A new formulation of relativistic quantum theory of fields with applications to particles travelling faster than light

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Introduction

The discovery of radiation oscillators by Planck (1) and of the statistics of photons by Bose (2) led us to believe that the proper relativistic description of quantum mechanical systems was by a "second-quantized" theory. The ideas of Planck and Bose found their logical completion in the operator formalism of Fock (3) which could describe a collection of symmetrized many-particle states with a variable number of particles. The method could be extended to cover the description of particles obeying Fermi statistics and to relativistic theories.

In the quantization of a Schrödinger field one started out by associating an independent harmonic oscillator with each solution of the Schrödinger equation. The second quantized field was then a linear sum of *all* the annihilation operators with coefficients which are the normalized wavefunctions for the corresponding mode. This field is not hermitian; and its hermitian conjugate is a linear sum of creation operators only. With a complete set of solutions of the Schrödinger equation included, the equal time commutation (or anticommutation) relations satisfied by the field are canonical.

On the other hand, the quantization of a relativistic field makes use of a complete set of positive energy solutions of the field. The field is then broken up into positive and negative frequency parts which are respectively associated with annihilation and creation operators for positive energy wave functions only. This decomposition is not "local" but does have the merit that we always deal with positive energy particles only. Such a method (or its equivalent) is universally employed to quantize fields. The restriction to only positive energy particles is so natural that general investigations of field theories have considered it to be axiomatic (4).

Nevertheless, it has been found desirable to reexamine this question and to study whether negative energy particles can be entertained in quantum field theory. On the one hand the possible existence of tachyons (5) (i.e. particles travelling faster than light) makes it necessary to develop a new method of second quantization since for the corresponding field the decomposition into positive and negative frequency parts is not relativistically invariant (6). On the other hand the study of infinite-component wave equations has shown that relativistic invariance does not guarantee both positive and negative frequency parts to a local wave field. What seems to be needed is a different formulation of quantum field theory which would apply uniformly to all relativistic fields.

We propose such a formulation of relativistic quantum field theory. It associates each of the complete set of free-particle solutions with an independent oscillator. No distinction is made between positive time-like, negative time-like or space-like solutions at this stage. The formulation is extended to interacting systems as well and rules for computation of the transition amplitudes for processes are developed. At this stage we make use of a reinterpretation principle which enables us to restrict attention to positive energy particles in any system. The method is applied to discuss the interactions of tachyons (7) as well as to the discussion of the discrete transformations involving time inversion. For the interaction of time-like (or light-like) particles treated by perturbation theory, the scattering amplitudes computed on the basis of this formulation coincide with the result obtained within the conventional formulation.

2. *Quantization of the free field*

We quantize a scalar field $\Phi(x)$ by associating the entire field with annihilation operators

$$\Phi(x) = (2\pi)^{-3/2} \int a(k)\, \delta(k^2 - m^2)\, e^{-i\mathbf{k}\mathbf{x}} d^4k$$

$$= (2\pi)^{-3/2} \int \{a(\mathbf{k}, \omega)\, e^{-i\omega x_0 + i\mathbf{k}\mathbf{x}} + a(\quad \mathbf{k}, \quad \omega)\, e^{i\omega x_0 - ikx}\} \frac{d^3k}{2\omega}$$

where

$$\omega = +\sqrt{m^2 + \mathbf{k}^2}$$

The operators satisfy the commutation relations

$$[a(\mathbf{k}, \pm\omega), a^{\dagger}(\mathbf{k}', \pm\omega')] = \pm 2\omega\delta(\mathbf{k} - \mathbf{k}')$$

All other commutators vanish. A standard state (the "vacuum") is chosen to obey:

$$a(\mathbf{k}, \pm\omega)|0\rangle = 0$$

It is now straightforward to give an interpretation of the quantized field as an assembly of bosons of both positive and negative energies. The quantization is relativistically invariant. The invariant commutator function is given by

$$[\Phi(x), \Phi^\dagger(y)] = i\Delta(x-y)$$

$$= -\frac{i}{(2\pi)^3}\int e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \sin \omega(x_0 - y_0)\frac{d^3k}{\omega}$$

and is manifestly causal. The contraction function

$$\tau_0(x, y) = \langle 0 | T(\Phi(x)\Phi^\dagger(y)) | 0\rangle$$

is given by

$$\tau_0(x, y) = \frac{i}{2}\varepsilon(x^0 - y^0)\, \Delta(x-y)$$

which is the time-symmetric Green's function:

$$(\Box_x^2 + m^2)\tau_0(x, y) = -\delta(x-y),$$

rather than the Stückelberg–Feynman causal Green's function.

