Proc. Roy. Soc. A. **305**, 319–343 (1968)
Printed in Great Britain

The nature of universal primary interactions of particles*

By E. C. G. Sudarshan
Syracuse University, Syracuse, New York,

(*Communicated by R. E. Peierls, F.R.S.—Received* 25 *September* 1967)

A theory of primary interactions between elementary particles is proposed. It is based on the hypothesis that there are three primary interactions: weak, electromagnetic, and strong; and each of them is characterized by a single coupling constant. The primary weak interactions couple the leptons to themselves and to vector and axial vector fields, but not to nucleons. The primary electromagnetic interactions couple the electromagnetic field to the charged leptons or to the neutral vector meson fields, but not to nucleons. The primary strong interactions couple the vector and axial vector fields to the nucleons. All the couplings are universal. On the basis of this theory it is possible to account quantitatively for the anomalous magnetic moments of the nucleons, the ratio of the Gamow–Teller and the Fermi β decay coupling constants, weak magnetism, absence of neutral lepton currents, pion-decay, pion-nucleon scattering lengths, and the principal features of the nuclear force. The theory when extended to strange particles, leads automatically to the suppression of weak decays of strange particles. Our older chirality invariant V–A four-fermion interaction is recovered as the effective interaction for small momentum transfers. No intermediate bosons in the conventional sense are required or expected. The application to the absolute calculation of electromagnetic mass shifts and non-leptonic decay rates yields finite answers which will be discussed in another paper.

1. Introduction

It has been known for a decade that weak interactions of non-strange particles have a simple universal structure (Sudarshan & Marshak 1957; Feynman & Gell-Mann 1958). For the leptons this takes the form of a chiral V–A coupling

$$2^{-\frac{1}{2}}\,G(\bar{\mu}\gamma^{\lambda}(1+\gamma_5)\,\nu_{\mu})^{\dagger}(\bar{e}\gamma_{\lambda}(1+\gamma_5)\,\nu_e) \tag{1·1}$$

and, to a first approximation, the nucleonic coupling to the leptons can be written in the form

$$2^{-\frac{1}{2}}\,G(\bar{\eta}\gamma^{\lambda}(1+\gamma_5)\,p)^{\dagger}\,(\bar{e}\gamma_{\lambda}(1+\gamma_5)\,\nu_e). \tag{1·2}$$

A similar expression holds for the muon-neutrino coupling to the neutron and proton. The approximate validity of (1·2) in nuclear β decay was somewhat puzzling in view of the possibility that the effective coupling constants of the nucleons would be affected by strong interactions. In the case of vector coupling this could be understood if the leptons were not coupled directly to the nucleon, but to an entity which was conserved in the presence of strong interactions (Gershtein & Zeldovich 1955; Feynman & Gell-Mann 1958). A similar situation was already encountered in connexion with the interaction of nucleons with photons. Here the numerical value of the electric charge is unaffected by the charge-conserving strong interactions to which the nucleons are subjected. We shall see below that the connexion between electromagnetism and weak interactions is far reaching. In fact, the theory developed in this paper is able to relate such diverse things as the absence of neutral

* Supported by U.S. Atomic Energy Commission. The principal results were presented at the Centennial Colloquium of the American University of Beirut (March 1967) and the joint CAP-SMF-APS meeting in Toronto (June 1967).

lepton currents, the universality of electric charge, the universality of the Fermi interaction, the principal features of the nuclear forces, and the electromagnetic properties of nucleons.

During the past years evidence has been accumulating that the vector isovector particle called the ρ meson is universally coupled to nucleons and pions (Sakurai 1966; Signell & Durso 1967), as well as to other baryons and mesons. This universal coupling makes up the third member of the class of universal vector couplings which already include the vector (Fermi type) weak β decay interaction and the vector (Dirac type) electromagnetic interaction.

We could understand all of these couplings to be aspects of *a single universal interaction*, if we identify the vector meson field as the entity to which the β decay lepton covariants as well as the electromagnetic vector potential are coupled. We are thus led to the hypothesis that the vector β interaction and the vector electromagnetic interaction are secondary phenomena—the primary phenomenon being the universal coupling of vector mesons with the baryons and mesons on one hand, and the direct coupling of the vector meson with the β covariants and the electromagnetic potentials on the other. Weak and electromagnetic interactions (of the vector type) thus become properties of the vector meson fields alone! The details of the coupling structures and their relation to empirical results are discussed later on in this paper.

The weak interactions, however, consist of an axial vector contribution in addition to the vector contribution. It is therefore suggested that there should be an axial vector meson field which is suitably coupled to the baryons and mesons. This axial vector meson field also should be coupled to the lepton covariants. If such an entity did not exist we shall have destroyed the universality of the weak interactions. Let us therefore *postulate* that we have a two-step process for the axial vector interactions in β decay: the Gamow–Teller weak interactions are then secondary (effective) interactions of the baryons arising solely out of the coupling of the axial vector meson field with the hadrons. *Weak interactions of hadrons are due entirely to the primary coupling of vector and axial vector meson fields.*

Electromagnetism does not have an axial vector component (see, however, the discussion of the non-relativistic limit in § 3), but the vector field through which electromagnetism is coupled must belong to the same multiplet as the one through which weak vector interactions are brought about. *Electromagnetism is not a primary property of the hadrons* but only of the vector meson fields.

It would be desirable to have some general principles which relate the coupling constants of the vector mesons among themselves and with the coupling constants of the axial vector mesons. The simple equality of the coupling constants is not expected to hold for hadrons; for leptons, however, it is true as evidenced by the chirality invariant combination $([1+\gamma_5]\psi)$ that occurs in weak interactions. Is it perhaps that the lack of equality is to be traced to the presence of internal symmetries for the hadrons and their occurrence as supermultiplets? It has been noted that the numerical ratio of Gamow–Teller to Fermi coupling constants for weak interaction is $1{\cdot}18 \pm 0{\cdot}02$ (Bühring & Schopper 1965) and it is practically the same as $\sqrt{(\frac{25}{18})}$. A simple framework in which this number appears quite naturally is

presented in the latter part of this paper (§ 3) where the nature of the universal coupling is spelled out.

Contact may be established with low energy meson-nucleon processes by considering the axial vector field to have two components, the 'transverse' spin-one component yielding a pseudovector particle (the A_1 particle?) and the 'longitudinal' spin-zero component yielding the pseudoscalar pion. The coupling of pions to baryons thus becomes an aspect of the coupling of the axial vector mesons to baryons. Further, the question of the universality of the axial vector coupling is equivalent to the question of the universality of the pion coupling (Schwinger 1967 *a*; Wess & Zumino 1967). We are thus led to predict the detailed manner in which the axial vector meson is coupled to the baryons. In this paper it is shown that the phenomenology of low energy pion physics is fully understood in terms of a single strong coupling constant: for convenience we may take this to be the pseudovector pion-nucleon coupling constant. The phenomena that could be correlated in terms of this single coupling constant include both *s*- and *p*-wave scattering lengths, the magnitude of the vector coupling constant, and a variety of indirect meson effects like the electromagnetic properties of the nucleons and the nature of nuclear forces.

The hypothesis of the direct coupling of the vector and axial vector mesons to the lepton covariants is subject to direct experimental verification. For the vector mesons, however, this is rather difficult because of the pronounced instability of the vector mesons against decay into pions. In the case of the axial vector mesons this could be verified by a direct computation of the coupling of the pion with the lepton covariants.

These considerations extend in a natural manner to the decays violating strangeness. For the leptonic strangeness changing weak interactions we could consider a coupling of a strange vector or axial vector meson field with the lepton covariants. Experimental results were usually interpreted (Cabibbo 1963) to require that this coupling strength is smaller than the strangeness conserving leptonic decay coupling by a factor of 10 or so. This would be a distressing feature if it were true. However, the hypothesis of a two-step transition within the universal vector-axial vector theory enables us to relate the apparent suppression of strangeness violating decays to the universal formulation of the theory of weak interaction‡ and show that the strange axial vector and vector fields can be understood to have the same universal coupling constant. This formulation yields absolute rate computations of the strangeness violating leptonic processes. Once the universal coupling responsible for the strangeness changing leptonic weak interactions is invoked, the decay rate for K mesons decaying into leptons can be directly computed.

Finally, one comes to a study of the non-leptonic decays involving strange particles. In this case we have to couple the vector and axial vector interactions by means of a direct weak coupling. By considering the manner in which this involves both the vector and axial vector fields in a parity violating chiral coupling we would expect both parity conserving the parity violating decays. It is shown that we can

‡ We were led to this formation of hadron leptonic decays by the ingenious work of T. Pradhan and M. Patnaick: 'Suppression of Strangeness-changing Leptonic Decays of Hadrons', *Saha Institute Report* 1967.

obtain an interaction, which can be used to calculate nonleptonic decay amplitudes. These questions are discussed in § 4 where it is shown that the interrelation between the various amplitudes expressed in terms of sum rules follows from these couplings independent of the numerical values of the coupling constants for strong interactions.

The electromagnetic properties of the nucleons can be computed without the introduction of any new parameters. We obtain $\mu_p/\mu_n = -1{\cdot}49$, in excellent agreement with the experimental value $-1{\cdot}46$. The absolute value of the magnetic moment is also predicted, but it is about 10 % too large. The isoscalar magnetic moment may be seen to be zero. These numerical values are shown to be intimately related to the nature of nuclear forces and the condition that the nucleon–nucleon interaction has no very singular tensor force contribution is related to the vanishing of the isoscalar nucleon magnetic moments. The absolute ratio of the ρ to π meson mass is computed on the basis of the vanishing of the very singular tensor force terms, and the result is in fair agreement with experiment.

