THE NATURE OF PRIMARY INTERACTIONS OF ELEMENTARY PARTICLES*

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The four-fermion interaction

Over thirty years ago E. Fermi[1] gave a theory of nuclear beta decay in which the neutrino–electron coupling to the nucleons was postulated as a new interaction. Patterned after the structure of quantum electrodynamics, Fermi constructed the fundamental interaction Lagrangian from the scalar product of the four-vector which is formed out of the nucleon spinors with the one formed from the lepton spinors. This is the so-called 'vector' interaction. This led, in the nonrelativistic limit, to spin-independent beta decay interactions. G. Gamow and E. Teller[2] showed that this choice was inadequate to explain the beta decay of the Thorium series, and they proposed a spin-dependent interaction. The differential energy spectrum of the electrons emitted in most beta transitions (the 'allowed' or 'statistical' shape) and the systematic assignment of the reduced lifetimes (the 'ft-values') to various degrees of forbiddeness soon established the essential correctness of the new interaction structure for nuclear beta decay. The succeeding decades saw the discovery of the muon and the processes of muon decay and muon capture by nuclei, and the Fermi interaction described these processes as well. About twenty years ago the outlines of the hypothesis of Universal Fermi Interaction[3] were formulated by O. Klein and G. Puppi, and were elaborated by several authors[4]. However the succeeding years found disparity between the theoretical inferences from beta decay data and the structure of the process of muon decay. It was only after the discovery of parity violation in beta decay that these questions could be resolved. Ten years ago I analyzed the experimental data then existing on weak interactions and I concluded that not all the experiments could be consistent[5]. On the basis of a critical examination of the experiments on beta decay and other weak interactions I was led to the choice of V−A interaction as the only possible universal four-fermion interaction. During the summer of 1957 R. E. Marshak and I developed the concept of chirality invariance as a guiding

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principle to search for a systematic derivation of this V–A interaction[6]. We showed that the four-fermion interaction so introduced shared chirality invariance with the canonical commutation relations as well as the electromagnetic interaction, and automatically incorporated the concept of the two-component neutrino. The V–A form disagreed with several experimental results, but these disagreements disappeared when these crucial experiments were repeated during the following year[7]. Feynman and Gell-Mann[8] showed almost immediately that the V–A structure could be inferred from the requirement of a nonderivative coupling within a framework where two-component spinor fields satisfying the Klein–Gordon equation were used. They also showed[8,9] that with the V–A theory it was possible to consider a conserved vector current as the source of the Fermi part of the nuclear beta interaction; in this case the strong nuclear interaction should not lead to any renormalization of the coupling constant for the Fermi type beta interaction[10]. This was very satisfactory since the observed values of the vector coupling constants in muon decay and (Fermi type) nuclear beta decay are practically equal. On the basis of the chiral V–A coupling it is possible to estimate the renormalization of Gamow–Teller beta decay coupling constant and the numerical value so obtained is in good agreement with the observed value[11]. This computation removed the last significant obstacle to the form of the four-fermion interaction that we had proposed for nuclear beta decay[6]. The ten years that have elapsed since this discovery have served to establish it as *the* theory of the weak interaction of nonstrange particles[12].

Need for a new theory

Several new experimental developments in particle physics[13] as well as a desire to find a more fundamental connexion between strong and weak interactions prompt us to reexamine the theoretical basis for the universal four-fermion interaction. Among the weak interaction processes we find that the strangeness violating leptonic decays of hyperons and kaons are slower by a factor of about ten as compared with the estimates made on the basis of a direct strangeness violating four-fermion coupling. There is also unmistakable evidence now for lack of invariance under combined inversion in weak interaction, while the chiral V–A interaction is invariant under combined inversion (though not under charge conjugation or space inversion separately). In the domain of strong interactions a whole collection of vector mesons have been discovered and they seem to play an important role in meson–nucleon and nucleon–nucleon interactions, as well as for understanding the electromagnetic properties of the nucleons. The discovery of the vector mesons by Maglic and coworkers following a method suggested by us and the tentative identification of pseudovector mesons suggests that vector and axial vector fields associated with these particles are related to the vector-axial vector structure of weak interactions. We shall take as a starting point the postulate that the weak and electromagnetic interactions of the nucleon are not primary interactions but are consequences of the weak and electromagnetic interactions of the vector and axial vector meson fields.