If we wish to quantize the Dirac field we choose a complete set of solutions of Dirac equation with either sign of energy and write:

$$\psi(x) = \sum_{r=1}^{2}\int\{a_r(\mathbf{k}, \omega)\, u_r(\mathbf{k}, \omega)\, e^{-i\omega x_0 + ikx} + a_r(-\mathbf{k}, -\omega)\, u_r(-\mathbf{k}, -\omega)\, e^{i\omega x_0 - ikx}\}$$

$$\times \frac{d^3k}{2\omega}$$

The creation and annihilation operators satisfy the anticommutation relations:

$$\{a_r(\mathbf{k}, \pm\omega), a_s^\dagger(\mathbf{k}', \pm\omega')\} = 2\omega\delta_{rs}\,\delta(\mathbf{k} \quad \mathbf{k}')$$

All other anticommutators vanish. We can then deduce

$$\{\psi(x), \psi^\dagger(y)\} = S(x-y) = \left(i\gamma^\mu \frac{\partial}{\partial x^\mu} + M\right)\Delta(x-y)$$

which is also causal.

3. *Interacting fields*

A heuristic method of discussing the interactions between these fields is to proceed to the interaction picture and consider the time ordered expression for the S-operator (8)

$$S = T\{\exp i\int W(x)\, d^4x\}$$

where $W(x)$ is the interaction in the interaction picture. If we rewrite this operator expression as a normal-ordered expansion, the coefficients of the various terms are the unrenormalized expressions for the transition amplitudes for various processes. This reduction is easily carried out using Wick's theorem. But before doing that we recall that there is some freedom in the definition of the asymptotic fields in terms of which the field theory gets a particle interpretation. This freedom corresponds to the freedom in the choice of the Green's function. We make the choice

$$\Phi_{in}(x) = \Phi_0(x) - \frac{1}{4}\int \Delta^{(1)}(x-y)\,\xi(y)\,d^4y$$

$$\Psi_{in}(x) = \Psi_0(x) - \frac{1}{4}\int S^{(1)}(x-y)\,\eta(y)\,d^4y$$

where $\Delta^{(1)}$ and $S^{(1)}$ are the even invariant functions which are solutions of the homogeneous equations; where Φ_0, Ψ_0 are the interaction picture fields; and ξ and η are the sources of the boson and fermion fields. The second terms on the right hand side satisfy the respective free-field equations. With this choice of the asymptotic fields, the effective contraction function for the boson becomes

$$\tau(x, y) = \tau_0(x, y) - \frac{i}{2}\Delta^{(1)}(x-y) = \Delta_c(x-y)$$

A similar relation holds for the fermion contraction function also. We have thus recovered the conventional unrenormalized expansion of the scattering amplitude as a power series in the coupling constant. The result so obtained contains all the familiar infinities of perturbation theory; the heuristic method of renormalization of the perturbation expansion of the conventional theory can be transplanted into the present formalism to provide a renormalized perturbation expansion in which each term is finite.

Since the renormalized perturbation series yields an amplitude which is unitary to the order of approximation desired, it follows that in our new formalism the unitarity relation is true when only the positive energy particles are included in the intermediate states, despite the formal introduction of negative energy states.

4. *Negative energy particles: the reinterpretation principle*

We now wish to interpret the negative energy particles. For this purpose we consider processes in which they participate. Classically if we stipulate that a particle has a displacement in the same direction as the momentum, then it follows that the elapsed time Δt would change sign as the energy changes sign.