2. Electromagnetism of hadrons

The conservation of electric charge is the fundamental principle of electrodynamics. The numerical electric charges of all hadrons seem to be integral multiples of the electron charge. Charge conservation and the associated gauge invariance of the first kind (that is, invariance under numerical phase transformations on the charged fields), when extended to gauge invariance of the second kind (where the gauges become space-time functions), demands the traditional coupling of the electromagnetic vector potential to the charge-current four-vector of the charged fields. The interaction between photons and nucleons would, for example, be through the Yukawa coupling

$$e\overline{N}\,\tfrac{1}{2}(1+\tau_3)\,\gamma^\lambda N\mathscr{A}_\lambda. \tag{2·1}$$

Without prejudice to charge conservation, or to gauge invariance of the second kind, we could add explicitly gauge invariant couplings with electromagnetic fields of the type

$$(e/2m)\,\overline{N}(\kappa_0+\kappa_1\tau_3)\,\sigma^{\lambda\nu}N\tfrac{1}{2}\mathscr{A}_{\lambda\nu}. \tag{2·2}$$

The couplings (2·1) and (2·2) are referred to as the Dirac and the Pauli type couplings. Instead of considering the electromagnetic couplings as primary, we have discussed in the Introduction the attractive hypothesis that, like the universal weak interactions, the universal electromagnetic interactions of the hadrons could be considered as primary attributes of the vector meson fields and are indeed secondary interactions for other hadrons. In this manner the universality of electromagnetism gets unified with the universality of weak interactions. The electromagnetic properties of hadrons become, then, direct probes of the coupling types of the vector mesons to the hadrons. Since the vector mesons are the sole mediators between the electromagnetic fields and the hadrons the observed isovector and isoscalar components for the electromagnetic moments of say, the nucleons demands that there exist isovector and isoscalar vector mesons. Such vector mesons ρ and ω have been found, with nearly equal masses.

The structure of electromagnetic interaction

The new theory of universal electromagnetic interactions of nucleons involves the following strong and electromagnetic interactions:

$$e'(m_\rho^2\, V^\lambda + m_\omega^2\, W^\lambda)\,\mathscr{A}_\lambda; \tag{2·3}$$

$$\bar{N}\{\tfrac{1}{2}g\boldsymbol{\tau}\,.\,\mathbf{V}^\lambda\gamma_\lambda + \tfrac{1}{2}g'\boldsymbol{\tau}\,.\,\mathbf{V}^{\lambda\nu}\tfrac{1}{2}\sigma_{\lambda\nu} + \tfrac{1}{2}g_0\, W^\lambda\gamma_\lambda + \tfrac{1}{2}g_0'\, W^{\lambda\nu}\tfrac{1}{2}\sigma_{\lambda\nu} + \tfrac{1}{2}g_{00}\,\Phi^\lambda\gamma_\lambda + \tfrac{1}{2}g_{00}'\,\Phi^{\lambda\nu}\tfrac{1}{2}\sigma_{\lambda\nu}\}N. \tag{2·4}$$

In addition we have the usual coupling of the photons with the electrons

$$e\bar{\psi}\gamma^\lambda\psi\mathscr{A}_\lambda. \tag{2·5}$$

In the above expressions e, g, g', g_0, g_0' are dimensionless coupling constants. The vector fields are described by ten-component Duffin–Kemmer objects. The four vector components $\mathbf{V}^\lambda$ and six tensor components $\mathbf{V}^{\lambda\nu}$ describe the isovector $\boldsymbol{\rho}$ field. The corresponding quantities for the isoscalar ω and ϕ fields are denoted by W^λ, $W^{\lambda\nu}$ and ϕ^λ, $\phi^{\lambda\nu}$ respectively. Since there are no scalar particles associated with the vector meson field, we may write

$$\partial^\lambda\mathbf{V}_\lambda = 0, \quad \partial^\lambda W_\lambda = 0, \quad \partial^\lambda\phi_\lambda = 0. \tag{2·6}$$

It immediately follows that the current to which the photon field $\mathscr{A}_\lambda$ is coupled is strictly conserved; and hence the Ward–Fradkin–Takahashi identities are automatically obeyed.

Once these couplings are known we can compute the effective photon–nucleon coupling

$$\bar{N}\left\{\frac{ge'm_\rho^2}{2(t-m_\rho^2)}\tau_3 + \frac{g_0\,e'm_\omega^2}{2(t-m_\omega^2)}\right\}\gamma^\lambda N\mathscr{A}_\lambda + \bar{N}\left\{\frac{g'e'm_\rho}{2(t-m_\rho^2)}\tau_3 + \frac{g_0'\,e'm_\omega}{2(t-m_\omega^2)}\right\}\sigma^{\lambda\nu}N\tfrac{1}{2}\mathscr{A}_{\lambda\nu}. \tag{2·7}$$

Comparing (2·7) with the standard expression (2·1) we deduce

$$e' = -e/g, \quad g_0 = g. \tag{2·8}$$

The anomalous magnetic moments are

$$\kappa_1 = -\frac{e'}{e}g'\frac{2m}{m_\rho} = \frac{g'}{g}\frac{2m}{m_\rho},$$
$$\kappa_0 = -\frac{e'}{e}g_0'\frac{2m}{m_\omega} = \frac{g_0'}{g}\frac{2m}{m_\omega} \tag{2·9}$$

independent of the absolute value of the coupling constants, since only their ratios enter (2·9). These calculations are in the spirit of the work of Salam, Delburgo & Strathdee (1965). In order to go beyond this we should have a theoretical framework from which the ratios of the strong coupling constants can be predicted. This framework should be one which treats vector and tensor couplings of the vector meson fields on the same footing.

The vector meson coupling scheme

The nucleon and the ($I = J = \frac{3}{2}$, even parity) nucleon resonance must be taken together in a quantitative theory of low energy pion-nucleon interactions. In such a treatment together they furnish a $2\times 2+4\times 4 = 20$ dimensional third rank symmetric tensor representation of $SU(4)$ (Gürsey & Radicati 1964). The group $SU(4)$ has 15 infinitesimal generators which consist of a scalar isovector, a pseudovector isoscalar and a pseudovector isovector. Considering the non-relativistic limit of (4) we find that one possible choice is to take the time-components of the ρ meson field as the scalar isovector, the space–space components of the ϕ meson field as the pseudovector isoscalar and the space–space components of the ρ meson field as pseudovector isovector. This choice is:

$$\left.\begin{aligned} &\text{scalar isovector} = \rho \\ &\text{pseudovector isoscalar} = (1/m_\phi)\,(\boldsymbol{\nabla}\times\boldsymbol{\varphi}), \\ &\text{pseudovector isovector} = (1/m_\rho)\,(\boldsymbol{\nabla}\times\boldsymbol{\rho}). \end{aligned}\right\} \tag{2·10}$$

It is in the spirit of (but different from) the scheme of Capps (1965) and of Belinfante & Cutkosky (1965). With this choice the coupling constants should be in the ratio of $1:1:\frac{5}{3}$, so that we get

$$g' = \tfrac{5}{3}g, \quad g'_{00} = g. \tag{2·11}$$

The predicted anomalous magnetic moments become

$$\left.\begin{aligned} \kappa_1 &= \tfrac{5}{3}(2m/m_\rho) = 4{\cdot}1 \quad &(\text{expt.: } 3{\cdot}7), \\ \kappa_0 &= 0 &(\text{expt.: } 0{\cdot}1). \end{aligned}\right\} \tag{2·12}$$

The corresponding value of the ratio of proton and neutron magnetic moment becomes

$$-(\mu_p/\mu_n) = 1+(g/g')\,(m_\rho/m) = 1{\cdot}49, \tag{2·13}$$

which is in excellent agreement with the experimental value‡

$$-(\mu_p/\mu_n) = (2{\cdot}79/1{\cdot}91) = 1{\cdot}46.$$

The absolute values of the nucleon magnetic moments predicted by (2·12) are several percent too large.

Several remarks are relevant: First, in choosing the ϕ field as the pseudovector isoscalar meson field we were led by the empirical observation that the isoscalar magnetic moment is nearly zero. Were we to choose ω instead of ϕ, we would get a large anomalous isoscalar magnetic moment ($\sim$ 2·5 magnetons) which is completely unacceptable. Secondly, we note that the overall strong coupling constant has cancelled out in the estimates (2·12) and (2·13). Of course there will be higher order corrections nonlinearly dependent upon the strong coupling constant. Finally, we note that in this approximation, both the Dirac and Pauli form factors are essentially identical; this is in fair agreement with experiment, though the fall-off provided by one-pole form factor is too slow. The experimental form factors may point to an additional 'kinematic' form factor, not arising from meson exchange. We have

‡ Schwinger (1967*a*) and Renninger & Videira (1967) have given another interesting explanation of this numerical value.

discussed elsewhere (Sudarshan 1967) the possibility of such a form factor arising out of an infinite component wave equation.

Having established the general framework of the nucleon electromagnetic form factors we can apply similar considerations to other hadrons. The electromagnetic properties are still mediated by the vector mesons ρ and ω. Consequently *all* hadron form factors arising out of the two-step process have the same form factor.‡

It is now of interest to compute the electromagnetic mass differences based on such a structure. The contributions are now convergent and can be calculated explicitly. These questions will be discussed in detail elsewhere.