Primary interaction Lagrangian

We now propose a theory of primary interactions[14] which retains most of the successes of our chiral V–A interaction[6], but extends it so as to include strange particle decays. It treats the weak and electromagnetic interactions of the strongly interacting particles as secondary interactions, included by virtue of their strong coupling to the vector and axial vector fields. The primary interactions are the direct coupling of the vector and axial vector meson fields with leptons and the photon. The primary interactions are listed below:

(i) *Electromagnetism*

The electrons and muons are coupled to the Maxwell field $\mathscr{A}_\lambda$ according to the standard interaction

$$-e\{\bar{\mu}\gamma_\lambda\mu + \bar{e}\gamma_\lambda e\}\mathscr{A}^\lambda.$$

The nucleons are *not* directly coupled to $\mathscr{A}_\lambda$ but the neutral vector fields ρ_λ and ω_λ are coupled to the Maxwell field according to the linear coupling:

$$-e'\left\{\frac{m_\rho^2}{g}\rho_\lambda + \frac{m_\omega^2}{g}\omega_\lambda\right\}\mathscr{A}^\lambda$$

where g is the strong coupling constant of the vector mesons to the nucleons and, by virtue of the squares of the meson masses, the coupling constant e' is dimensionless. The absolute conservation of electric charge in neutron beta decay implies that

$$e' = e.$$

The vector fields ρ_λ and ω_λ must be divergence-free to ensure the conservation of the electric charge. This implies, in turn, that there are no scalar particles associated with the vector fields ρ_λ, ω_λ.

(ii) *Strong Interactions*

The leptons do not have any strong interactions. The strong interactions involve the Yukawa couplings of the strongly interacting particles with the vector and axial vector fields. The vector fields are divergence-free and therefore describe only vector mesons:

$$\partial^\lambda V_\lambda = 0.$$

The axial vector fields do not have this property, but we may write:

$$A_\lambda = B_\lambda + (\xi/m)\partial_\lambda\phi; \qquad \partial^\lambda B_\lambda = 0;$$

where B_λ describes the pseudovector meson and ϕ describes a pseudoscalar meson of mass m. The dimensionless parameter ξ is a characteristic constant of the theory. The coupling of the (nonstrange) meson fields to the nucleon fields is described by the strong interaction Lagrangian:

$$\begin{aligned}\tfrac{1}{2}\bar{N}\{g\gamma^\lambda\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\lambda + g'\sigma^{\lambda\nu}\tfrac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_{\lambda\nu} + g_0\gamma^\lambda\omega_\lambda + g'_{00}\sigma^{\lambda\nu}\tfrac{1}{2}\phi_{\lambda\nu}\\ + f\gamma^\lambda\gamma_5\boldsymbol{\tau}\cdot\mathbf{A}_\lambda + f'\sigma^{\lambda\nu}\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{A}_{\lambda\nu} + f_0\gamma^\lambda\gamma_5 E_\lambda + f'_{00}\sigma^{\lambda\nu}\gamma_5\tfrac{1}{2}D_{\lambda\nu}\}N\end{aligned}$$

where the symbols stand for the vector and tensor field components of the respective mesons ρ, ω, ϕ, A, E and D.

(iii) *Weak Interactions*

The purely leptonic weak interactions are of the form:

$$\frac{G}{\sqrt{2}}(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e)(\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)^+$$

with possibly a term involving the self-coupling of the $(e\nu_e)$ pair with itself. The nucleons do not have a primary coupling either to the leptons or among themselves. The other primary weak interactions involve the vector and axial vector fields weakly coupled to the baryons or the leptons. The meson–lepton weak interaction is:

$$-\frac{G}{\sqrt{2}}(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e + \bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)\times\left(\frac{m_\rho^2}{(g/\sqrt{2})}\rho^\lambda + \frac{m_A^2}{(g/\sqrt{2})}A^\lambda\right).$$

The meson-baryon weak interaction is

$$-\frac{G}{\sqrt{2}}(m_A^2 A_\lambda + m_V^2\rho_\lambda)J^\lambda$$

where J^λ is a suitable expression which is bilinear in the baryon fields.