In other words, negative energy particles travel backward in time. Consider an apparatus I which emits a particle at time t_1 and another apparatus II which absorbs it at time t_2. For the case of a positive energy particle, t_2 is later than t_1 and energy is transferred from I to II. On the other hand, in the case of a negative energy particle t_2 is earlier than t_1 and apparatus II registers a net loss of energy. It is more satisfactory to interpret this as the emission of a positive energy particle at t_2 by apparatus II which is subsequently absorbed by apparatus I at time t_1. For negative energy particles we should interchange emissions and absorptions and consider only positive energy.

Similar considerations apply in quantum theory also since, by Ehrenfest's theorem, on the average the particle moves in the direction of the momentum. But in quantum mechanics an interchange of emission and absorption processes is given by "crossing". We interpret negative energy particles in the initial state as positive energy antiparticles in the final state.

It is essential to note that crossing can be applied to transition amplitudes, not to quantum mechanical states. This implies in turn that while we can restrict attention to only transition amplitudes with positive energy objects in the initial and final states, we cannot make such a choice on the states. The present method is therefore somewhat different from Dirac's hole theory of the positron (9). And we have already seen that the method applies to both boson and fermion systems.

5. *Time inversion and strong reflection*

The present formulation of quantum field theory is relativistically invariant but we have not discussed the discrete transformations. For the spin 0 field the space reflection transformation can be viewed purely geometrically; on the annihilation operators it is

$$a(\mathbf{k}, \pm\omega) \rightarrow \eta_p a(-\mathbf{k}, \pm\omega)$$

where η_p is the intrinsic parity. This is consistent with the commutation relations and is the same as in the usual formulation. On the other hand, time reflection transformation cannot be so implemented in the usual formalism for the simple reason that there are no negative energy particles. But in the present formulation we have no such difficulty. We postulate accordingly:

$$a(\mathbf{k}, \pm\omega) \rightarrow \eta_c a(\mathbf{k}, \mp\omega)$$

where η_c is a phase factor still to be fixed. Since a positive energy particle would become a negative energy particle by this transformation we have to invoke our reinterpretation principle (6). As far as scattering amplitudes are concerned we may take a negative energy particle in the initial state as being a positive energy antiparticle with opposite momentum in the final state. For a selfconjugate particle η_c is thus simply the charge conjugation parity. More generally the purely geometric transformation that we have defined is not the conventional Wigner time reversal (10), but that followed by charge conjugation.

Related comments apply to the strong reflection transformation also. We define it geometrically by

$$a(\mathbf{k}, \pm\omega) \to a(-\mathbf{k}, \mp\omega)$$

for a scalar field. More specifically we consider the purely geometric transformation

$$\Phi(x) \to \Phi(-x)$$

$$\Psi(x) \to \gamma_5 \Psi(-x)$$

By the reinterpretation principle we see that this transformation is equivalent to the conventional TCP operation (11).

It is interesting to note that in the space of states both the TC and TCP operations are linear operations unlike in the conventional formulation.

We may postulate that the entire Action function is invariant under the strong reflection geometric transformation. We can use the imposition of this invariance, which may be called the *S*-principle, to deduce the fundamental relation between spin and statistics (12).

6. *Application to interactions of tachyons*

It has generally been believed that the theory of special relativity forbids the existence of particles travelling faster than light. Both Poincaré and Einstein have explicitly stated such a belief (13). Closer examination reveals that the usual arguments against their existence are invalid. On the basis of the reinterpretation principle discussed above, all the puzzles and paradoxes put up in connection with the existence of tachyons can be resolved. The quantization method discussed above is appropriate for such a situation since no decomposition of the field into positive and negative frequency solutions is involved. The change in the sign of the energy of a particle under a suitable change of frame is to be interpreted as a change in the physical identification of the process from one frame to another frame. Thus for example, what may be viewed as elastic scattering of a tachyon in one frame would appear as the dissociation of one particle into three in a suitable frame. These questions have been analyzed extensively elsewhere (5).

The present method of quantization leads to a simple expression for the *S*-matrix elements involving tachyons either in the internal or external lines.

It is obtained by the analytic continuation of the usual result to a suitable negative value of the square of the mass (7). We have to remember that tachyons of zero spin alone may be obtained in this fashion and they obey Bose statistics. On the basis of this result we could suggest several methods of experimental detection of tachyons.