We note, in passing, that all these interactions are invariant under space inversion, particle conjugation and time reversal.

3. Theory of weak interactions

The purely leptonic chirality invariant weak interaction responsible for the decay of the muon may be written

$$2^{-\frac{1}{2}}\, G(\bar{\mu}\gamma^{\lambda}(1+\gamma_5)\,\nu_{\mu})^{\dagger}\,(\bar{e}\gamma_{\lambda}(1+\gamma_5)\,\nu_e) \tag{3·1}$$

with perhaps the related terms

$$2^{-\frac{1}{2}}\, G(\bar{e}\gamma_{\lambda}(1+\gamma_5)\,\nu_e)^{\dagger}\,(\bar{e}\gamma_{\lambda}(1+\gamma_5)\,\nu_e) + 2^{-\frac{1}{2}}\, G(\bar{\mu}\gamma_{\lambda}(1+\gamma_5)\,\nu_{\mu})^{\dagger}\,(\bar{\mu}\gamma_{\lambda}(1+\gamma_5)\,\nu_{\mu}). \tag{3·2}$$

From the interaction (1) the muon decay lifetime can be computed

$$\tau(\mu) = (192\pi^3/G^2 m_{\pi}^4)\, m_{\pi}^{-1}\,\text{second}.$$

The observed lifetime is $(2{\cdot}198 \pm 0{\cdot}001)\times 10^{-6}$ second (Lee & Wu 1965) which yields

$$G = 1{\cdot}432\times 10^{-49}\,\text{erg cm}^3.$$

Radiative corrections revised this to yield

$$G = 1{\cdot}435\times 10^{-49}\,\text{erg cm}^3 = 2{\cdot}43\times 10^{-7}\, m_{\pi}^{-2}. \tag{3·3}$$

There are a multitude of leptonic weak processes involving hadrons. The most familiar is the nuclear beta decay process

$$n \rightarrow p + e + \bar{\nu}_e \tag{3·4}$$

which contains both vector and axial vector transitions. The best estimates for the vector coupling constant (after the radiative corrections) yield

$$G_V = 1{\cdot}403\times 10^{-49}\,\text{erg cm}^3 = 2{\cdot}38\times 10^{-7} m_{\pi}^{-2}. \tag{3·5}$$

while the axial vector coupling constant is somewhat larger. The ratio of the vector to axial vector coupling constants is experimentally found to be

$$-G_A/G_V = 1{\cdot}18 \simeq \sqrt{(\tfrac{25}{18})}. \tag{3·6}$$

We shall later on see a framework in which this number emerges naturally.

‡ These calculations are in the spirit of the work of Salam, Delburgo & Strathdee (1965).

The theoretical attempts to account for the nuclear β decay started with Fermi's theory of nuclear β decay (Fermi 1934), amended by Gamow & Teller (1936) to include spin-dependent interactions and found its logical completion in the chiral V–A interaction (Sudarshan & Marshak 1957). In these theories one envisages a direct four-fermion coupling of the hadron and leptons similar to the coupling (3·1). Fermi's original suggestion included only a vector coupling

$$G\bar{N}\gamma^{\lambda}P\bar{e}\gamma_{\lambda}\nu \tag{3·7}$$

and included neither the axial vector interaction nor the parity violation which was unknown at that time. A more serious shortcoming was that it made no contact with the basic strong interaction mechanism. This became all the more pronounced after the universal chiral V–A structure was discovered.

As discussed briefly in the Introduction, Yukawa had made the suggestion that the meson field providing the nuclear force should also serve to transmit the beta interaction (Yukawa 1935). In succeeding years this suggestion was largely ignored, partly on the basis of the hypothesis of the universal Fermi interaction. In the chiral V–A interaction it is entirely suppressed. Yukawa himself considered at one time that his suggestion for nuclear β decay was no longer valid (Yukawa 1949). It now appears, on the basis of experimental and theoretical information not available ten years ago, that the theory of universal interactions requires that the nuclear β decay interaction be interpreted as a two-step transition, with the electromagnetic and β interactions being primary couplings of the meson fields alone.

The structure of hadron weak interactions

The new theory of universal interactions of nucleons requires the coupling of the vector and axial vector fields with the lepton currents; accordingly we supplement the muon decay interaction (3·1) with the hadron–lepton coupling

$$G'(m_A^2 A^{\lambda}+m_\rho^2 V^{\lambda})(\bar{e}\gamma_{\lambda}(1+\gamma_5)\nu_e) \tag{3·8}$$

with a related term involving muons (to account for the muon capture reaction). By virtue of the mass squared factors G' has the same dimensions as the muon decay coupling constant G. Here V^{λ} is the charged vector meson (ρ^+) field which satisfies

$$\partial^{\lambda}V_{\lambda} = 0, \tag{3·9}$$

assuring us that there are no scalar (spin-zero, even-parity) quanta that are associated with this field. The axial vector field does not satisfy this condition, but may be written in the Stuckelberg form

$$A^{\lambda} = B^{\lambda}+\xi\frac{1}{m_\pi}\partial^{\lambda}\phi_\pi \tag{3·10}$$

with

$$\partial^{\lambda}B_{\lambda} = 0, \quad \partial^{\lambda}A_{\lambda} = (\xi/m_\pi) \quad \phi_\pi. \tag{3·11}$$

For momenta on the pion mass-shell we get

$$\partial^{\lambda}A_{\lambda} = \xi m_\pi \phi_\pi, \tag{3·12}$$

which is the PCAC condition (Gell-Mann & Lévy 1960; Nambu 1960). Thus the axial vector field carries two kinds of quanta: the pions and the spin-one quanta of

the divergence-free field. We now find the absolute value of G by requiring that the effective nuclear β decay Fermi coupling constant be equal (within 2 % only!) to the muon decay coupling constant. In the preceding section we have already considered the coupling of the vector meson field V^λ with nucleons

$$\tfrac{1}{2}g\overline{N}\{\boldsymbol{\tau}\,.\,\mathbf{V}^\lambda\gamma_\lambda+(g'/g)\,\boldsymbol{\tau}\,.\,\mathbf{V}^{\lambda\nu}\tfrac{1}{2}\sigma_{\lambda\nu}\}N. \tag{3·13}$$

Recalling that the fields involved in (3·8) are charged fields, we can write down the effective interaction of the Fermi type

$$\frac{G'm_\rho^2}{t-m_\rho^2}\frac{1}{\sqrt{2}}g(\overline{N}\gamma^\lambda\tau^\dagger N)\,(\bar{e}\gamma_\lambda(1+\gamma_5)\,\nu_e). \tag{3·14}$$

Comparing (3·1) and (3·14) we deduce

$$G' = -(G/g). \tag{3·15}$$

Coupling constants, form factors, weak magnetism

As a consequence of the 'magnetic' coupling of the vector meson field, the effective β decay interaction differs from the pure local-coupling in two ways. First, we have the form factor of the vector vertex,

$$F_V(t) = \{1-(t/m_\rho^2)\}^{-1}, \tag{3·16}$$

coming from the momentum transfer dependence in (3·14). By comparing with the electric form factor in the preceding section, we see that they are identical. *This universality of the vector form factor* is a reflexion of the underlying unity of the various processes brought about by vector exchange. Secondly, we have an anomalous moment contribution to the vector coupling coming from the second term in (3·13). It leads to an effective interaction of the type

$$2^{-\frac{1}{2}}G\,\kappa_1(m_\rho/2m)\,(\overline{N}\sigma^{\lambda\nu}q_\nu\,\tau^\dagger N)\,(\bar{e}\gamma_\lambda(1+\gamma_5)\,\nu_e) \tag{3·17}$$

where κ_1, is the isovector nucleon magnetic moment,

$$\kappa_1 = (2m/m_\rho)\,(g'/g) \simeq 4.$$

This 'weak magnetism' term has been considered before (Gell-Mann 1958). The present theory has some resemblance to the scheme involving a conserved vector current (Gershtein & Zeldovich 1955; Feynman & Gell-Mann 1958), where the vector β interactions involve a direct coupling of the isospin current to the leptons. In such a theory it is possible to relate the weak magnetism to the anomalous magnetic moment (Gell-Mann 1958) though the absolute value of either cannot be computed. In the present theory the coupling is to the vector meson field, which is 'conserved' by virtue of (3·9); but the vector meson field is *not* a multiple of the isospin density. Nevertheless, the relation between the weak magnetism and the anomalous magnetic moment is still valid. In the present theory, we can directly compute the absolute value of the weak magnetism term. The estimate (3·17) seems to have been verified experimentally (Lee, Mo & Wu 1963).

