced with great success.

We have outlined here a theory of universal primary interactions of particles, including strong, electromagnetic and weak interactions. The most important idea is that the primary interactions of the baryons consist of the strong universal coupling to vector and axial vector fields only. Both electromagnetic and weak interactions of baryons are acquired characteristics. Thus this theory is the logical completion of the idea that the beta decay of the nucleon arose only by virtue of its coupling to the meson. Such diverse items of particle phenomenology such as nuclear magnetic moments, ratio of the Gamow-Teller and Fermi coupling constants, weak magnetism, absence of neutral lepton currents, apparent suppression of strange particle leptonic decays, pion–nucleon scattering lengths and the salient features of the nuclear force are quantitatively accounted for by present theory. Such an array of satisfactory predictions leads us to study this theory seriously. A more detailed account of this theory will be published elsewhere.

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