The direct method of determination of the square of the mass by measuring the momentum and energy has been tried already by Alväger & Erman (14) in an experiment. The alternate method of looking for Čerenkov emission as a direct measure of the velocity has been tried by Alväger *et al.* (15). Both these experiments presupposed that the tachyons are electrically charged; and their negative result may be interpreted to mean that tachyons, if they exist, carry no electric charge. The following methods can be used to search for neutral tachyons as well:

1. Search for "decays in flight" of a stable particle. If we see a proton decay in flight we can be sure that at least one tachyon is among the decay products.
2. Fixed poles in the scattering amplitudes. If the scattering amplitude between two particles exhibits a peak in the invariant momentum-transfer variable for space-like values we can conclude that a tachyon is being exchanged.
3. Large angle scattering. If fast particles scatter through large angles with a pronounced resonance in the invariant momentum-transfer, a tachyon is being emitted.
4. Effective mass plots: A peaking in the effective mass of a collection of pions computed with some of the pions in the initial state and some in the final state would indicate the formation of a tachyon resonance.
5. Missing mass spectroscopy: as extended to negative values for the effective squared mass for the missing four-momentum.

It would be very surprising in view of the construction of a quantum theory of scalar tachyons if such particles are not found in any systematic experimental search.

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Discussion

Streater

Would you not expect the vacuum to split up under a small perturbation into two such particles, one of positive and one of negative energy?

Sudarshan

Depending on the manner of speaking, both yes and no. Of course, the vacuum can go into, say, several positive and negative energy particles. However, at least one of the particles must have negative energy because energy and momentum has to be conserved. So we cannot draw a diagram in this fashion:

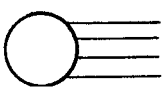
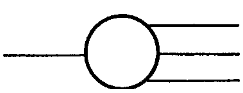
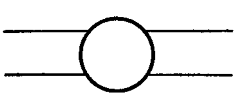


Fig. 1

with only positive energy particles; some of them must have negative energy. To interpret it physically, we should consider the diagram to be:



Now this process would not be possible if the masses are not right, because it would mean one particle decaying into three particles, if only one of the particles had negative energy. Or perhaps two of them happen to have negative energy in which case you would really be watching the elastic scattering process:



The physical reinterpretation principle according to which only positive energy particles are to be considered, would show that rather than seeing the decay of the vacuum into four particles, we are really witnessing either two particles going into two particles or one particle decaying into three particles.

Nambu
In this connection, why cannot the vacuum decay into two particles, each having zero energy?

Sudarshan
This would resemble a free, zero energy tachyon propagating, since for a zero energy particle the sense of propagation can be reversed at will provided we change the sign of the momentum.

Omnès
I do not quite understand the question of fixed poles. If you have a pole with fixed negative squared mass, would not the scattering amplitude be infinite in the physical region?

Sudarshan
Well, perhaps it would become finite in a more satisfactory computation; my calculation was in the lowest approximation.

Let me further explain my answer. There are two ways out of this difficulty. One of them is to say that a strictly well-defined mass is not possible, and this happens for the space-like solutions of the Majorana equation (or any of these infinite-component equations which contains space-like solutions). If you look at them, you will not find a well-defined mass, but a continuous spectrum of imaginary masses. So in this case, instead of a pole, you find in the Born approximation a certain kind of branch cut, but in a more exact calculation I don't know what one would find. The second possibility is that for the precise mass case we calculate the scattering amplitude to a better approximation. Unitarity taken into account in a reasonable fashion should then essentially limit the scattering amplitude so that it is prevented from becoming infinite in the physical region. Something must happen which is not taken into account in the lowest order of approximation.

Haag
Could you say something about the reasons for believing in the existence of such things as tachyons?

Sudarshan
Every infinite-component equation which does not have infinite mass degeneracy seems to lead to space-like solutions (with real momenta). Any theory which gives internal structure to particles, like non-local field theory, also gives rise to such particles. So I suspect that such particles are an essential ingredient for strong interaction physics.