Let us now consider the axial vector field contribution. To compute this we must introduce a strong axial vector-nucleon coupling of the form

$$f\overline{N}\{\gamma^\lambda\gamma_5\,\tfrac{1}{2}\boldsymbol{\tau}\,.\,\mathbf{A}_\lambda+(f'/f)\,\tfrac{1}{2}\sigma^{\lambda\nu}\gamma_5\,\tfrac{1}{2}\boldsymbol{\tau}\,.\,\mathbf{A}_{\lambda\nu}\}N. \tag{3·18}$$

The leading term leads to an effective axial vector coupling constant

$$G_A = G'f = -G_V(f/g), \tag{3·19}$$

so that

$$-G_A/G_V = f/g. \tag{3·20}$$

The numerical value (3·6) of $-G_A/G_V$ shows that f and g cannot be equal. To determine the absolute values of f and g we should extend our strong interaction framework to enable us to obtain the ratios of the coupling constants f, f' to g. For this purpose we note that in the $SU(4)$ multiplet furnished by N and N^* we had to consider a 15-component meson matrix: for the purpose of fixing the electromagnetic parameters we choose a meson matrix of vector fields only. We must now consider another meson matrix containing axial vector mesons also. For this purpose we note that the low-energy limit of (3·18) contains the terms

$$\tfrac{1}{2}fN^\dagger\boldsymbol{\tau}\sigma_j\{\mathbf{A}^j + (f'/f)\,\mathbf{A}^{0j}\}N. \tag{3·21}$$

Both of these terms are isovector pseudovector quantities. We have to make a choice between the relative strengths of the couplings to $\mathbf{A}_j$ and $\mathbf{A}_{0j}$. In this connexion we note that if f'/f is real the $\mathbf{A}_{0j}$ term violates charge conjugation invariance as well as combined inversion (CP) invariance but not parity. In view of the observed CP violation (Christenson, Cronin, Fitch & Turlay 1964) in neutral kaon decay, we must not put f'/f to be zero. The 'maximal violation' would be when $f'/f = \pm 1$. In this case the normalized meson matrix would have the components (cf. Capps 1965; Belinfante & Cutkosky 1965):

$$\left.\begin{aligned} &\text{scalar isovector} &&= \boldsymbol{\rho} \\ &\text{pseudovector isoscalar} &&= Z_j \\ &\text{pseudovector isovector} &&= 2^{-\frac{1}{2}}(\mathbf{A}_j + \mathbf{A}_{0j}) \end{aligned}\right\} \tag{3·22}$$

In these expressions Z_λ is an isosinglet axial vector meson field whose pseudoscalar component is the η

$$Z^\lambda \to \xi\left(\frac{1}{m_\eta}\right)\partial^\lambda\phi_\eta. \tag{3·23}$$

The coupling constants become,

$$f = f' = (\tfrac{5}{3}\sqrt{2})\,g. \tag{3·24}$$

The axial vector/vector ratio (3·20) now becomes:

$$-G_A/G_V = \sqrt{(\tfrac{25}{18})} = 1{\cdot}18 \tag{3·25}$$

in excellent agreement with experiment (Lee 1966; Schwinger 1967*b*; Freund 1967).

We can now calculate the effective axial vector interaction. The leading term is the usual chiral axial vector coupling

$$2^{-\frac{1}{2}}G_A(\overline{N}\gamma^\lambda\gamma_5\tau^\dagger N)\,(\bar{e}\gamma_\lambda(1+\gamma_5)\,\nu_e). \tag{3·26}$$

This interaction has a form factor

$$F_A(t) = \{1-(t/m_A^2)\}^{-1}, \tag{3·27}$$

where m_A is the mass of the axial vector meson. If we identify it with the A_1 particle m_A^2 is approximately $2m_\rho^2$. In addition to the axial vector interaction (3·26) we have

an induced pseudoscalar term that comes from the pion part of the axial vector field. This is of the form

$$2^{-\frac{1}{2}}G_A(2m_e/m_\pi)\,\xi(m_A/m_\pi)^2\,(\bar{N}\gamma_5\,\tau^\dagger N)\,(\bar{e}(1+\gamma_5)\,\nu_e) \tag{3·28}$$

with an effective pseudoscalar coupling constant (to electrons!) (Goldberger & Treiman 1958; Wolfenstein 1958)

$$G_P = (2m_e/m_\pi)\,(m_A/m_\pi)^2\,\xi G_A \tag{3·29}$$

and a form factor

$$F_P = \{1-(t/m_\pi^2)\}^{-1}. \tag{3·30}$$

Time reversal violation

We also have a momentum dependent term

$$(G_A/\sqrt{2}m_A)\,(\bar{N}\tau^\dagger\sigma^{\lambda\nu}\gamma_5\,q_\nu N)\,(\bar{e}\gamma_\lambda(1+\gamma_5)\,\nu_e). \tag{3·31}$$

This is a 'weak electric dipole' coupling (which, however, does not contribute to electromagnetic interactions!) analogous to the 'weak magnetism' terms. But this term violates charge conjugation and time reversal invariance. It is worth pointing out that this term has no contribution from the pion exchange. Such a large violation of time reversal invariance should be viewed with caution, but the time reversal violation is *not* present in purely pion phenomena. The relevance of 'weak electric dipole' coupling in the present context is that it provides for a natural vehicle for time reversal violation in weak interaction without having it in electromagnetic or strong (pion) phenomena. But the time reversal violation is expected in nuclear β decay, although, being proportional to the momentum, the effect is less pronounced in small momentum transfer reactions. In a typical β decay reaction the energy release is of the order of several keV; but even for a reaction with 1 MeV energy release, the contribution of (3·22) is a CP violating amplitude whose ratio to the normal amplitude is of the order

$$(f'/f)\,(m_e/m_A) \simeq 10^{-3}. \tag{3·32}$$

This is the ratio of the *amplitudes*. The present upper limits on the degree of CP violation in β decay is of the order of several parts per cent in amplitude; the expected effect is more than an order of magnitude smaller! For a corresponding hyperon β decay the effect would be much more pronounced; we would expect an effect of the order of 10 % which may be measurable in the near future.

While these effects are 'small', they are of great interest in view of the observed CP violation in neutral kaon decay (Christenson *et al.* 1964). Since the effect is dependent on momentum transfer it could be enhanced by considering large momentum transfer reactions like neutrino initiated processes. The reactions with electrons or muons at large angles with respect to the incident neutrino direction would show the most pronounced effect; the momentum transfer dependence of this term is governed by the form factor (3·27) which falls off only rather slowly.

Decay of the pion

We now consider the decay of the pion. In the present theory this is a direct process governed by (3·8), (3·10) and (3·12). The decay interaction is of the form

$$(Gm_A^2/g)\,(\xi/m_\pi)\,\partial^\lambda\phi_\pi(\bar{\mu}\gamma_\lambda(1+\gamma_5)\,\nu_\mu) = (h/m_\pi)\,\partial^\lambda\phi_\pi(\bar{\mu}\gamma_\lambda(1+\gamma_5)\,\nu_\mu) \tag{3·33}$$

with

$$h = (Gm_A^2\,\xi/g). \tag{3·34}$$

The quantity h is a dimensionless parameter that can be determined from the decay of the pion. The pion decay lifetime is related to this quantity by the relation

$$\tau(\pi) = \frac{h^2}{4\pi}\left(\frac{m_\mu}{m_\pi}\right)^2\left[1-\left(\frac{m_\mu}{m_\pi}\right)^2\right]^{-2} m_\pi^{-1}, \tag{3·35}$$

where we have taken account of the dominance of the decay mode

$$\pi^\dagger \to \mu^\dagger + \nu_\mu.$$

Using the observed lifetime

$$\tau(\pi) = 2{\cdot}551\times 10^{-8}\,\text{second}$$

we get

$$h = 1{\cdot}48\times 10^{-7}. \tag{3·36}$$

Since G is known, and since m_A^2 is identified with the square of the A_1 particle mass we can use this to deduce

$$\xi/g = (m_\pi/m_A)^2\,(h/Gm_\pi^2) = 1{\cdot}02\times 10^{-2}. \tag{3·37}$$

It is worth pointing out that this computation is completely unaffected by the presence or absence of the term coupling $A_{\lambda\nu}$ with the coupling f' to the nucleons.

There are three other meson decay modes of interest. The first concerns the decay of the A_1 meson according to

$$A_1 \to \mu + \nu.$$

If ϵ denotes the polarization of the A_1 the decay proceeds through the interaction (3·8) with the matrix element

$$M(A_1 \to \mu\nu) = (Gm_A^2/g)\,\bar{\mu}\,\not{\epsilon}\,(1+\gamma_5)\,\nu_\mu. \tag{3·38}$$

This decay could be easily ignored as far as branching ratios are concerned since there exists the fast decay

$$A_1 \to \rho + \pi$$

via strong interactions. A similar process is the decay of the vector meson according to

$$\rho \to \mu + \nu$$

with the decay matrix element

$$(Gm_\rho^2/g)\,\bar{\mu}\,\not{\epsilon}\,(1+\gamma_5)\,\nu_\mu. \tag{3·39}$$

This decay is again negligible compared with the fast decay

$$\rho \to \pi + \pi.$$

The electronic decay modes of A_1 and ρ can also be written down similarly.