Induced electromagnetism

Given these primary couplings we can compute the induced electromagnetic and weak interaction effects. The effective nucleon electromagnetic interaction may be seen to be of the form

$$\begin{aligned}\frac{e}{2}\bar{N}\{(g_0/g)\Gamma_V(t)\{1-(t/m_\omega^2)\}^{-1}\gamma^\lambda\mathscr{A}_\lambda + \Gamma_V(t)\{1-(t/m_\rho^2)\}^{-1}\gamma^\lambda\mathscr{A}_\lambda\\ + (g'/m_\rho g)\Gamma_T(t)\{1-(t/m_\rho^2)\}^{-1}\tau_3\tfrac{1}{2}\sigma^{\lambda\nu}\mathscr{A}_{\lambda\nu}\}N\end{aligned}$$

where $\Gamma_v(t)$, $\Gamma_T(t)$ are the form factors of the vector meson–nucleon vertex. The vanishing of the neutron charge is assured if $g_0 = g$. The nucleon isovector anomalous magnetic moment is given by

$$\mu_1 = (2m_N/m_\rho) \cdot (g'/g) \cdot \Gamma_T(0)$$

The isoscalar magnetic moment vanishes identically. Hence the ratio of the proton and neutron magnetic moments is

$$\mu_P/\mu_n = -\left\{1 + \left(\frac{m_\rho}{m_n}\right)\left(\frac{g}{g'\Gamma_T(0)}\right)\right\}$$

The $SU(4)$ model of strong interactions (see below) gives $\Gamma_T(0)g'/g = 5/3$ and we are then led to predict:

$$\mu_1 = \begin{cases} 4.1 & \text{(theory)} \\ 3.7 & \text{(experiment)} \end{cases}$$

$$\mu_p/\mu_n = \begin{cases} -1.49 & \text{(theory)} \\ -1.46 & \text{(experiment)} \end{cases}$$

which is exceptionally good. The electric and magnetic form factors are given by

$$F_{V,T}(t) = \Gamma_{V,t}(0)\{1 - (t/m_\rho^2)\}^{-1}.$$

If the vector meson vertex falls off in a manner characterized by a single pole structure, the electromagnetic form factors would have dipole structures. This is in agreement with observations[13].

Induced weak interactions

Similar calculations for the effective nucleon beta decay interaction yield:

$$\frac{G}{\sqrt{2}}\bar{p}[\Gamma_V(t)\{1 - (t/m_\rho^2)\}^{-1}\gamma^\lambda + (f/g)\Gamma_A(t)\{1 - (t/m_A^2)\}^{-1}\gamma^\lambda\gamma_5]n$$
$$\times \{\bar{\mu}\gamma_\lambda(1 + \gamma_5)\nu_\mu + \bar{e}\gamma_\lambda(1 + \gamma_5)\nu_e\}.$$

The lack of renormalization of the vector beta coupling and its numerical equality with the muon beta coupling immediately follow. Comparison with beta decay experiments suggests

$$\Gamma_T(0)f/g = g_A \simeq 1.2$$

When we consider the momentum dependent term we obtain the familiar induced pseudoscalar term[15] and the weak magnetism term[9] with the usual *numerical* values.

By virtue of the fundamental principle of the (differential) conservation law of electric charge-current, the neutral components of ρ and ω must remain divergence-free. Hence, we do not expect that they can be coupled to neutral chiral (massive) lepton currents. This is very satisfactory because such neutral lepton currents seem to be totally absent in weak interactions.

It is worth pointing out that the combined inversion (CP) is violated by the 'magnetic' coupling of the axial vector field. This does not lead to any CP violating pion–nucleon coupling, but it does lead to a certain degree of CP violation in nuclear beta decay. But present experiments are not sensitive to this CP violation; for a beta transition with 1 MeV energy release the ratio of the CP violating amplitude to the CP conserving amplitude is about

$$(f'/f)(m_e/m_A) \simeq 10^{-3}.$$

The values of $\Gamma_T(g'/g)$, $\Gamma_A(f/g)$ and $\Gamma_T(f'/g)$ can all be derived from the following line of reasoning. In the low energy limit the vector and axial vector mesons reduce to Fermi type and Gamow–Teller type couplings. The 'electric' coupling of the vector meson is of the Fermi type while the 'magnetic' coupling (i.e., through $\rho_{\lambda\nu}$) is of the Gamow–Teller type. The vector meson couplings may then be identified with the generators of a noninvariance $SU(4)$ group with the nucleon and the $I = J = \frac{3}{2}$ nucleon resonance treated as constituting a representation of the same group[16]. This yields the ratio