It is interesting to note that while (3·38), (3·39) predict comparable rates (the axial vector decay is somewhat faster!) for the electronic and muonic decay modes, this is in sharp contrast with the two-body decay of the pion. It is well known that the axial vector interaction suppresses the electronic mode significantly. This result is recovered in the present theory also. One finds for the branching ratio (Ruderman & Finkelstein 1949)

$$R_2 = \frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left[\frac{1-(m_e/m_\pi)^2}{1-(m_\mu/m_\pi)^2}\right]^2 = 1{\cdot}2 \times 10^{-4}, \tag{3·40}$$

which is to be compared with the experimental number

$$R_2 = (1{\cdot}21 \pm 0{\cdot}01) \times 10^{-4}.$$

We could also consider the three-body decay ('β decay') of the pion

$$\pi^+ \to \pi^0 + e^+ + \nu_e. \tag{3·41}$$

This involves the zero-momentum transfer coupling of the vector meson with pions. If we consider that the vector coupling of the vector meson with hadrons is universal we could calculate the effective matrix element for this process

$$M(\pi \to \pi e\nu) = 2m_\pi G\bar{e}\gamma^0(1+\gamma_5)\,\nu_e. \tag{3·42}$$

The strong vector coupling constant cancels out in the final expression for the matrix element, which we have written down in the lepton rest frame. The branching ratio for this mode can be calculated giving (Feynman & Gell-Mann 1958)

$$R_3 = \frac{\Gamma(\pi \to \pi e\nu)}{\Gamma(\pi \to \mu\nu)} = 1{\cdot}07 \times 10^{-8}, \tag{3·43}$$

in good agreement with the observed value

$$R_3 = (1{\cdot}12 \pm 0{\cdot}08) \times 10^{-8}.$$

Neutral lepton current

One of the most interesting questions related to β decay is whether there are neutral lepton currents. For example, is it possible to have an interaction of the form

$$G'(m_A^2 A_0^\lambda + m_\rho^2 V_0^\lambda)\,(\bar{\mu}\gamma_\lambda(1+\gamma_5)\,\mu), \tag{3·44}$$

where A_0^λ, V_0^λ are neutral vector fields? Because of the chiral nature of the weak vector coupling, if such a neutral lepton current exists we would expect it to be coupled to both neutral vector and axial vector fields. However, since the neutral vector isovector field is coupled to the electromagnetic field, all currents coupled to V_0^λ must be strictly conserved. But

$$\partial^\lambda(\bar{\mu}\gamma_\lambda(1+\gamma_5)\,\mu) = 2m_\mu(\bar{\mu}\gamma_5\,\mu) \neq 0, \tag{3·45}$$

hence the coupling (3·44) is forbidden. Similar considerations extend to the neutral current of electrons also. Since the right-hand side is proportional to the lepton mass a neutral current of neutrinos is not forbidden. Such a current, however, cannot lead to the two-body leptonic decay of a pseudoscalar neutral meson. The present experimental indications are that there are no neutral lepton currents in weak

interactions; we point out that a three-body leptonic mode with two neutrinos could easily be confused for a radiative decay mode. It would be of great interest to know if there are neutral currents of neutrinos or not.

It is remarkable that in the present theory, as far as hadron weak (or electromagnetic!) interactions are concerned, there is an 'intermediate boson'. For weak interactions of the nucleons we have in fact three of them, ρ, A_1, and π. We have thus restored Yukawa's hypothesis (Yukawa 1935) that the β interactions are a primary property of the mesons.

4. Weak interactions of strange particles

The considerations of the β decay of ordinary particles may be extended to β decay (and the analogous process involving muons) of the strange hadrons. In this case we would expect to have to extend simultaneously our strong interaction framework also to cover weak interactions. It has become clear that grouping together particles into multiplets which furnish representations of $SU(3)$ leads to a further systematization of the particles. The stable spin $\frac{1}{2}$ even-parity baryons can be identified as an octet and the mucleon resonance together with Y_1^*, Ξ^* and Ω would provide a spin $\frac{3}{2}$ even-parity decimet of $SU(3)$ (Gell-Mann 1962; Néeman 1961). These objects together immediately furnish the 56-dimensional representation of $SU(6)$. It would then be interesting to ask whether the vector-axial vector coupling scheme for $SU(4)$ could be extended to $SU(6)$. When proceeding from $SU(2)$ to $SU(3)$ we can see that the hypercharge becomes a generator; this is a simplification over the $SU(2)$ case where the electric charge contained both isoscalar and isovector components.

The meson fields are also now $SU(3)$ multiplets (Ikeda, Ogawa & Ohnuki 1959; 1960; Salam & Ward 1961). The observed vector mesons constitute a nonet which is the sum of an octet and a singlet. The same is true of the pseudoscalar mesons; and presumably of the associated axial vector fields. There are no scalar particles associated with the vector fields. We should therefore continue to write:

$$\partial^\lambda V_\lambda^\alpha = 0, \tag{4·1}$$

$$A_\lambda^\alpha = B_\lambda^\alpha + \xi(1/m_\alpha)\,\partial_\lambda \phi^\alpha. \tag{4·2}$$

There are two essential difficulties involved in the passage from isospin to unitary symmetry. The first one is a purely technical one: there are two ways of invariantly coupling three octets. They may be written in terms of the so-called D and F type couplings. The vector couplings of the vector mesons are expected to be of the F type while the axial vector meson couplings are expected to have a D/F ratio of $\frac{3}{2}$ according to the $SU(6)$ model. But we have to appeal eventually to experiment to give the coupling constants.

The second difficulty concerns the large mass differences between particles in the same multiplet and this 'broken symmetry' is a fundamental difficulty in the appeal to any simple framework of unitary symmetry. We shall see below that in our theory the absence of scalar particles for the vector fields prevents them from being vector coupled to the baryons.

Leptonic decays: dynamical origin of Cabibbo suppression factors

The primary weak interaction is obtained by the direct chiral coupling of the strange vector and axial vector meson fields with lepton currents

$$-(Gm_A^2/g)(A^\lambda + A'^\lambda)(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e), \tag{4·3}$$

where A and A' correspond to the non-strange and strange charged axial vector meson fields. The vector meson fields have similarly the coupling

$$-(Gm_\rho^2/g)(V^\lambda + V'^\lambda)(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e), \tag{4·4}$$

where V, V' are the non-strange and strange charged vector meson fields. Given these couplings we can proceed to calculate the decay of vector and axial vector mesons into leptons. These are quite straightforward; compare (3·38) and (3·39). The decay rates depend essentially on phase space.

The more interesting decay comparison is that between the non-strange and strange pseudoscalar meson decays. The decay matrix element for the pion was already computed

$$\begin{aligned} M(\pi \to \mu\nu) &= h(1/m_\pi)\,\partial^\lambda\phi_\pi(\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu); \\ h &= -m_A^2\,\xi(G/g). \end{aligned} \tag{4·5}$$

By an identical calculation we get

$$M(K \to \mu\nu) = h\frac{1}{m_\kappa}\,\partial^\lambda\phi_\kappa(\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu). \tag{4·6}$$

We can now calculate the kaon lifetime in terms of the pion lifetime, since the constant h cancels in this ratio. We find

$$\tau(K^+)/\tau(\pi^+) = (m_\kappa/m_\pi)\left[\frac{1-(m_\mu/m_\pi)^2}{1-(m_\mu/m_\kappa)^2}\right]^2 R, \tag{4·7}$$

where R is the branching ratio of the two-body leptonic mode in K^+ decay. Experimentally,

$$R = 0{\cdot}65,$$

$$\tau(\pi^+) = 2{\cdot}55\times 10^{-8}\,\text{second}$$

so that

$$\tau(K^+) = 1{\cdot}17\times 10^{-8}\,\text{second}, \tag{4·8}$$

which is to be compared with the experimental value

$$\tau(K^+) = 1{\cdot}22\times 10^{-8}\,\text{second},$$

is found to be in good agreement. It is interesting to write the pion and kaon decay amplitudes in the form:

$$\left.\begin{aligned} M(\pi^+ \to \mu\nu) &= h'\cos\theta_M\,\partial^\lambda\phi_\pi(\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu), \\ M(K^+ \to \mu\nu) &= h'\sin\theta_M\,\partial^\lambda\phi_\kappa(\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu). \end{aligned}\right\} \tag{4·9}$$

Then θ_M is called the Cabibbo angle (Cabibbo 1963). We have the explicit value, according to (4·5) and (4·6) (Gell-Mann 1962; Pradhan & Patnaik 1967)

$$\tan\theta_M = (m_\pi/m_\kappa) = 0{\cdot}28. \tag{4·10}$$

This angle is therefore no longer a new parameter for strange particle weak interactions; we have a dynamical origin for the Cabibbo angle for two-body leptonic decay of mesons.

Let us now see if the situation persists for the axial vector leptonic decays of hyperons. In this case also, experimentally we have a suppression over and above that given by the simple 'phase space' estimate. Since, according to (4·3), the strange axial vector meson is coupled equally strongly to leptons as the non-strange axial vector meson, any such suppression would come only from the modification of the coupling of the strange axial vector meson as compared with the non-strange axial vector meson. The simple symmetry predictions are not likely to be satisfactory. However we do know that the strange pseudoscalar mesons are strongly coupled to the baryons as seems to be suggested by self-consistency calculations involving the octet and the decimet. A direct analysis of the kaon-nucleon data also seems to verify this. We shall accordingly deduce the effective kaon-hyperon vertex by restricting attention to the longitudinal part of the axial vector field and requiring $SU(3)$ symmetry for it.

Let $d/(1-d)$ denote the D/F ratio; we shall eventually choose $d = 0{\cdot}6$ suggested by $SU(6)$ or by $SU(3)$ bootstrap calculations involving octets and decimets. Let us all consider the hyperon leptonic decay:

$$\Lambda \to p + e + \bar{\nu}. \tag{4·11}$$

The axial vector part of the decay vertex is given by a general expression of the form

$$\Gamma_A^\lambda = F_1 \gamma^\lambda \gamma_5 + F_2 \gamma_5 q^\lambda. \tag{4·12}$$

The 'longitudinal' part of this vertex is proportional to the coupling of the pseudoscalar mesons by virtue of the PCAC relation,

$$\left.\begin{aligned} \partial_\lambda A'^\lambda &= \xi m_K \phi_K, \\ \partial_\lambda A_\lambda &= \xi m_\pi \phi_\pi. \end{aligned}\right\} \tag{4·13}$$

The coupling of the pseudoscalar mesons is assumed to be according to $SU(3)$. For the ratio of the ΛpK vertex and $np\pi$ vertex we have the ratio

$$(3-2d)/\sqrt{6}.$$

Computing the longitudinal parts of the axial vector vertices for the strange and non-strange cases we get

$$(m_\Lambda + m)\, m_K F_1 = 2m\, m_\pi\, g_A (3-2d)/\sqrt{6}. \tag{4·14}$$

In deriving this relation we have ignored the term $tF_2(t)$ since it is small compared with $(m_\Lambda + m)$. If we now define a Cabibbo angle for the axial vector hyperon decays by

$$F_1 = 6^{-\frac{1}{2}}(3-2d)\, g_A \tan\theta_B, \tag{4·15}$$

then

$$\tan\theta_B = \frac{2m}{m_\Lambda + m}\,\frac{m_\pi}{m_K} = 0{\cdot}26. \tag{4·16}$$

The partial decay rate for Λ hyperons (as well as the rate for leptonic decays of other hyperons) computed from this value of the F_1 is in good agreement with

experiment; incidentally we note that the baryon angles θ_B are all equal and practically equal to the meson angle θ_M. Since all decays are suppressed by the same amount the $SU(3)$ *octet property* (Cabibbo 1963) *of the vertex will be preserved.* For the vector decays these considerations do not apply but there are simpler considerations pointing to their suppression. We have already pointed out that the vector fields are divergence free,

$$\partial^\lambda V'_\lambda = 0. \tag{4·17}$$

The corresponding vector vertex is given by

$$\Gamma^\lambda_V = G_1 \gamma^\lambda + G_2 \sigma^{\lambda\nu} q_\nu. \tag{4·18}$$

The vanishing of the longitudinal part of the vertex (4·18) yields

$$(m_\Lambda - m)\, G_1 = 0.$$

Because the first factor is not zero it follows that

$$G_1 = 0. \tag{4·19}$$

Hence the strangeness violating vector decays must proceed entirely by virtue of the momentum-dependent 'magnetic' coupling G_2 and of the departure of $G_1(t)$ from $G_1(0)$. It is not possible to express this suppression in terms of a simple angle. Experimentally the leptonic decays via vector interaction of the hyperons are suppressed by somewhat more than the decays via the axial vector interactions.

Similar considerations apply also to the three-body leptonic decays of pseudoscalar mesons. The matrix element for the decay

$$K \to \pi + e + \nu \tag{4·20}$$

has a contribution only from the vector coupling. The general matrix element may be expressed in terms of a vector and a tensor coupling in the form

$$\Gamma^\lambda_V = H_1(q^\lambda + k^\lambda) + H_2(q^\nu - k^\nu)\,(q^\lambda k^\nu - q^\nu k^\lambda). \tag{4·21}$$

The vanishing of the longitudinal part of this vertex gives

$$(m_\kappa^2 - m_\pi^2)\, H_1 = 0,$$

where k^λ, q^λ are the kaon and pion four-momenta. Hence the leading terms for the vector decay must vanish;‡ the decay must proceed entirely by the 'irregular term' $H_2(t)$ and the departure of $H_1(t)$ from $H_1(0)$. The vector decay of kaons is also suppressed though it has not been possible to express it in terms of an angle.

It is important to point out that while the leptonic decays of the kaon and the hyperons are suppressed the leptonic decays of the strange vector or pseudovector particles is *not* suppressed. It is a crucial difference from the hypothesis of Cabibbo that the strangeness violating 'currents' all enter with a factor $\tan\theta$ weaker than the strangeness conserving 'currents'. Experimental verification of these rates are therefore very important.

We have essentially demanded that the vector K^* particle have only magnetic coupling with baryons or the $K\pi$ pair. This should be subject to experimental test. At the present time we have little direct information on these questions.

‡ Some consequences of this assumption were studied by Weinberg, Marshak, Okubo, Sudarshan & Teutsch (1958).

The continued validity of gauge invariance for the isovector electromagnetic coupling together with the coupling of the linear combination of the fields V, V', A, A' essentially requires that neutral currents be absent from strange particle decays; the current of two neutrinos is an exception.

Non-leptonic weak couplings

There is a third kind of weak interaction in which no leptons take part; these are the non-leptonic transitions. These may be brought about by a self-coupling of the strange vector and axial vector currents with non-strange vector and axial vector currents. It has been found satisfactory to make the hypothesis of an $SU(3)$ octet component for the transformation property of the non-leptonic weak interaction (Okubo 1963; Gell-Mann 1964). This assumes that the current contains both charged and neutral currents; this is essential, though no neutral lepton currents are involved. If we write the charged field coupling, we have

$$2^{-\frac{1}{2}}G(\sqrt{2}/g)^2\,(m_A^2\,A^\lambda+m_V^2\,V^\lambda)\,(m_A^2\,A'_\lambda+m_V^2\,V'_\lambda). \tag{4·22}$$

If the total interaction is to be made isospin $\Delta I=\frac{1}{2}$ we must add a 'neutral' contribution

$$\tfrac{1}{2}G(\sqrt{2}/g)^2\,(m_A^2A_0^\lambda+m_V^2\,V_0^\lambda)\,(m_A^2A'_{0\lambda}+m_A^2\,V'_{0\lambda}). \tag{4·23}$$

To assure the octet transformation proporty this has to be further amended. But the important point to be made is that all the constants entering the interactions are already specified by other interactions and we should seek a zero-parameter fit to the data on non-leptonic decays.

We shall not attempt to make any detailed calculations here. Instead we will refer to a detailed calculation of the three-pion decays of kaons by Graham & Yun (1967). They have used the couplings (4·22) and (4·23) with the identification (4·2) for the axial vector fields. They obtain good agreement with transition rates and reasonable predictions for the ratio of s-wave to p-wave contributions. Related computations can be made for the nonleptonic decays of hyperons and for the kaon decay into two pions. Crude estimates using the simplest processes give $\sim 10^{-10}$ s. For the lifetime, which is to be contrasted with the observed value estimated of $2{\cdot}5\times10^{-10}$ s for the hyperon while the observed value is $0{\cdot}9\times10^{-10}$ s for the kaon. While these numbers are not to be taken seriously they do indicate that the universal theory of interactions can predict the non-leptonic decays also.‡

We thus find that the more important aspects of weak interactions are in accordance with the proposed theory. Since the strangeness changing vector current is not coupled to baryons by a vector coupling it proceeds through magnetic terms as well as through parity violating self-mass terms. To the extent these self-masses, like the parity conserving self-masses, are resulting from strangeness violating meson–meson couplings of vector-axial vector meson fields, they would both *transform as octets*. The suppression of the leptonic decays therefore does not affect the non-leptonic decays. This is in accordance with experiments.

‡ This mechanism for parity violating decays is different from the K^* exchange model of Schwinger (1964) and of Lee & Swift (1964).

It is to be remembered that in the usual theory the Cabibbo suppression of leptonic decays and the apparent lack of suppression of the non-leptonic decays make it difficult to write down any simple current-current interaction. In the present theory the same problem does not arise since the two kinds of decay proceed by entirely different mechanisms. Such a possibility does not obtain in the framework in which the four-fermion interaction of hadrons is considered primary. The question whether the strange vector mesons have no vector couplings with the hadrons would be of great interest as far as strong interaction phenomenology is concerned. It is amusing to note that, provided we restrict our attention to only Yukawa couplings of the vector mesons, the meson masses get *depressed* by the strong coupling; it is therefore attractive to consider the idea that this is the mechanism that depresses the ρ and ω masses relative to the K^* mass. If we postulate the suppression of the (axial) vector coupling of the strange axial vector mesons, which we have derived in (4·15) and (4·16) above, we have a qualitative understanding of the axial vector (and pseudoscalar) masses.

We see that the present theory provides a satisfactory framework for all weak interactions in terms of a universal coupling constant. To the extent that the pion decay rate can be deduced from the Goldberger–Treiman relation, the parameter ξ can itself be determined from the formula

$$\xi^2 = \tfrac{1}{2}(m_\pi/m_A)^2 = 0{\cdot}8\times 10^{-2} \tag{4·24}$$

from the A meson and pion masses; it is therefore *not* a new constant. Only the strong interaction coupling constant g and the weak coupling constant G are to be considered as free parameters.