$$\Gamma_T(0)\cdot(g'/g) = \tfrac{5}{3}$$

which we have used above. Similarly the low energy limit of the axial vector mesons yield Gamow–Teller interactions for both the axialvector and pseudotensor couplings. We may therefore obtain:

$$\sqrt{\Gamma_A^2 f^2 + \Gamma_T^2 f'^2}/g = \tfrac{5}{3}.$$

If in addition we assume

$$\Gamma_A f \simeq \Gamma_T f'$$

we get

$$\Gamma_A f = \Gamma_T f' = \frac{5}{3\sqrt{2}}\, g$$

so that

$$g_A = \Gamma_T\cdot(f/g) = 1.2$$

Strange particle weak interaction and the suppression of leptonic decays

For the leptonic decays of the strange particles we extend our theory by the replacement:

$$V_\lambda \to V_\lambda + V'_\lambda$$
$$A_\lambda \to A_\lambda + A'_\lambda$$

where V'_λ, and A'_λ are the strange vector and axialvector fields. We shall continue to demand that the vector field remain divergence free and that the longitudinal part of the axial vector field be proportional to the pseudoscalar field:

$$\partial^\lambda V'_\lambda = 0; \qquad \partial^\lambda B'_\lambda = 0.$$

$$A'_\lambda = B'_\lambda + (\xi/m')\partial_\lambda \phi',$$

with the same numerical parameter ξ. We can now calculate the ratio of pion and kaon decay rates into $(\mu\nu_\mu)$ final states:

$$\frac{\Gamma(\kappa \to \mu\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{m_\pi}{m_\kappa} \cdot \left\{\frac{1 - (m_\mu/m_\kappa)^2}{1 - (m_\mu/m_\pi)^2}\right\}^2 \simeq 1.4$$

Using the experimental value of the pion lifetime of 2.55×10^{-8} sec and the branching ratio 0.65 for the two-body leptonic mode of the kaon we predict for the kaon lifetime

$$\tau(K^+) = \begin{cases} 1.17 \times 10^{-8}\ \text{sec} & \text{(theory)} \\ 1.22 \times 10^{-8}\ \text{sec} & \text{(experiment)} \end{cases}$$

which is in good agreement with the observed value. It is of course essential to note that no new 'smallness parameter' was introduced for describing this interaction.

For the axial vector decays of hyperons we could make use of the relation[17]

$$\partial^\lambda A'_\lambda \simeq \xi m' \phi'$$

It turns out that the effective hyperon Gamow–Teller beta coupling is smaller than the nucleon beta decay coupling by the factor

$$\frac{m_\pi}{m_\kappa}\left(1 + \frac{M - m_n}{2m_n}\right)^{-1} \simeq 0.26,$$

where M is the hyperon mass. On the other hand, as far as the vector beta coupling is concerned, the strange vector meson field has no electric coupling and hence the leading term for the vector beta coupling should vanish. This comes about because a divergence-free vector field cannot couple to two fields with different masses. Hence the vector decay must proceed through the smaller momentum-dependent terms and it is also expected to be significantly suppressed. We see that our theory automatically accounts for the suppression of the strange leptonic decays without any new smallness parameter being introduced.

Consequences for strong interactions

It is possible to make use of the present theory to compute strong interaction effects. Since the calculation involves purely strong interaction phenomena only the lowest order calculations are to be viewed with a certain amount of caution. But it is interesting to observe that we can get good results for *s* and *p* wave pion–nucleon scattering lengths by considering nucleon and nucleon resonance exchange and vector meson exchange. The pion–nucleon interaction is not to be postulated anew but is itself an aspect of the axial vector–nucleon coupling; we have the primary pion–nucleon interaction

$$(f_1/m_\pi)\bar{N}\gamma^\lambda\gamma_5\partial^\lambda(\tau\cdot\phi)N;$$

$$f_1 = \tfrac{1}{2}gg_{\mathscr{A}}\xi$$

A simple calculation for the *s*-wave scattering lengths yields[18] the values (measured in inverse pion masses)

$$\mathscr{A}_1 = \begin{cases} +0.20 \text{ (theory)} \\ +0.183 \text{ (experiment)} \end{cases} \qquad \mathscr{A}_3 = \begin{cases} -0.10 \text{ (theory)} \\ -0.109 \text{ (experiment)} \end{cases}$$