5. Strong interactions

From the study of the electromagnetic and weak interaction properties of the hadrons we have been led to propose a meson–nucleon coupling structure of the form

$$\begin{aligned}\tfrac{1}{2}\overline{N}\{&g\gamma^\lambda\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\lambda+g_0\gamma^\lambda\omega_\lambda+g'\sigma^{\lambda\nu}\boldsymbol{\tau}\cdot\tfrac{1}{2}\boldsymbol{\rho}_{\lambda\nu}+g'_{00}\sigma^{\lambda\nu}\tfrac{1}{2}\phi_{\lambda\nu}\\&+f\gamma^\lambda\gamma_5\boldsymbol{\tau}\cdot\mathbf{A}_\lambda+f_0\gamma^\lambda\gamma_5 Z_\lambda+f'\sigma^{\lambda\nu}\gamma_5\boldsymbol{\tau}\cdot\tfrac{1}{2}\mathbf{A}_{\lambda\nu}+f'_0\sigma^{\lambda\nu}\gamma_5\tfrac{1}{2}Z_{\lambda\nu}\}N,\end{aligned} \tag{5·1}$$

with

$$\left.\begin{aligned} g' &= \tfrac{5}{3}g, & g_0 &= g'_{00} = g,\\ f = f' &= \sqrt{\tfrac{25}{18}}g, & f_0 &= f'_0 = 2^{-\frac{1}{2}}fg.\end{aligned}\right\} \tag{5·2}$$

This involves the coupling of the pseudoscalar mesons in view of the decomposition

$$A^\lambda = B^\lambda + \xi\left(\frac{1}{m_\pi}\partial^\lambda\phi_\pi\right), \tag{5·3}$$

etc. The coupling structure (5·1) was obtained by considering an $SU(4)$ non-invariance group for which the nucleons and nucleon resonance constitute a 20-dimensional representation and for which the 'charge' and 'magnetic field' components of the vector fields constitute a meson matrix with 15 components; and an alternative meson matrix is obtained from considering the vector and axial vector meson fields. We have found that this scheme gives a quantitative understanding of the electromagnetic properties of the hadrons; and of weak interaction

phenomena when it is supplemented by universal meson–photon and meson–lepton interactions. The quantitative predictions were essentially independent of the absolute magnitudes of the strong coupling constants. But the strong interaction phenomena depend upon the magnitude of the strong coupling constant g. We shall content ourselves with a brief list of predictions.

Scattering lengths for S-waves

Given the interaction structure (5·1) we can calculate the low energy pion-nucleon scattering parameters, in particular the s- and p-wave scattering lengths. The s-wave scattering proceeds via two mechanisms: the dominant one is vector meson exchange; the other one is the exchange of the nucleon and the nucleon resonance. A direct calculation (Pradhan, Sudarshan & Saxena 1967) yields the values:

$$a_1 = -2a_3 = (a_\rho + a_n)\,(1+(m_\pi/m))^{-1}, \tag{5·4}$$

where

$$\left.\begin{aligned} a_\rho &= +\frac{g^2}{4\pi}\left(\frac{m_\pi}{m_\rho}\right)^2 m_\pi^{-1}, \\ a_n &= -\frac{f^2}{4\pi}\,2\xi^2\left(\frac{4m}{M}-1\right)m_\pi^{-1}, \end{aligned}\right\} \tag{5·5}$$

where m, M are the masses of the nucleon and the nucleon resonance respectively. With the numerical values

$$g = 9{\cdot}0, \quad f = \sqrt{\tfrac{25}{18}}\,g, \quad \xi = 0{\cdot}09 \tag{5·6}$$

we get

$$a_\rho = 2{\cdot}15 m_\pi^{-1}, \quad a_n = -0{\cdot}31 m_\pi^{-1}, \tag{5·7}$$

so that

$$\left.\begin{aligned} a_1 &= +0{\cdot}160 m_\pi^{-1}, \\ a_3 &= -0{\cdot}080 m_\pi^{-1}. \end{aligned}\right\} \tag{5·8}$$

These values are in good agreement with the experimental values

$$\left.\begin{aligned} a_1 &= +0{\cdot}183 m_\pi^{-1}, \\ a_3 &= -0{\cdot}109 m_\pi^{-1}. \end{aligned}\right\} \tag{5·9}$$

Scattering lengths of p-waves

A corresponding calculation of the p-wave scattering length is most easily carried out in the static limit. This is the case that has been extensively studied using the non-invariance group model (Kuriyan & Sudarshan 1967; Fairlie 1967; Deshpande 1967). In this case, independent of the absolute value of the coupling constant it is possible to show that (Deshpande 1967)

$$a_{13} = a_{31} = \tfrac{1}{4}a_{11} \tag{5·10}$$

which is in reasonable agreement with experiment. To make a more quantitative estimate we calculate the nucleon and nucleon isobar exchange to get

$$\left.\begin{aligned} a_{11} &= \left(\frac{f^2}{4\pi}\right)(\tfrac{1}{2}\xi^2)\left[1+\left(\frac{m_\pi}{m}\right)\right]^{-1}\left\{-\frac{8}{3}+\frac{16}{9}\left(\frac{f^*}{f}\right)^2\left(\frac{m_\pi}{M-m+m_\pi}\right)\right\}m_\pi^{-1}; \\ a_{13} &= \left(\frac{f^2}{4\pi}\right)(\tfrac{1}{2}\xi^2)\left[1+\left(\frac{m_\pi}{m}\right)\right]^{-1}\left\{\frac{4}{3}+\left(\frac{f^*}{f}\right)^2\left(\frac{m_\pi}{M-m-m_\pi}\right)\right. \\ &\qquad\qquad \left.+\frac{1}{9}\left(\frac{f^*}{f}\right)^2\left(\frac{m_\pi}{M-m+m_\pi}\right)\right\}m_\pi^{-1}, \end{aligned}\right\} \tag{5·11}$$

where $(f^*/f) = \sqrt{\frac{3}{2}}$ is the ratio of the $N^*N\pi$ to $NN\pi$ coupling constant. Substituting the numerical values we find

$$\left.\begin{aligned} a_{11} &= -0{\cdot}115 m_\pi^{-1}, \\ a_{13} = a_{31} &= -0{\cdot}029 m_\pi^{-1}, \\ a_{33} &= +0{\cdot}177 m_\pi^{-1}. \end{aligned}\right\} \tag{5·12}$$

These numbers are to be compared with the observed values

$$\left.\begin{aligned} a_{11} &= -0{\cdot}101 m_\pi^{-1}, \\ a_{13} &= -0{\cdot}029 m_\pi^{-1}, \\ a_{31} &= -0{\cdot}038 m_\pi^{-1}, \\ a_{33} &= +0{\cdot}215 m_\pi^{-1}, \end{aligned}\right\} \tag{5·13}$$

and they are in reasonable agreement.

It has been already pointed out that for reactions like nucleon resonance production in meson–nucleon collisions, one could deduce a 'crossing symmetry sum rule' (Kuriyan 1966)

$$M(\pi^+ p \to N^{*+}\pi^+) = M(\pi^- p \to N^{*+}\pi^-) \tag{5·14}$$

which is identical with the relation

$$A_1 = \sqrt{10}\, A_3 \tag{5·15}$$

The experimental data has been analysed (Olsson 1965) to yield

$$A_1/A_3 = +3{\cdot}34 \tag{5·16}$$

in good agreement‡.

Nuclear forces

The most primitive manifestation of strong interactions are of course in the nuclear force. Ever since Yukawa postulated a meson theory attempts have been made to deduce a quantitative expression for the nuclear force. We shall content ourselves with several observations about the basic consequences of our interaction structure (5·1). The nuclear forces now consist of three distinct ranges,§ the longest range 'tail' governed by the pion Compton wavelength. This force consists of the familar spin-dependent central force plus the tensor force. The tensor force is singular as r^{-3} and should therefore be subject to cancellation by other such singular terms. This feature was recognized almost two decades ago,¶ but we now have such contributions coming from the vector and axial vector fields. The second range is determined by the vector meson wavelength; and finally we have the axial-vector meson wavelength as the third range. These are in qualitative agreement with the

‡ *Note added in proof, May* 1968. Biswas & Saxena (1968) have employed the present theory to calculate the decay widths of axial vector mesons. For the A_1 width they predict 103 MeV, in good agreement with experiments. They predict also the s- and d-wave coherence parameter for this decay.

§ The Japanese Group has discussed such a classification of the 'regions' for nuclear forces.

¶ See Rosenfeld (1949) and Schwinger (1942) for a discussion of early approaches to the problem.

nature of the nuclear force, but a quantitative analysis has to be carried out to see the extent to which we could understand the nuclear forces on the basis of the present framework.

It is amusing to note that the required cancellation of the highly singular tensor force terms enables us to relate the meson masses. Within our interaction structure this relation simplifies to yield a very simple relation between the pion and vector meson masses. We have thus the crude estimate

$$m_\pi/m_\rho = \xi f/g \simeq 0{\cdot}13, \tag{5·17}$$

for which the observed value is 0·18. This estimate is not particularly good, but is of the right order of magnitude.

6. Summary and outlook

We have constructed and analysed a theory of universal primary interactions of particles which encompass strong, electromagnetic and weak interactions. The vector-axial vector structure, which was first deduced from an analysis of weak interaction data, has been shown in this theory to be equally important in strong interactions. On the basis of this crucial observation we are able to define a unique strong interaction structure in which the only new parameter is the effective p-wave pion nucleon coupling constant which has the experimental value of $f = 1{\cdot}0$.

We have shown that this unique structure is in accord with low energy nucleon–nucleon interaction, with low-energy s- and p-wave pion–nucleon interaction and the magnitudes of nucleon magnetic moments. We have also obtained a crude estimate for the mass of the ρ meson from the requirement of a non-singular tensor force.

Applied to weak interaction theory we can predict the ratio of vector to axial vector interactions in excellent agreement with experiments. The relationship of this number to pion–nucleon scattering amplitude is now a simple dynamical connexion.

The theory seems to yield several useful results when applied to weak interactions of strange particles. The apparent suppression of the leptonic decays of kaons and hyperons, parametrized by the Cabibbo angle, is here deduced from the universal vector-axial vector theory. For non-leptonic decays we could get quantitative predictions. We could now calculate the absolute parity conserving and parity violating decay amplitudes for hyperons. In both cases we could not only get the ratio of the amplitudes but the absolute rates can be predicted. Incidentally the absolute pion decay rate is also computed in agreement with experiment.