Similarly for p-wave scattering lengths we get

$$a_{11} = \begin{cases} -0.091 & \text{(theory)} \\ -0.101 & \text{(experiment)} \end{cases}$$

$$a_{13} = \begin{cases} -0.022 & \text{(theory)} \\ -0.029 & \text{(experiment)} \end{cases}$$

$$a_{31} = \begin{cases} -0.022 & \text{(theory)} \\ -0.039 & \text{(experiment)} \end{cases}$$

$$a_{33} = \begin{cases} +0.133 & \text{(theory)} \\ +0.215 & \text{(experiment)} \end{cases}$$

With the exception of the resonant channel, these numbers show satisfactory agreement.

In these calculations[18] we have used $g = 9.0$; $\xi = 0.16$. We could relate these parameters to other strong interactions, particularly to meson mass ratios.

Some qualitative features of nucleon–nucleon interaction can be discerned; first of all, the nuclear force consists of three distinct contributions from pseudoscalar, vector and pseudovector particles with their characteristic ranges. The longest range term comes from pion exchange and this 'tail' of the nuclear force has long been known to be consistent with the central and tensor forces coming from pion exchange. Vector meson exchange gives the intermediate range potentials and this includes the leading contribution to the spin-orbit potential The pseudovector meson exchange leads to the shortest range potentials. Since all the coupling constants are now uniquely specified the nucleon–nucleon interaction can be computed.

As an additional strong interaction consequence we know that the nucleon–nucleon potential should not have any γ^{-3} singularity at the origin which is known to result if only a pseudoscalar exchange is considered. If we require a cancellation[19] to occur between the π and ρ contributions we are led to a prediction of the ratio of their masses:

$$(m_\pi/m_\rho) = \begin{cases} 0.188 & \text{(theory)} \\ 0.182 & \text{(experiment)} \end{cases}$$

This shows remarkable agreement.

Summary and outlook

We have discussed a theory which attempts to identify specific primary interactions obeying simple rules and leading to a unified treatment of strong, electromagnetic and weak interactions. The vector-axial vector structure, which was first deduced from an analysis of weak interaction data, is seen to be equally important in strong interactions.

The electromagnetic interactions now consist of two types: the familiar 'minimal' interaction of the leptons with the photons, and a different kind of coupling of the neutral vector meson with the photon. Electromagnetism is a primary property of the neutral vector meson, but is only a derived property for the other hadrons. The absence of scalar mesons and the divergence-free nature of the vector field guarantee that the source of the electromagnetic field is always conserved. It is an immediate manifestation of this two-step nature of electromagnetism of nucleons that the nucleon magnetic moments deviate so markedly from those for Dirac particles. A similar mechanism applies for weak interactions also. The suggestion which Yukawa[20] made and later abandoned in connection with meson theory has been resurrected and amplified in the present theory. Beta decay is a derived property of the nucleon but a primary property of the meson. In a manner of speaking the vector and axial vector mesons are intermediate vector bosons for hadron weak interactions. Purely leptonic interactions are direct couplings in the present theory; no intermediate vector bosons are required or expected in muon decay.

The self same difficulties of the local field theory that arise for lepton electrodynamics and for lepton four-fermion interactions continue to be present in this theory. Nor does this theory offer any suggestion as to why there are three categories of interactions. A solution to the problem of divergences of local field theory is outlined in my report on 'Indefinite Metric and Non-local Field Theory' to this Conference.

Such diverse items of particle physics phenomenology as nucleon magnetic moments, beta decay couplings, weak magnetism, absence of neutral lepton currents, absence of scalar mesons, apparent suppression of strange particle decays, pion–nucleon scattering lengths, and the general features of the nucleon–nucleon interaction are correlated in this theory. Such correlations suggest that a fundamental Lagrangian describing the primary interactions is another step forward in our understanding of the study of matter.

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