The structure of the p-wave meson–baryon interactions has already been discussed extensively in connexion with compact non-invariance group models. The relations between strong interaction processes are discussed there; they were found to be in excellent agreement with experiment. In the present theory we are able to calculate the absolute values of the various strong interaction amplitudes as well. It is also to be pointed out that the non-invariance group intrinsic to the present theory is compact.

Even-parity spin-one particles seem to play an important role in the formulation. It would be very satisfactory if such particles are discovered and established.

The electromagnetic interactions now consist of two types: the familiar Dirac type ('minimal') coupling of the leptons with photons; and a different kind of interaction of the neutral vector mesons linearly with the photons. Electromagnetism is a primary property of the vector mesons; for the hadrons it becomes an acquired characteristic. The requirements of current conservation demand that the vector meson fields be divergence-free. No scalar mesons are therefore expected to be associated with these. It is an immediate manifestation of this two-step nature of electromagnetism of hadrons that the nucleon magnetic moments deviate so markedly from the values for Dirac particles.

Weak interactions of leptons still have the familiar chirality invariant form. But hadrons participate in weak interactions only by virtue of the coupling of the vector meson fields with lepton pairs; and by virtue of a bilinear coupling of the vector and axial vector mesons with themselves. There is only one universal coupling constant. The suggestion which Yukawa made and later abandoned in connexion with meson theory has been resurrected in the present theory. Weak interaction of the hadrons is an acquired characteristic; β decay is a property of the meson.

Within this framework the absence of neutral lepton currents, at least in strangeness conserving decays, is inevitable by virtue of the requirement of current conservation for electromagnetism. If the ρ or ω field were coupled to neutral chiral lepton currents, this would destroy current conservation. In the self coupling of vector mesons leading to non-leptonic decays are led to demand that the strange vector fields also should be divergence-free (and that parity violating interactions proceed by non-strange axial currents coupled to strange vector currents). The divergence-free nature of the strange vector current would be destroyed if they were coupled to neutral chiral lepton currents. We have thus a reason for not having any neutral lepton currents in vector decays. In the chirality invariance limit the pseudoscalar mass becomes zero and the neutral axial current is also conserved provided the axial current is not coupled to a neutral chiral lepton current. This suggests that the neutral axial current is also not coupled to neutral lepton currents. Apparently no experimental evidence is there for neutral lepton currents in either vector or axial vector decays.

In a manner of speaking, the vector and axial vector mesons are 'intermediate vector bosons' for hadron weak interactions. Purely leptonic interactions are direct couplings in the present theory and *no* intermediate vector bosons are required or expected.

The hadron form factors for weak as well as electromagnetic processes have an essentially universal form with a momentum transfer dependence governed by the axial vector and vector meson masses.

The CP violation in weak interaction is required by the present theory and there is a natural place for it in the theory in the 'magnetic' coupling of the axial vector field. It will make no contribution, under this situation, to pion phenomena in the low energy region or to the electric dipole moment of the neutron. No quantitative

measure of this possible CP violation has been calculated though estimates show it to be a very small fraction of a per cent in nuclear β decay.

The strange vector mesons are not coupled via the vector coupling to baryons. This leads to a strong mass difference in the right direction, since purely Yukawa interactions tend to depress the physical meson masses, the non-strange vector mesons are lighter than the strange ones.

There are still many unexplained features of particle physics; the remaining ones, however, deal with the existence of a particle spectrum, and the existence of a hierarchy of interactions. There is some hope that the baryon and meson mass spectra could be quantitatively understood, but the existence of the three categories (weak, electromagnetic, strong) of interactions remains unfathomed. There is also the perennial question of the electron-muon duality.

The author takes pleasure in thanking Professor J. Mehra for his interest in this work and Professor J. J. de Swart for his critical reading of the manuscript.

References

Belinfante, J. G. & Cutkosky, R. E. 1965 *Phys. Rev. Lett.* **14**, 33.
Biswas, S. N. & Saxena, R. P. 1968 *Phys. Rev.* (in the Press).
Bühring, H. & Schopper, H. 1965 quoted in Lee & Wu (1965).
Cabibbo, N. 1963 *Phys. Rev. Lett.* **10**, 531.
Capps, R. H. 1965 *Phys. Rev. Lett.* **14**, 31.
Christenson, J. H., Cronin, J. W., Fitch, V. L. & Turlay, R. 1964 *Phys. Rev. Lett.* **13**, 138.
Deshpande, N. G. 1967 *Phys. Rev.* (in the Press).
Fairlie, D. B. 1967 *Phys. Rev.* **155**, 1964.
Fermi, E. 1934 *Z. Phys.* **88**, 161.
Feynman, R. P. & Gell-Mann, M. 1958 *Phys. Rev.* **109**, 193.
Freund, P. G. O. 1967 *Phys. Rev. Lett.* **17**, 1021.
Gamow, G. & Teller, E. 1936 *Phys. Rev.* **49**, 895.
Gell-Mann, M. 1958 *Phys. Rev.* **111**, 362.
Gell-Mann, M. 1962 *Phys. Rev.* **125**, 1067.
Gell-Mann, M. 1964 *Phys. Rev. Lett.* **12**, 155.
Gell-Mann, M. & Lévy, M. 1960 *Nuovo Cim.* **16**, 705.
Gershtein, S. S. & Zeldovich, J. B. 1955 *J. exp. theor. Phys.* (*USSR*) **29**, 698.
Goldberger, M. L. & Treiman, S. B. 1958 *Phys. Rev.* **110**, 1178.
Graham, R. H. & Yun, S. K. 1967 Syracuse University Report.
Gürsey, F. & Radicati, L. 1964 *Phys. Rev. Lett.* **13**, 173.
Ikeda, M., Ogawa, S. & Ohnuki, Y. 1959 *Progr. Theor. Phys.* **22**, 715.
Ikeda, M., Ogawa, S. & Ohnuki, Y. 1960 *Progr. Theor. Phys.* **23**, 1073.
Kabir, P. K. 1963 *The development of weak interaction theory*. New York: Gordon and Breach.
Kuriyan, J. G. 1966 Doctor's Thesis, Syracuse.
Kuriyan, J. G. & Sudarshan, E. C. G. 1967 *Phys. Rev.* (in the Press).
Lee, B. W. 1966 *Proc. Thirteenth Int. Conf. High Energy Physics.*
Lee, B. W. & Swift, A. R. 1964 *Phys. Rev.* **136**, B 228.
Lee, T. D. & Wu, C. S. 1965 *Ann. Rev. Nucl. Sci.* **15**.
Lee, Y. K., Mo, L. W. & Wu, C. S. 1963 *Phys. Rev. Lett.* **10**, 253.
Nambu, Y. 1960 *Phys. Rev. Lett.* **4**, 380.
Néeman, Y. 1961 *Nucl. Phys.* **26**, 222.
Okubo, S. 1963 *Phys. Lett.* **4**, 14.
Olsson, M. G. 1965 *Phys. Rev. Lett.* **15**, 710; **15**, 768 (E).
Pradhan, T. & Patnaik, M. 1967 *Saha Inst. Rep.*
Pradham, T., Sudarshan, E. C. G. & Saxena, R. P. 1968 *Phys. Rev. Lett.* **20**, 79.

Renninger, G. H. & Videira, A. 1967 *Phys. Rev.* (in the Press).
Rosenfeld, L. 1949 *Nuclear forces*. Amsterdam, North-Holland.
Ruderman, M. & Finkelstein, R. J. 1949 *Phys. Rev.* **76**, 1458.
Sakurai, J. J. 1960 *Annls Phys.* **11**, 1.
Sakurai, J. J. 1966 *Phys. Rev. Lett.* **17**, 1021.
Salam, A., Delburgo, R. & Strathdee, J. 1965 *Proc. Roy. Soc.* A **284**, 146.
Salam, A. & Ward, J. C. 1961 *Nuovo Cimento* **20**, 419.
Schwinger, J. 1942 *Phys. Rev.* **61**, 387.
Schwinger, J. 1964 *Phys. Rev. Lett.* **12**, 630.
Schwinger, J. 1967*a* *Phys. Lett.* **24** B, 473.
Schwinger, J. 1967*b* *Phys. Rev. Lett.* **18**, 923.
Signell, P. S. & Durso, J. W. 1967 *Phys. Rev. Lett.* **18**, 185.
Sudarshan, E. C. G. 1967 *Proc. Int. Conf. Particles and Fields.*
Sudarshan, E. C. G. & Marshak, R. E. 1957 *Proc. Padua-Venice Conf. on Mesons and Newly Discovered Particles;* reprinted in Kabir (1963).
Weinberg, S., Marshak, R. E., Okubo, S., Sudarshan, E. C. G. & Teutsch, W. B. 1958 *Phys. Rev. Lett.* **1**, 25.
Wess, J. & Zumino, B. 1967 *New York Univ. Rep.*
Wolfenstein, L. 1958 *Nuovo Cim.* **8**, 882.
Yukawa, H. 1935 *Proc. Phys-Math. Soc.* (*Japan*) **17**, 48; reprinted in *Suppl. Progr. Theor. Phys.* (1957–8).
Yukawa, H. 1949 *Rev. Mod. Phys.* **21**, 474.